Predictable and Unpredictable in Quantum Mechanics*

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The theory of quantum mechanics interpreted by the Copenhagen school consists of a set of principles that contradict the principle of locality and determinism present in classical and relativistic mechanics. These contradictions pose some difficulties to the formulation of a quantum theory of gravity. Alternatively, we also have deterministic interpretations of quantum mechanics such as de Broglie-Bohm pilot-wave theory and 't Hooft's superdeterministic cellular automaton. Recently, three experiments of quantum entanglement convincingly violate Bell's inequalities and renounce local realism in the quantum world. Moreover, a recent double-slit experiment also demonstrated trajectories of entangled photons in a *deterministic* manner. In this essay, we debate over the orthodox interpretation and question whether the double-slit experiment could deterministically be produced by chaotic dynamics of the many-body problem and random initial conditions. We also discuss how two spacelike separated entangled particles, whose quantum states are unpredictable before locally measured by one of two observers, are strongly correlated only via either *nonlocal* or *superdeterministic* hidden variables.

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I. Indeterministic Copenhagen Interpretation

Quantum mechanics developed by Niels Bohr [1, 2] and Werner Heisenberg [3, 4] at the University of Copenhagen, and Max Born [5, 6] at the University of Göttingen in the 1920s has provided us with the most widely accepted interpretation of phenomena in the quantum world. The so-called Copenhagen interpretation includes the principle of superposition aimed at explaining Young's double-slit experiment with light [7] that revealed the wave nature of photons (see Figure 2). According to the superposition principle, the quantum state of a photon is superposed on two different pure states. Similarly, the Thomson–Germer electron diffraction experiment [8] also reported the same wave behavior in electrons that confirmed the hypothesis of wave-like nature of quantum particles proposed by de Broglie [9]. The temporal evolution of the wave function of an elementary particle in quantum physics is governed by the Schrödinger equation [10] that is a deterministic formulation. However, the Born rule [5] in Copenhagen quantum mechanics relies on the statistical interpretation of the wave function. Bohr's correspondence principle [1] states that the quantum state of a wave function is mapped to its corresponding classical behavior in large numbers of particles. In the Copenhagen interpretation, Heisenberg's uncertainty principle [4] also plays an essential role that puts a fundamental limit on the measurement of physical quantities of a subatomic particle. Hence, the *Copenhagen interpretation* can be regarded as *indeterministic* mechanics that is foreign to deterministic classical mechanics.

The collapse of a wave function evolving according to the Schrödinger equation as a superposition of different states raised the so-called *measurement problem.* Heisenberg first mentioned the concept of the wave function collapse, but did not explicitly explain how it happens [4]. According to John von Neumann's view in 1932 [11], the collapse of a wave function occurs upon the performance of an observation, while the temporal evolution of the wave function maintains obeying the Schrödinger equation as long as no observation is performed. However, Paul Dirac in 1935 proposed a jump into an eigenstate of the observed quantity as a measurement is being made [12]. Both von Neumann's and Dirac's views agree that the collapse is a result of either an observational intervention or a measurement performance. Accordingly, the wave function collapse randomly assigns an eigenstate to an *observable* particle when it is being observed. The quantum state of an observable is therefore *unpredictable* before any observation that leads to *contextuality* in quantum mechanics.

The contextual attributes of a pair of quantum observables in an entangled state examined through a thought experiment by Albert Einstein, Boris Podolsky, and Nathan Rosen (EPR) in 1935 [13] asserted that the description of physical *reality* in quantum mechanics is incomplete. The EPR thought experiment involves two particles, $\mathcal{A}$ and

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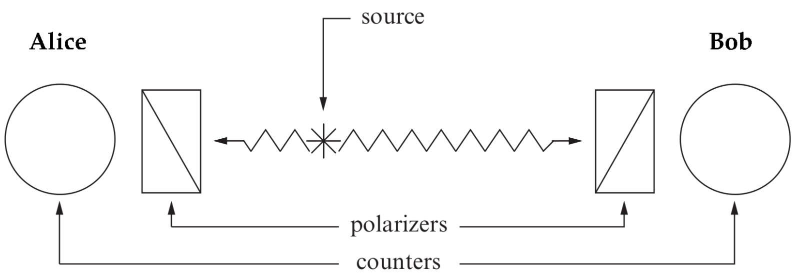

FIG. 1: EPR thought experiment (adopted from [18], p. 241).

