# An Infinite Universe - An Analog Reality

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## 1 Introduction

The infinitesimal has fascinated and perplexed natural philosophers for millennia, from Zeno to Zermelo. Does Nature ever deal in perfectly continuous quantities or is it constrained, for whatever reason, to only ever handle numbers up to a certain level of precision? Is it a 'perfect calculator' or finite and thus comparable to a large desktop computer? In this essay, I shall focus on a particular case that can be made for reality being analog. I choose this in part because it seems *a priori* the harder case to make and so the more interesting, the usual trend being to quantize everything in the world. Clearly reality has the capacity to perform some discrete, all-or-nothing calculations (as in the conservation of quanta of charge), but this in no way suggests that *all* its calculations are grainy. It would be far more remarkable in my opinion if the universe ever managed to perform a single calculation to infinite precision.[1]

[1]Richard Feynman expressed similar intrigue over this possibility: "It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny

## 2 The Computational Analogy

Let me begin by explicating this 'perfect calculator' analogy. Somehow, the universe exhibits an extraordinary capacity to keep track of numbers. In the Newtonian representation each particle at any given moment requires six real numbers to specify its position and velocity with respect to absolute space. These numbers get updated from one moment to the next by the deterministic and continuous solutions to Newton's 2nd Law.

It has long been noted that, when viewed this way, the universe appears to be strikingly analogous to an ordinary computer. If the observable universe were Newtonian, its hard drive would therefore need to have at least $\sim 6 \times 10^{80}$ addresses. A modern desktop computer (what one might call an isolatable subroutine of the cosmic computer), by comparison, has a meagre $\sim 10^{12}$ storage spaces in its hard drive. As impressive as such a universe may sound, if it is truly Newtonian, then the computational prowess of just a *single* particle coordinate would, literally, be infinitely more information-rich than all $10^{12}$ binary spaces on the desktop. For if the universe were truly Newtonian, then every single coordinate would store an infinity of digits and thus an infinite number of 'bits of information'. We know of course that the

a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny piece of space? Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do? So I have often made the hypothesis that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed, and the laws will turn out to be simple, like the chequer board with all its apparent complexities. But this speculation is of the same nature as those other people make - 'I like it', 'I don't like it', - and it is not good to be too prejudiced about these things." R. Feynman, p.57-8, *The Character of Physical Law*, 22nd printing, MIT Press 1995.

universe is not Newtonian, but quantum mechanics and general relativity both involve continuous fields over smooth space-time, so are no different in this sense.

Viewed from this perspective, the question at hand can be rephrased in the following way: does the universe manage to store an infinite number of bits of information for every particle coordinate? If so, then reality is analog. But if it is truly analog, could we ever discover this? I believe not - the best we could do is find indirect clues that lend favourably to this conclusion.

The reason why we could never confirm directly that space is a perfect continuum can be seen from the structure of the mathematical expression of the infinitesimal. One can never describe what the infinitesimal *is*, only that you can never reach it. For example, the separability axiom states, roughly, "for every $x < y$ one can always find $z$ such that $x < z < y$." In other words, no matter how hard we try to probe the microscopic, we can always be trumped by Nature *if it is a continuum*. On the other hand, if Nature only deals with discrete sets of numbers up to a certain precision, then it might be possible, in principle, to discover this fact through round-off errors (though in practice a digital reality could well be precise *enough* to hide its graininess beyond our technological reach).[2]

We are therefore left with underdetermination of an asymmetric sort. If reality is digital, it is possible that we discover this one day by pushing our measurements of microscopic quantities to greater and greater accuracies;

[2]If, for example, we predicted a set of observable quantities in Nature and we found, upon measurement, that Nature always disagreed by rounding in the direction implied by the next predicted digit, then we could be confident that Nature is truly digital.

but if reality is analog, we could never discover this by such means because we'd never be able to rule out that digital reality *just* managed to avoid detection. However, I shall point out an indirect approach to lend credence to the analog - not by the microscopic, but the macroscopic.

## 3 Flexible Information Capacity

To see this, let us return to the analogy of the universe as a computer. Let us initially differentiate between a computer's storage space and the memory capacity of that space. It will turn out that this distinction is rather superficial. A desktop performing an N-body simulation of a 3-dimensional system will assign each particle 6 storage spaces to specify its position and velocity, each space with enough memory for, say, 12 significant figures. We can represent this memory allocation as in Figure 1.

Now let the memory allocation depicted in Figure 1 somehow acquire an infinite number ($\aleph_0$) of such storage spaces thus allowing the simulation to have $\aleph_0$ additional particles with the same level of precision (12 significant figures).[3] Alternatively (but not additionally) one could use the extra $\aleph_0$ spaces to specify the coordinates of two particles to infinite precision. This is depicted in Figure 2 where we have rearranged the data structures so that the digits of the $\aleph_0$ storage spaces are used to specify the phase-space coordinates to infinite precision.[4]

[3] I am of course ignoring the fact that the Desktop would take an infinite amount of time to process an infinite amount of bits of information.

