Is the Theory of Everything Lurking in Uncomputability?

Irek Defée

irek.defee@protonmail.com

Abstract. Development of Theory Of Everything (TOE) has a fundamental issue of *baggage* which is understood as a set of notions and assumptions underlying a theory. TOE should at best be baggage-free but this is problematic with theories based on algorithmic representations. We show how uncomputability may provide a new conceptual framework for the TOE without baggage. Special uncomputable sequences are introduced and it is shown how a looped self-referential system of symbols from sequences and sequences from symbols is formed. Properties of this system are analyzed emphasizing symmetry requiring operation of infinite permutation groups. It is shown how mathematical structures emerge and how it is possible to tackle nothingness and its relation to uncomputability. Our development points that TOE is a special mathematical structure arising from the uncomputable substrate intrinsically tied with nothingness and dissolving symmetry of this structure results in physics. This indicates that uncomputability forms ultimate foundation of physics enabling to answer deepest questions about the Universe.

In any field, find the strangest thing and then explore it
John Archibald Wheeler

1 Introduction

This essay aims to show that within the concept of uncomputability there is a potential for foundational ideas on the deepest level of physics which is called Theory Of Everything (TOE). Current problems in fundamental physics are reaching beyond of what is understood as 'standard' theory. Quantum gravity would be operating at the Planck-level of time, length, energy, which are also corresponding to conditions at the beginning of the Universe. The TOE might be even below this level. If the TOE is of a 'standard' kind it would use sophisticated mathematics and equations. Than the question about the origin of the sophistication will be inescapable putting doubt if that is the ultimate TOE. This points to a need for new thinking about foundations, in particular that somewhere at depth the complexity of theory should be reducing. Belief that truly foundational ideas should be simple resonates with more recent realization that theories are formulated using certain background concepts and descriptions in human and symbolic languages, raising questions of the background complexity, so-called "baggage" problem[1]. These were considerations which have led us to a curious signpost of *Uncomputability* as the potential direction towards the TOE, not at least due to its unorthodox conceptual scope. Its unorthodoxy is wide ranging and going against many established canons of physics. First, uncomputability deals with things which are not algorithmic and physics is concerned with algorithmic descriptions, it is the equations with predictive power which receive highest reverence. With no algorithms and equations what kind of physics there could be? Second, uncomputability is tied with infinity which is supposed to have no place in physics. Already Hilbert exorcised infinity from physics by his authority [2], and later other eminent figures provided strong arguments against it [3], [4]. Third, uncomputability and physics are meeting in the most peculiar way in the field of real numbers, physics without real numbers seems impossible but on the other hand almost all real numbers are uncomputable and random

[5]. But since in practice calculations with approximations and quantizations at any desired precision are working, the real numbers are relegated from physics as an idealization [6] or an outright fiction [7]. Infinity is treated in similar way, there is no place for it but infinities are a common view in physics: in equations where counting extends to infinity, in formulas which are diverging to infinity but made converging by tricks, in questions what happens at $r = 0$ in equations describing forces and charges [8]. And quantum mechanics is inherently relying on infinite-dimensional Hilbert Space. Infinity exorcists say to this all it is just due to the artefacts of mathematics or our yet imperfect theories and we should not bother about it.

These concepts rejected but stubbornly present point out that there is something strange here. Maybe at the deepest level there are no algorithms, infinity is common, real numbers are basic? The Uncomputability signpost looks then not so surely pointing into a blind alley but maybe to something foundational. At least it fits perfectly into the Wheeler line of thinking which makes its exciting to exploration. That provides motivation for undertaking our journey into the uncomputability, keeping in mind that baggage reduction is our main objective.

2 Sequences and computability

What could be the lightest baggage one should allow when looking for the TOF? From the reasons listed above it seems that even including (real) numbers maybe questioned nowadays. That is leaving us just with the set theory, which means that all mathematical structures above it can not be taken for granted but rather expected to be resulting from some buildup. Here our initial baggage will thus be only a finite set and with it we will proceed towards uncomputability. Let X be a finite set of elements called symbols, think about Chinese, Latin, Hebrew characters, or digits as examples. For utmost simplicity but without loss of generality we take just two symbols marking them by 0,1 and noticing that they are not digits here though obviously they remind us about the smallest unit of information bit and about binary representation of numbers. Let $X^\star$ denotes infinite set made by concatenation of these symbols, think about words and numbers composed of characters and digits. Finite concatenations are called strings and infinite ones sequences, then the $X^\star$ set looks like $X^\star = \{0, 1, 00, 01, 11, 10, 111, \ldots 1111010101\ldots\}$. Basic feature of the $X^\star$ is its invariance under permutation of symbols, exchange the symbols and the set stays the same. We say that each string and sequence have a *mirror* under the symbols exchange and the $X^\star$ set can be split into two disjoint subsets under *mirroring*.