$\mathcal{B}$, that are in an entangled state due to a previously brief interaction, and are traveling in two opposite directions. The particle $\mathcal{A}$ is traveling toward an observer named "Alice", and the particle $\mathcal{B}$ toward an observer named "Bob"[1] (see Figure 1). According to the Copenhagen doctrine, the quantum states such as polarization or spin components (up: $\uparrow$ and down: $\downarrow$) of these two particles are unpredictable before any measurement by Alice or Bob. Due to the past interaction, each of these two particles are expected to have a quantum state opposite to each other. A quantum state is recorded by Alice as the wave function is collapsed to a random eigenstate of the observable $\mathcal{A}$ during the observation, while Bob subsequently observed a quantum state of the observable $\mathcal{B}$ that is necessarily opposite to the quantum state of the observable $\mathcal{A}$ recorded by Alice. It seems that the wave function collapse of the particle $\mathcal{A}$ should instantaneously affect the wave function of the particle $\mathcal{B}$, which is in contradiction to the principle of *causality* in special relativity, so called the *EPR paradox*. Einstein referred to it as "spooky action at a distance" in a letter to Born in 1947 [15]. However, Erwin Schrödinger coined the word *entanglement* in a two-part article [16] that extended the EPR argument. Following the EPR argument, Schrödinger's famous *cat* paper [16] also presented a thought experiment of a hypothetical cat and a killing radioactive source in a sealed box that could be simultaneously in both the "*alive* and *dead*" cat superposition states, and questioned how the collapse of a wave function in a superposition of two different states (e.g. $\uparrow$ and $\downarrow$) is ended to an *unpredictable* final state. To solve the EPR paradox in indeterministic measurements, Einstein, Podolsky and Rosen proposed (local) *hidden variables* associated with elements of *reality* [13]. Later in 1935, Bohr replied to EPR in a paper [17] and argued for completeness of quantum mechanics using the principle of complementarity.[2]

II. Deterministic Bohmian Mechanics

The wave-like nature of light in double-slit interference was also explained by Louis de Broglie [19] in the 1920s using a well-defined trajectory of each particle, whose velocity is constrained by a *guiding equation*, that is ontologically traveling through each of two slits without any superposition. In an attempt to explain hidden variables in the EPR argument, David Bohm in 1952 [20, 21] incorporated de Broglie's approach into the so-called *pilot-wave theory* that is considered as an alternative deterministic interpretation of quantum mechanics. In the de Broglie-Bohm pilot-wave theory, a single-particle is described by: (1) its wave function evolving according to Schrödinger's equation, and (2) its actual position evolving according to a velocity constrained by the guiding equation (there is no superposition over space). Similarly, this pilot-wave description is extend to a system of many particles, whose velocities are governed by the guiding equation, and its overall wave function of all particles is evolving based on Schrödinger's equation. It can be seen that the position of each particle in a many-particle system is determined by the velocity given by the guiding equation, so the position of any particle depends upon the positions of the other particles involved in the system that manifests *nonlocal deterministstim*. John Bell in 1966 [22] demonstrated that hidden variables in any quantum mechanics such as pilot-wave theory must be irreducibly nonlocal. The guiding equation in the de Broglie-Bohmian pilot-wave theory (also called Bohmian mechanics) supports the aspect of nonlocality, as Bell [23] mentioned "the guiding wave, in the general case, propagates not in ordinary three-space but in a multidimensional-configuration

[1] Two observers named "Alice and Bob" were first introduced in a paper on a digital communication method between two cryptosystems by Rivest, Shamir, and Adleman in 1978 [14].