[4] Rearranging bits into different data structures is a common task for a computer.

| | | Twelve Significant Figures | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st Particle | $x_1$ | 1 | 4 | 9 | 2 | 1 | 9 | 1 | 2 | 1 | 2 | 9 | 7 |
| | $y_1$ | 4 | 8 | 2 | 5 | 8 | 7 | 6 | 4 | 9 | 6 | 5 | 8 |
| | $z_1$ | 0 | 5 | 8 | 2 | 5 | 3 | 8 | 5 | 8 | 7 | 2 | 3 |
| | $\dot{x}_1$ | 4 | 4 | 7 | 8 | 6 | 6 | 3 | 8 | 7 | 9 | 9 | 5 |
| | $\dot{y}_1$ | 2 | 9 | 4 | 9 | 4 | 1 | 8 | 6 | 5 | 5 | 8 | 1 |
| | $\dot{z}_1$ | 8 | 0 | 9 | 1 | 7 | 0 | 9 | 8 | 2 | 2 | 0 | 0 |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| nth Particle | $x_n$ | 8 | 2 | 7 | 0 | 9 | 8 | 1 | 8 | 5 | 4 | 2 | 2 |
| | $y_n$ | 5 | 3 | 1 | 8 | 1 | 6 | 9 | 9 | 7 | 8 | 7 | 7 |
| | $z_n$ | 6 | 4 | 7 | 4 | 0 | 4 | 7 | 1 | 8 | 0 | 1 | 3 |
| | $\dot{x}_n$ | 2 | 5 | 3 | 7 | 1 | 2 | 4 | 4 | 2 | 1 | 8 | 8 |
| | $\dot{y}_n$ | 1 | 1 | 2 | 8 | 9 | 6 | 3 | 6 | 1 | 7 | 3 | 5 |
| | $\dot{z}_n$ | 9 | 9 | 8 | 3 | 2 | 7 | 1 | 8 | 9 | 3 | 9 | 9 |

Table 1: Memory schematic for the phase-space of a 3D N-Body simulation using data structures of 12 significant figures. (Another number would have to specified giving the location of the decimal place but we ignore that here).

Similarly, if we were to add $\aleph_1$ extra storage spaces, we could add an infinite number of particles to the simulation with phase-space coordinates specified to an infinite degree of precision. The fact that the computer memory can be used either for particle number or particle precision suggests, strictly by analogy, that the universe can do something similar. This brings us to the key hypothesis of this essay: if the universe is infinite in particle number, then it is infinite in particle precision.

This, of course, cannot be strictly proven though there is something intuitive about it. If the universe does have 'infinite information capacity,' as it would if it contained an infinite number of particles (even if they were each

| | Particle 1 | | | | | | Particle 2 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $y_1$ | $z_1$ | $\dot{x}_1$ | $\dot{y}_1$ | $\dot{z}_1$ | $x_2$ | $y_2$ | $z_2$ | $\dot{x}_2$ | $\dot{y}_2$ | $\dot{z}_2$ |
| $\aleph_0$ Significant Figures | 1 | 4 | 9 | 2 | 1 | 9 | 1 | 2 | 1 | 2 | 9 | 7 |
| | 4 | 8 | 2 | 5 | 8 | 7 | 6 | 4 | 9 | 6 | 5 | 8 |
| | 0 | 5 | 8 | 2 | 5 | 3 | 8 | 5 | 8 | 7 | 2 | 3 |
| | 4 | 4 | 7 | 8 | 6 | 6 | 3 | 8 | 7 | 9 | 9 | 5 |
| | 2 | 9 | 4 | 9 | 4 | 1 | 8 | 6 | 5 | 5 | 8 | 1 |
| | 8 | 0 | 9 | 1 | 7 | 0 | 9 | 8 | 2 | 2 | 0 | 0 |
| | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

Table 2: Memory schematic for the phase-space of a 3D N-Body simulation with $\aleph_0$ storage spaces of 12 significant figures. Instead of using the extra memory for more particles of finite precision, the memory is used to specify *two* particles with infinite precision (i.e. a continuum).

specified with finite precision), then it would not be difficult to suppose that it could go one cardinality further with respect to particle position. For the great barrier is actualizing *any* infinity whatsoever. If the universe is extraordinary enough to realise or, in some way difficult to imagine, get around the paradoxes of Hilbert's Hotel, then additional 'infinities' don't seem so problematic. In fact, one could argue that it would be *less* surprising that particles be specified with infinite precision than that there be an infinite number of particles because Hilbert took his paradox to imply that *actual* infinities are impossible (and particles are arguably more 'actual' than are their positions, which are *abstract* relations between 'real' particles).

## 4 Cosmological Clues

We must then turn to ask whether the universe is in fact infinite. It is probably enough just to ask whether it is infinite in spatial extent since the quantum vacuum permeating it produces virtual particles, and these would surely be enough to require of the universe an infinite information capacity. Most cosmologists probably do think that the universe is spatially infinite though their motivations are quite varied. Obviously we can only see a finite distance into the universe, so any inference to infinity can only be made through appeals to 'naturalness.' The most common of such appeals derives from the supposed flatness of space. If space is Euclidean, then it is fairly natural to suppose that it be infinite in extent though it could of course have a non-standard topology like a torus.

It turns out that the measured curvature of the universe is tantalisingly close to zero and could very possibly have slight positive curvature rendering its volume finite. But the very fact that spatial curvature is so close to zero creates a fine-tuning problem going back to the Big Bang singularity. If there had been even the slightest departure from the critical density, the universe would have rapidly expanded or collapsed soon after the Big Bang rendering the universe uninhabitable. The most widely held 'solution' to this and other problems is the inflationary scenario, but this too probably requires fine-tuning and so is often taken to imply an inflationary multiverse scenario conjoined with the anthropic principle to 'explain' why we observe spatial flatness. In this case, the universe might again be infinite in extent.

## 5 Summary

The very Nature of the infinitesimal is such that it can never be probed directly. If reality is digital, we might discover its graininess, but if it is truly analog - precise all the way down - then we can never know this for sure. However, cosmology might be able to offer clues. If the universe could plausibly be supposed to be of infinite extent, then its capacity to handle the infinitesimal would likewise seem quite plausible. To put it another way, it would be somewhat surprising that the universe had $\aleph_0$ storage spaces in its hard drive, but only finite memory for particle position. If it can traverse the infinite once, why not twice?