This construction of the $X^\star$ and its subsets looks rather minimal but we make it now even leaner. In the $X^\star$ set there is distinctive symbol ordering in strings and sequences. Looking first into strings, some symbols are at the beginning or end, some others at specific positions inside strings and thus it is not only the symbol itself but also its position within which has meaning. This can be seen as a kind of theoretical baggage and we eliminate it by putting all symbols on equal footing. It is done by joining the beginning and end of strings, strings become circular which equals to imposing equivalence relation under cyclic shift, e.g. four different strings of the $X^\star$set now belong to a single equivalence class $[1000, 0100, 0010, 0001]$. Information about symbol position with respect to the beginning and end is lost and left is only information about mutual positions of symbols. For sequences the procedure is bit different since they are infinite and thus only have beginning but no end. In this case we can think about eliminating the dependence on the symbols position in two ways. Imagine circular sequences with length increasing up to infinity, or imagine that the beginning of a sequence extending to the right is shifted to infinity on the left, the sequence is now expanding to infinity in both directions. For such sequences equivalence class if formed by taking all their infinite number of shifts. Note that under the shift equivalence not all circular strings and sequences will have mirrors, think about periodic ones. Our interest will be focused on sequences in equivalence classes for which the mirrors exist. We will use the notation $X^\star$ for the set of such sequences as the general $X^\star$ set will not be used in the sequel.

2.1 Uncomputable sequences

Dealing with strings or sequences of two symbols $0, 1$ immediately brings to mind computers. Computers produce strings all the time by executing programs which are also represented by strings. Without getting into details we note that an important chapter of computability theory deals with the relations between strings and sequences and the minimal length of computer programs producing them [9]. It should be evident that any string can be produced by a program having roughly its length, the string is just stored in full in the computer and outputted. But for many strings the program for producing them may have shorter length, this will be possible if there are some regularities which a program can exploit. Some strings will not have such regularities meaning that there is no way to produce them other than store and output, such strings are said to have maximal complexity. Computability theory establishes that with the increasing length of strings the number of those having maximal complexity is quickly growing and among long strings are maximally complex. In the case of sequences which are infinite, they obviously can not be stored in any computer but some may have algorithms describing them and thus a computer can in principle produce them, think e.g. about the number π in binary representation.

From this there is only one step to our main target, sequences called *uncomputable* which have no algorithmic description. We can think about such sequences as a limiting case of maximally complex strings when their length is increasing to infinity. Infinite length means there is no way of storing and no description means there is no way of producing them. Uncomputable sequences can be thought as having symbols randomly distributed but randomness is a delicate concept with multitude of aspects so this is only a rough idea [10]. What is truly amazing is size of the set of uncomputable sequences. The set of all strings and sequences which have algorithmic description is finite since the set of all algorithms is finite, this is *computable* set. In turn the set of *all* strings and sequences is in correspondence to the set of binary representations of real numbers which has uncountable cardinality denoted by $\aleph_0$. It follows then that the set of uncomputable sequences also has $\aleph_0$ cardinality, and thus so big it is impossible to count these sequences.

Previously we imposed equivalence relation on sequences under shift operation, let us see what is the impact of this on uncomputable sequences. Uncomputable sequence has countably infinite length and thus countably infinite number of shifts. There will be thus still uncountably many equivalence classes of uncomputable sequences. We will call set of these sequences in equivalence classes $X^\star_{un}$, and this set can be also be divided into two disjoint subsets under mirroring operation on symbols.

Next we will reveal certain magic we did in preparation for eliminating some baggage from uncomputable sequences.

2.2 Deconstructing magical trick

In the above we fast forwarded from two symbols to uncomputability and by this a peculiar trick of mathematical magics was covered up. In mathematics things can appear out of an empty hat but here we are very sensitive to any baggage. Seeing in slow motion one can notice that we defined two symbols, concatenated them in the hat, added a definition and voilá, uncomputable set $X_{un}^{\star}$ came out. The trick is that there were initially assumed two symbols but somehow they were cloned to uncountably many ones and all perfectly identical. Then under the disguise of 'concatenation' these symbols were glued together into sequences in a strangely ordered manner creating this uncountably big set of uncomputable sequences. From pure mathematics point this is fine but from our theory baggage point the out-of-nowhere appearance of infinity of identical symbols begs for some explanation. Thinking about this one can note curious fact that there is a kind of asymmetry here, to create uncountably many *different* sequences there are uncountably many *identical* symbols used and they are cloned from just two. This leads us to to the basic idea.