[2] In [17], Bohr concluded: "The dependence on the reference system, in relativity theory, of all readings of scales and clocks may even be compared with the essentially uncontrollable exchange of momentum or energy between the objects of measurements and all instruments defining the space-time system of reference, which in quantum theory confronts us with the situation characterized by the notion of complementarity."

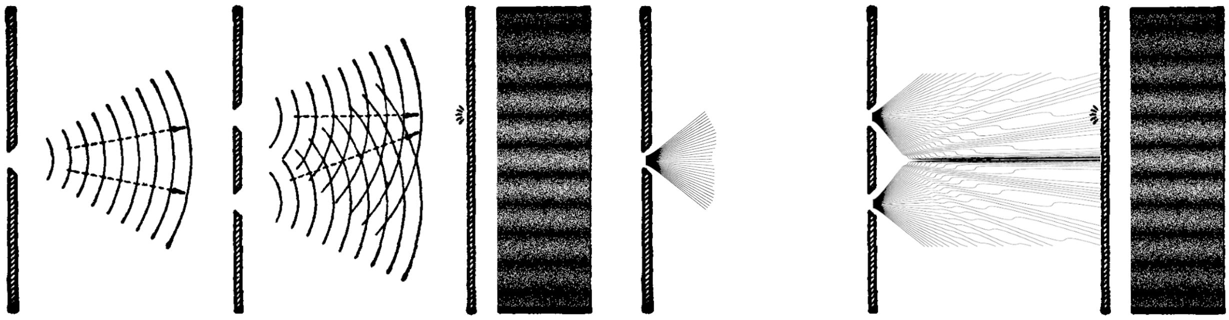

FIG. 2: Illustrations of the interference pattern in Young's double-slit experience according to the "wave *or* particle" Copenhagen interpretation (*left* panel; adapted from [26], p. 216), and the "wave *and* particle" interpretation in Bohmian mechanics (*right* panel; adapted with some modifications from [27], p. 23).

space is the origin of the notorious *nonlocality* of quantum mechanics." Thus, Bohmian mechanics can be recognized as a deterministic quantum theory with *nonlocal* hidden variables that contradicts the principle of causality in special relativity.

Bohmian mechanics incorporates the concept of particle trajectory, which was abandoned by Copenhagen mechanics. Based on Bohr's interpretation, a quantum object such as a photon can be either a *wave or particle*, and it cannot be both at the same time. However, a quantum object in Bohmian mechanics can have both *wave and particle* properties at the quantum scale by incorporating a pilot wave that guides the particle trajectory. In the Copenhagen interpretation, interference patterns in Young's double-slit experiment were explained by the probability of the wave simultaneously passing through both slits (see left panel in Figure 2). But, Bohmian mechanics incorporates both wave and particle descriptions, and the wave function of each particle has a well-defined trajectory while traveling through one of slits (see right panel in Figure 2). According to Bohmian mechanics, the unpredictable final positions of particles in the two-slit experiment are associated with unknown initial positions of particles as emitting from the source, so such uncontrolled initial conditions of the trajectories introduce the appearance of randomness in the interference pattern. Nevertheless, the Copenhagen "wave or particle" interpretation is based on Heisenberg's uncertainty principle specifying that the exact position and momentum (velocity) of a particle cannot simultaneously be measured, so its trajectory (position) and quantum state cannot ontologically be determined at any time before the collapse of its wave function due to an observation. In the Copenhagen doctrine, observables are nonontological *superposables* rather than ontological *beables*. The word 'beable' was introduced by Bell [24, 25] to distinguish between *physical* (beable) and non-physical (observable) quantities, and he said that "observables must be made, somehow, out of beables", so the *being* of a quantum object should be ontologically separated from whatever it is observed. While the Copenhagen doctrine deals with superposable observables, Bohmian mechanics is aimed at describing ontological beables in *non-local determinism*.

The Copenhagen interpretation relies heavily upon the principles of uncertainty and superposition at the quantum level, but on the other hand, Bohmian mechanics ontologically describes the origin of the *quantum uncertainty* in a deterministic manner according to unavoidable randomness and technical difficulties in the initial conditions of the experiment. Randomness of a quantum system can also be described by the quantum equilibrium distribution [28, 29] that is roughly comparable to the classical thermodynamic equilibrium associated with fluctuations in density or velocity (also see [30] for quantum non-equilibrium). According to the so-called *quantum equilibrium hypothesis* [29] proposed in Bohmian mechanics, initial configurations of trajectories are distributed according to their initial wave functions [30, 31], and our most detailed *possible* knowledge about present configurations of particles is conveyed by the quantum equilibrium hypothesis, in what is called *absolute uncertainty* [29]. As debated by Dürr, Goldstein, and Zanghí [29], there are irreducible limitations on the possibility of obtaining knowledge, i.e. absolute uncertainty, which precisely emulates Heisenberg's uncertainty principle.

Other alternative solutions to randomness and unpredictability in Bohmian mechanics are rooted in the so-called *many-body problem* in classical mechanics (see e.g. [32]). The *three-body problem*, which is the simplest case of the N-body problem, is a well-known example of a chaotic system governed by deterministic laws. While Isaac Newton successfully solved a two-body problem in gravitational physics, he pointed to difficulties in solving a three-body gravitational problem such as a Sun-Earth-Moon system in his *Principia* in 1687 [33]. In 1885, the journal *Acta Mathematica* (volume 7) announced a prize competition established by King Oscar II of Sweden and Norway for anyone who could solve the three-body problem. The prize was eventually awarded to Henri Poincaré on King's 60th birthday in 1889. A paper containing the solution was scheduled to be published in *Acta Mathematica* in 1889, but was withdrawn by Poincaré due to some errors. A revised version was finally published by Poincaré in 1890 [34], which was regarded as the establishment of a new branch of mathematics, called *chaos theory*. Poincaré found that there is an infinite number of periodic solutions to the three-body problem, and their possible trajectories are sensitive to initial conditions [35]. Similarly, the many-body problem can occur in a quantum system governed by deterministic laws. Recently, a many-body approach in Bohmian mechanics has been proposed to describe quantum dynamics of entangled bosonic systems [36]. Nevertheless, many-body Schrödinger equation is in general extremely difficult to be treated, and analytically impossible to be solved. Once upon a time, Max Born[3], one of the founding fathers of indeterministic Copenhagen mechanics, said that "It would indeed be remarkable if Nature fortified herself against further advances in knowledge behind the analytical difficulties of the *many-body problem*."

Bohmian mechanics that explicitly describes the quantum state of particles in a deterministic manner should be able to solve the EPR paradox, but according to nonlocality imposed by the guiding equation. Bell in 1964 [38] (reprinted in [25]) considered the EPR paradox and proved the incompatibility of quantum mechanics with *local* hidden variable theories. In a following paper by John Clauser, Michael Horne, Abner Shimony, and Richard Holt (CHSH) in 1969 [39], accompanied by subsequent papers by Bell [24, 25, 40], it was proven that a theory of quantum mechanics cannot satisfy some locality conditions, so called *Bell's inequality* and *CHSH inequality*. Considering two detectors named Alice and Bob that measure the spin components of particles along two different directions $\vec{a}$ and $\vec{b}$ of the detector, respectively (see Figure 1). Each of these detectors are designed to observe two different spin states ($\uparrow$ and $\downarrow$), and produce a binary output (+1 and −1 corresponding to $\uparrow$ and $\downarrow$). Bell's inequality that holds under local realism is as follows (see Ch. 4 in [25]):

$$S \equiv |E(\vec{a},\vec{b}) - E(\vec{a},\vec{b}') + E(\vec{a}',\vec{b}) + E(\vec{a}',\vec{b}')| \leq 2, \tag{1}$$