3 Sequences from symbols and symbols form sequences

Denoting the mirror operation on symbol by $\overline{0} = 1, \overline{\overline{0}} = 1$, each uncomputable sequence from the $X_{un}^{\star}$ set obviously has its mirror by applying mirror operation to each symbol. Sequences and their mirrors can be separated into disjoint subsets $X_{0un}^{\star}$ and $X_{1un}^{\star}$, it is not important which sequence is a mirror and goes to which set.

Now we will perform something outside of the standard toolbox. Note formal similarity between the mirror on a symbol and mirror on sequences. We can look at sequences as new symbols and denote these as by $s_{un}, \overline{s_{un}}$ where the subscript un denotes the sequence expressed in binary format. Uncountable infinity of symbols suddenly appears, as many as sequences. An idea strikes that we can use these new symbols to substitute for binary symbols in sequences. Imagine all binary uncomputable sequences and substituting the symbols 0,1 in sequences with the new symbols $s_{un}, \overline{s_{un}}$ in such a way that each of the new symbols is used only once and the symbol originating from a sequence s_{un} is not used in it. It is not important where the symbols $s_{un}, \overline{s_{un}}$ are put within a sequence, only that is done once. Since we do have as many symbols as sequences and these are uncountable sets, each symbol in each sequence will be now different.

Wait, you can say, that looks crazy as you are substituting binary symbols by symbols which are infinite sequences. That will be equivalent to the creation of new strange infinite sequences? Yes, this is true but what are those new strange sequences? We are dealing with infinite sequences and substitution does not change cardinality. We can even forget about the initially assumed binary symbols, the two symbols and cloning were just a rough approximation, in essence there are unique symbols which are originating from sequences. Moreover, after the substitution and by the construction of the substitution the equivalent sequences are still uncomputable which means we are within the same $X_{un}^{\star}$set. We can be even repeat this procedure again and think what happens if it is repeated iteratively. The sequence length will always remain countably infinite but since we have uncountably infinite number of symbols this means that the substitution has to return somewhere to the initial point, becoming circular.

What this all means? By substituting identical binary symbols with unique symbols from sequences a symmetry is restored between symbols and sequences as the magic trick with symbol cloning is eliminated. This leads to the creation of a loop in which sequences are made of symbols and symbols are made of sequences. All is possible due to manipulations on elements of uncountable sets and uncomputability which keeps sequences irreducibly infinite. For our development it looks like in a way we have reduced the baggage by eliminating the magic of symbols cloning and we can now answer where the symbols are coming from. Our baggage left is uncomputability plus hiding in the background is the Axiom of Choice which we have to accept when manipulating elements of uncountable sets but this baggage is rather uncontroversial for most mathematicians.

There is however still a question, is the uncomputable baggage composed of sequences-from-symbols and symbols-from-sequences really light and what is its origin? Such question is reaching to the very bottom, forcing us to look what is there.

3.1 Symmetry and nothingness

As described before we are not considering single sequences but rather their their equivalence classes under shifts. This has no impact on the looped system introduced above but it has consequence on how such uncomputable sequences will look from specific viewpoints. Since there is no preference for any member of the equivalence class and there is no imposed structure the sequences in one class have to be treated as a *bundle*. There is no prescribed order or structure of any kind within the bundle, that can be seen as requiring imposition of symmetry equaling to the action of full symmetric permutation group operating on sequences in the bundle. Sounds strange but it must be so since there is nothing else, of course no time, space, forces and structures whatsoever, the abstract symmetry condition imagined as a permutation group acting with infinite speed changing relative positions of infinite but countable number of bundle members. It will also be the same for all bundles of uncomputable sequences as there also is no structure imposed on the bundles and thus this too has to look like an action of permutation group acting on all bundles. In this case it will be symmetric permutation group acting on uncountably infinite set of bundles.

There landscape of sequences is completely changed by such symmetry requirements. To see how it looks we have to take a specific bird views, call it ∞-bird since it can take a look on local and global infinities. From the global ∞-bird all sequences are seen, all bundles and all permutations. Locally, the ∞-bird is focused on a specific symbol positions in a single bundle with its permutation group action, and that reveals curious effect. These permutations make impossible to establish locally to which sequence individual symbols belong at any given position. The symbols are just seen as separating into two subsets $X^{\star}_{0un}$ and $X^{\star}_{1un}$, in effect looking like there are only two symbols 0 and 1. This will be seen locally for any position in a bundle and for bundles of all sequences. Bundles will look locally identical with permutations acting on them effecting in loosing their identity, like merging into a single bundle. Symbols in the bundle will be like merging due to the identity loss and mirroring will make it even more fuzzy. Everything will be (locally) disappearing from view with nothing left to be seen.