where $E(\vec{a},\vec{b}) = \int \varrho(\lambda) A(\vec{a},\lambda) B(\vec{a},\lambda) d\lambda$ represents the correlation coefficient that describes the expected value of the product of the two binary outputs when measurements on the two particles are done by Alice and Bob using the detector configurations $\vec{a}$ and $\vec{b}$, respectively, λ is the hidden variables, $\varrho(\lambda)$ is the probability distribution of the hidden variables, $A(\vec{a},\lambda)$ (and $B(\vec{a},\lambda)$) is the binary output of the measurement by Alice (and Bob) that depends on the hidden variable λ and on the detector configuration $\vec{a}$ (and $\vec{b}$). The alternative configurations of the detectors are shown by $\vec{a}'$ and $\vec{b}'$. The correlation coefficient of the measurements of the two particles are estimated according to the four binary output probabilities for given measurement directions $\vec{a}$ and $\vec{b}$:

$$E(\vec{a},\vec{b}) = P_{++}(\vec{a},\vec{b}) + P_{--}(\vec{a},\vec{b}) - P_{+-}(\vec{a},\vec{b}) - P_{-+}(\vec{a},\vec{b}). \tag{2}$$

where is $P_{xy}(\vec{a},\vec{b})$ is the probability that Alice measures the binary output x and Bob measures the binary output y when they use the detector configurations $\vec{a}$ and $\vec{b}$, respectively. Considering the binary output probabilities with the alternative configurations of $\vec{a}'$ and $\vec{b}'$ respectively used by Alice and Bob, we have the so-called *CH-Eberhard inequality* [41, 42]

$$P_{++}(\vec{a},\vec{b}) - P_{+-}(\vec{a},\vec{b}') - P_{-+}(\vec{a}',\vec{b}) - P_{++}(\vec{a}',\vec{b}') - P_{+0}(\vec{a},\vec{b}') - P_{0+}(\vec{a}',\vec{b}) \leq 0. \tag{3}$$

The binary outcomes of particle detection labeled by '+', '−', and '0' mean that being detected in the first state, being detected in the second state, and remaining undetected, respectively. Avoiding the second state such that all '−' events lead to '0' events, the CH-Eberhard inequality is reduced to the following exclusive one-side CH-Eberhard inequality [43] (see also Supplemental Materials in [44, 45])

$$J \equiv P_{++}(\vec{a},\vec{b}) - P_{+0}(\vec{a},\vec{b}') - P_{0+}(\vec{a}',\vec{b}) - P_{++}(\vec{a}',\vec{b}') \leq 0. \tag{4}$$

The local realist preserves at $J \leq 0$.

Bell's inequality is violated when S is higher than 2 (or J higher than 0), implying that the quantum system does not obey *local realism*. A value of $S = 2\sqrt{2}$ means that the system is fundamentally incompatible with the combination of locality and realism. In 1982, Alain Aspect, Philippe Grangier, and Gérard Roger [46] measured polarization states of pairs of entangled photons, and reported $S = 2.697 \pm 0.015$, so they concluded "our experiment yields the strongest violation of Bell's inequalities .... Since it is a straightforward transposition of the ideal Einstein–Podolsky–Rosen-Bohm scheme, ..., and needs no auxiliary measurements as in previous experiments with single-channel polarizers. We are thus led to the rejection of realistic local theories". However, they also mentioned two loopholes (detector efficiency and static character) in their experiment that remain open for endorsements of local realism.

[3] Max Born quoted in [37], p. 40

III. Superdeterministic 't Hooft's Cellular Automaton

All physical phenomena in the entire universe could be the computational outcomes of cellular automata governed by deterministic laws. The idea that the universe could be a cellular automaton was first pioneered by Konrad Zuse in his book "Calculating Space" in 1969 [47]. John Wheeler famously aphorized "It from Bit" [48], and argued that any *it*, in other words, any item of the physical world comes into existence entirely from a discrete *bit* of information characterized by "yes" or "no" binary choices, i.e. the digits 1 or 0. Applications of cellular automata for solving complex physical problems have been extensively explored by Stephen Wolfram in the 1980s [49]. In his book "A New Kind of Science" published in 2002 [50] (Ch. 12, pp. 715–846), Wolfram proposed the *principle of computational equivalence* and declared that "all processes, whether they are produced by human effort or occur spontaneously in nature, can be viewed as computations." He further hypothesized that our universe is basically a simple program with some underlying rules that yield great complexity ([50], Ch. 9), and "it is the computational equivalence of us as observers to the systems in nature that we observe that makes these systems seem to us so complex and unpredictable" ([50], p. 845). Recently, Gerard t' Hooft utilized the idea of computational universality and complexity of cellular automata in his book "The Cellular Automaton Interpretation of Quantum Mechanics" published in 2016 [51], and proposed that quantum mechanics with locality can emerge from a cellular automaton model with locality that is controlled by deterministic laws, the so-called *superdeterminism*. This proposal can be traced back to his earliest works on deterministic quantum mechanics with cellular automaton in 1988–92 [52].

The concept of superdeterminism was first introduced by Bell in the 1980s when he was interpreting Aspect's EPR experiments that depicted a strong violation of Bell's inequality. He pointed out that there are two possible solutions to this problem: nonlocality or superdeterminism. While nonlocality contradicts the principle of causality in special relativity, the alternative resolution that maintains causality and locality involves absolute determinism, i.e. superdeterminism. In an interview for a documentary program for BBC Radio 3 in 1985, Bell said ([53], p. 47): "one of the ways of understanding [Aspect's experimental results] is to say that the world is *superdeterministic*. That not only is inanimate nature deterministic, but we, the experimenters who imagine we can choose to do one experiment rather than another, are also determined. . . . In the analysis it is assumed that free will is genuine, and as a result of that one finds that the intervention of the experimenter at one point has to have consequences at a remote point, in a way that influences restricted by the finite velocity of light would not permit. If the experimenter is *not free* to make this intervention, if that also is *determined in advance*, the difficulty disappears." The idea of superdeterminism can resonate with the inclusion of superdeterministic hidden variables in quantum mechanism. To falsify the non-deterministic nature of quantum mechanics, Sabine Hossenfelder in 2011 [54] proposed an experiment setup involving a semi-transparent mirror, two detectors (Alice and Bob), and a fully-reflective mirror (see Figure 2 in [54]). These two mirrors creates a loop for entering photons, so Alice and Bob measure polarization states of photons in the loop. According to the Copenhagen interpretation, the collapse of one observable wave function by Alice destroys its quantum state, so a measurement by Bob after Alice's measurement should not have access to the same information. We should also note that Paul Dirac strangely explained his *jump* interpretation of the wave function collapse in a deterministic way ([55], p. 36): "When we measure a real dynamical variable ξ, the disturbance involved in the act of measurement causes a jump in the state of the dynamical system. From physical continuity, if we make a second measurement of the same dynamical variable ξ immediately after the first, the result of the second measurement must be the same as that of the first. Thus after the first measurement has been made, there is *no indeterminacy* in the result of the second." However, Bohr supported his view that observables have no specific quantum states before any measurement and they are in superposition (see e.g. [26]). It seems that Hossenfelder's proposed experiment cannot lead to a confirmation of superdeterministic hidden variables, as we may also consider Dirac's interpretation of the collapse in the orthodox interpretation.