Nothing??? What really means nothing here? It means that locally the action of permutation groups will lead to the disappearance due to the identity loss which could be also be seen that locally such uncomputability has no meaning or even misleds that computability is there, and thus it disappears.

If that is not strange enough we are now able to tell something about nothing. Nothingness would typically be thought of like an empty set in the category of sets. This sounds like an algorithmic description but nothing should not be algorithmic because algorithms describe 'something'. Nothingness thus should be somehow non-algorithmic. Besides it should be global in the sense there is nothing apart of it, but in a rather peculiar way. On one hand it can not be global without any boundary whatsoever since in this case it would be just like an empty set. On the other hand it can not have any clear boundary to 'something' since then it would not be global.

What can be seen here in the bundles of uncomputable sequences locally disappearing due to permutations required by symmetry is nothingness which is appearing locally and it has no clear boundary since identical symbols are everywhere and they disappear whenever in sight. But there is boundary somewhere since from the global ∞-bird perspective uncomputable sequences are still there and there is no nothingness seen. Which means that this nothingness could be in a way seen like a 'hole' made by taking out of sequences finite segments whose lengths are growing to infinity. There is then nothing in sight but sequences are still there in infinity, there is a boundary somewhere but it is non-algorithmic. Such nothingness is even more tricky since it maybe exists or maybe not depending on the global or local ∞-bird view. Sounds odd but it is in fact good, answering to a philosophical paradox: if nothingness exists then it is "something"- which means it is not nothing. To which the reply here is that this nothingness exists or maybe not.

And that makes possible explanation where is the ultimate baggage of our uncomputable sequences originating from. It is originating from nothingness. Nothingness has a boundary made by uncomputable sequences and thus both concepts are intrinsically related, there is no nothingness without uncomputability and where there is uncomputability nothingness appears in its curious way. Hence the theory baggage is reduced to really nothing by pointing that in the nothingness our uncomputable sequences are. So we got to the ultimate where nothing else can be beneath it and we can start looking if something might be still build upon it.

4 Mathematical structures emerging

4.1 Real numbers

Our ∞-bird takes now a vantage point of global infinity, all sequences and all bundles are in a clear view, those 0 and 1 symbols appearing in the bundles can be seen too as a fleeting overlay. Another effect then shows up in the ∞-bird perspective. For each of the symbols 0 and 1 emerging from the bundles, both symbols $(0, 1)$ at the next position will be visible and then $(0, 1)$ again for each 0 and 1 and so on, continuing the view it will be like a tree of symbols seen extending from a symbol position up to infinity:

$$\begin{array}{ccc} 0 & \quad & 1 \\ 0 \;\; 1 & & 0 \;\; 1 \\ 01 \;\; 01 & & 01 \;\; 01 \\ \cdots\cdots & & \cdots\cdots \end{array}$$

This tree of symbols is reminiscent of binary representation of real numbers, from this ∞-bird view it will be like real numbers seen in both directions of the bundle, and that will be for each position like in a real line. The question is then, since sequences have countable number of symbols how the tree representing real numbers can fit into this? The answer is that the real numbers will be only seen apparent in perspective as expanding to infinity but in the end there is no place for all of them forming dense set of reals, either there is place to see some of them only or see all ot them only by approximating the tree to any countable accuracy. The sequence positions will serve as quantization marks, like a real number line with integers marked. This illusion of real numbers will be absolutely prefect unless the ∞-bird would want to see them all at uncountable depths which is really not needed. That real line illusion will hold for each bundle producing in effect uncountable infinity of identical real lines, though from the global ∞-bird view it will be evident that they originate from different sequences and thus are different.

How these lines will look with respect to each other? As said earlier symmetry requires action of permutation group on bundles. But since the real lines look identical and there is no mutual relation on them imposed, yet another permutation group has to act, the real lines will be shifting with respect to each other. From a fixed position on one real line, another real line will be looking like undergoing permutations by shift of the form which can formally be described as $a \times r + b$ with $a, b, r \epsilon R$, in other words that will look like having operations on the real number field $\mathbb{R}$ at hand! That sounds incredible, from nothing to mathematics laying down a big foundation stone for the TOE but the continuation is much easier.