In 2016, t' Hooft proposed a cellular automaton model of quantum mechanics that is constrained by the principle of locality and controlled by superdeterminism [51]. According to t' Hooft's interpretation, superposition of an observable does not correspond to an ontological state of a beable, and there are some "template states" that describe probabilistic distributions of states. These template states can be considered as superpositions of ontological states. t' Hooft mentioned that ([51], p. 81) "*Superpositions are man-made.* Our templates are superpositions, but that is because they represent only the very tiny sector of Hilbert space that we understand today. The entire universe is in only one ontological state at the time, and it of course cannot go into superpositions of itself." t' Hooft used superdeterminism to interpreted violation of Bell's inequality in EPR experiments ([51], p. 42): "in a deterministic hidden variable theory, Alice and Bob can only change their minds about the setting of their polarizers, if their brains follow different laws than they did before, and, like it or not, Alice's and Bob's actions *are determined* by laws of physics, even if these are only *local* laws." and these (super-)deterministic hidden variables are rooted in the time and place, when and where all the elements of the universe were together in a singularity, the *Big Bang*. But, this looks like a kind of "conspiracy".

Perceptions of knowledge and information can be classified as *endophysics* and *exophysics* according to observations from *inside* and *outside* physical systems, respectively. Endophysical perception corresponds to knowledge acquired by the observer that is part of the system, whereas exophysical perception is associated with information obtained by the experimenter form outside the system that may be treated as a black box. Endophysical perception is constrained by self-referential attributes of observations since observers and observables are parts of the physical system under observation. For example, imagine an artificial observer is simulated inside a program on a supercomputer governed by deterministic laws developed by a programmer. This artificial observer develops its own physical theory by performing experiments and observations inside the program (computational universe). While deterministic laws in the program are seen by the extrinsic programmer who developed the program, the intrinsic observer's limitations in perceptions of knowledge lead to a conclusion of indeterministic laws. Considering Kurt Gödel's second incompleteness theorem [56] in mathematics (Proposition XI in [57], p. 70), "If c be a given recursive, consistent class of *formulae*, then the propositional formula which states that c is consistent is not c-*provable*; in particular, the consistency of P is unprovable in P, it being assumed that P is consistent (if not, of course, every statement is provable)", so consistency of a program P controlled by formulae c is not provable in P itself. In a computational universe, whose laws are deterministic, behaviors of a physical system are unpredictable by the observer within the universe. Unpredictability in computational outcomes can be interpreted as "free will" by the artificial observer living inside the simulated universe. Karl Popper in 1950 [58] also questioned the completeness of endophysical perception limited by self-referential nature. He employed a kind of Zeno's arguments and specified that "a predictor[4] cannot predict itself" ([58] p. 119), concluding *intrinsic indeterministic* perception by an observer living in a deterministic world.

IV. *Experimental* Bell's Inequality Violation and Deterministic Trajectories

Aspect's EPR experiment in 1982 [46] was the first demonstration of strong Bell's inequality violation. Aspect and his collaborators reported an average value of $S = 2.697 \pm 0.015$ for five tests that exceeds about 35% more than the maximum value of $S = 2$ permitted by local realism. In 1998, Gregor Weihs *et al.* [59] again observed strong violation of Bell's inequality in an EPR experiment with $S = 2.73 \pm 0.02$ for 14,700 coincidence events collected in 10 seconds. However, Aspect's and Weihs' experiment had some open loopholes: communication, detection, and freedom-of-choice. The open *communication* (or locality) loophole means that the spacelike separation of two measurement bases are not far enough to prevent any possible local inference or communication between two observables. The *detection* (or fair-sampling) loophole is exploited if only a subensemble of particles violating Bell inequality is observed and considered as the entire ensemble [60]. Adequate detector efficiency can close this loophole by detecting the entire ensemble of emitted particles. The open *freedom-of-choice* loophole refers to a possible interdependence between the choices of two detector configurations. This loophole can be closed when the detector configurations are generated independently at the measurement bases and are spatially far from the source of the entangled particles. Recently, three separate experimental tests of Bell's inequalities in 2015 [44, 45, 61] eliminated our doubt about Aspect–Weihs experiments by closing these loopholes. The correlation between measurement outcomes they found are $S = 2.42 \pm 0.02$ [61] and $J = 7.27 \times 10^{-6}$ [44], which should be $S \leq 2$ and $J \leq 0$ to support local realism. These experimental results imply statistically strong rejection of *local* hidden variables and local realism. More recently in 2018, the BIG Bell Test Collaboration [62] totally closed the *freedom-of-choice* loophole (any possibility that the choices in detector configurations are affected by hidden variables to correlate with the quantum states of observables) by utilizing random choices generated from about 100,000 human participants playing an online video game, and they found strong violations of Bell's inequalities (see Table 1 in [62]) through 13 independent experiments done in different locations (Australia, China, Austria, Italy, Germany, Switzerland, France, Spain, Argentina, Chile, USA). Assuming the existence of *free will*, globally distributed human participants should strictly yield unpredictable choices for the detector settings. Violations of Bell's inequalities with the human 'free will' assumption under the causality principle implies faster-than-light 'spooky action at a distance', i.e., *nonlocality* of quantum states. Otherwise, sticking to causality (no faster-than-light action), one would expect predetermined choices and outcomes imposed by initial conditions from the farthest past in the universe, i.e., *superdeterminism*.

In 2016, Mahler *et al.* [63] detected deterministic trajectories of two entangled photons going through a double-slit apparatus. This experiment suggested that the trajectory of the first group of photons separated by the first slit strongly depends on the position of the second group of photons passing via the second slit, implying nonlocality according to Bohmian mechanics. We should note that Rozema *et al.* in 2012 [64] from the same research group conducted an experiment involving weak measurements to characterize photons before and after interaction with an apparatus, and reported a violation of Heisenberg's measurement-disturbance relationship commonly taught as the

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[4] In [58], a classical mechanical calculating and predicting machine is called a 'predictor'.

broader uncertainty principle, but they confirmed a measurement-disturbance relationship formulated by Ozawa in 2003 [65]. Their experiment demonstrated how uncertainties of quantum states can be fundamentally different from our measurement limitations in quantum mechanics. In 2016, Magaña-Loaiza *et al.* [66] also reported a possible looped trajectory of photons in a three-slit experiment that is in contradiction to the superposition principle in the Copenhagen interpretation. This possible exotic trajectory of photons was explained by strongly confined electromagnetic fields of plasmons resident on the surface of the metallic three-slit structure used in their interference experiment.

V. Concluding Remarks

In this essay, I have reviewed and discussed those advancements that helped us develop our understanding of quantum mechanics. The double-slit experiment in the 19th century revealed the wave-like characteristics of particles that were explained by a set of principles in the Copenhagen interpretation. Quantum mechanics and general relativity are two of the most fundamental theories developed in the 20th century. However, we still do not have any convincing quantum theory of gravity. Those principles adopted by the Copenhagen interpretation are too tricky to provide us with "quantum gravity". It is dfficult to image quantized space and time elements that easily embrace the principle of superposition An alternative approach to quantum gravity might be found in a deterministic interpretation of quantum mechanics. Recently, a double-slit experiment [63] on entangled photons revealed that their trajectories are not indeterministic. Furthermore, strong violation of Bell's inequality in several recent EPR experiments [44, 45, 61, 62] closed the loopholes of Aspect's and Weihs' experiments [46, 59] that provided new insights into the quantum world. It looks like that we are witnessing the rise of a deterministic theory of quantum world that might be accompanied by either *nonlocal* or *superdeterministic* hidden variables. Regarding the extension of de Broglie theory using hidden variables by Georges Lochak in 1975 [67], John Bell commented that ([18],p. 65) "But if his extension is local it will not agree with quantum mechanics, and if it agrees with quantum mechanics it will not be local. This is what the theorem says." Finally, as argued by Karl Svozil ([68], p. 190), there are correspondences beween *unpredictable* observables in deterministic quantum mechanics, Gödel *incompleteness* in mathematics, *Epimenides' paradox* in logic, and *halting problem* in algorithmics, so there could be a kind of causal (or mild) determinism in the quantum world due to undecidability in physics.

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