4.2 Fields and spaces

We now turn attention to the permutation group acting on bundles and those apparent real lines fleeting on them. The full symmetric permutation group acting on uncountably many elements is colossal, we call it $\Pi(X^{\star}_{uns})$. Its cardinality is exceeding the $\aleph_0$ cardinality of real numbers. Think about it acting on all subsets of $\mathbb{R}$, the cardinality is then seen to be $\aleph_1$. Actions of the $\Pi(X^{\star}_{uns})$ will permute elements in every imaginable way and subgroups will be forming too. We can get an idea where this may lead by taking permutations of few elements. A pair of bundles with real lines permuting in transposition and shifting with respect to each other will look like forming a direct product $\mathbb{R} \otimes \mathbb{R}$. Action of that group can be geometrically identified as Euclidean plane. Another possibility is when sequences in bundles will form pairs and the resulting pairs of reals will be shifting according to a common permutation group of the form $(a, b) \times (x, y), (a, b) \epsilon \mathbb{R} \otimes \mathbb{R}$ for all $(x, y) \epsilon \mathbb{R} \otimes \mathbb{R}$. That will form complex numbers and geometrically the pairs will represent complex plane.

This process can be iterated with bigger permutations. Taking pairs of complex number representation by transpositions, the field $\mathbb{Q}$ of quaternions will emerge. Repeating this for pairs of quaternions will result in the field $\mathbb{O}$ of octonions. The field structures are exhausted at this step since there are no bigger permutation groups acting due to zero divisors which stems from the fact there are only four normed division algebras [11].

There are however no limits on the formation of direct products for any number of copies of the fields using the bundle permutations. By this spaces of any dimensionality can be produced. In addition we mentioned there is quantization overlay on the real line which naturally leads to the imposition of distance and norms. Large variety of mathematical structures may emerge in this way: real Euclidean and Hilbert spaces, complex Hilbert spaces, spaces formed with quaternions and octonions. There are many more possibilities for the group action on uncountable number of elements.

We thus got a glimpse how mathematics on which fundamental physical theory is based appears. There only remains last question to be answered.

5 Where is the TOE lurking?

Or, you can ask, where is the physics here? If anything, we could only see fleeting appearance of some basic mathematical structures on top of uncomputable sequences. Where is then the usual stuff of physics, fields, forces, waves, particles, matter and the Universe? The answer is that we only covered the foundational TOE level from nothingness to simple mathematical structures which are indispensable for physics. We also see that there are colossal permutation groups in action which makes possible emerging of mathematical structures with much higher complexity.

So where the TOE is lurking? At its deepest level the non-algorithmic part of the TOE is seen since the substrate is formed by uncomputable sequences accompanying nothingness. One can predict that the higher level of the TOE must be within the $\Pi(X^{\star}_{uns})$ group and since by the Cayley theorem of algebra every group is isomorphic to a subgroup of a permutation group, within this group of permutations there are thus unlimited opportunities for the appearance of mathematical structures. One of such structures should be the TOE and then in a sense Physics≡TOE. The mathematical structure called physics has to be uniquely special as typical simple structures will be intermittent in the actions of the permutations groups. One can imagine physics as having very high initial symmetry operating on a huge number of elements. The symmetry will be then trapped in such a way that its unwinding will be possible only through a huge number of group actions. That will require local (internal) symmetries tied up with a global symmetry and acting like a brake in unwinding. Emergence of the global symmetry should be then simpler and can be seen as incidental formation of a special huge permutation group but it unwinding will be proceeding in much more complicated way. This makes obvious why we see so many and increasing symmetries in the progressing of physics theory, we are now still somewhere on the bottom-up track towards the TOE. According to what we see here the level of symmetry considered has to be still significantly increased but the ultimate end is in uncomputability.

6 Final remarks

Paraphrasing Feynman's *"There's Plenty of Room at the Bottom"* [12] we aimed here to see what is at the *very* bottom and we found that within uncomputability there is plenty of room to accommodate mathematics and TOE. Pursuing uncomputability allowed us to eliminate theory baggage and getting up to the nothingness. Then we found how mathematics may emerge and physics is also very likely to emerge as a special mathematical structure since room for this is uncountably huge. There is a long history of thinking about relations between mathematics and physics [13]. Ingenious minds were fascinated how the mathematical abstraction fits to the description of reality, summarized by Wigner's phrase of *unreasonable effectiveness of mathematics in the natural sciences*. Mathematics has been seen as the invention of the human mind which incidentally fits as a tool for physics in describing the world. We indicate here that mathematics exists, mathematical structures emerge virtually out of nothingness tied with uncomputability. Due to the richness of possibilities in our framework physics appears then as a very special mathematical structure which provides strong support to the notion that physics can be completely reduced to mathematics and all mathematical structures exist[14]. But from our point most of them have only very fleeting existence in the transient actions of colossal permutation groups.

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