Can we see into a black hole?

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This essay outlines a number of developments in physics that together permit an estimate of the cosmological constant. Through an examination of strings which enter a black hole, and connections to the Jordan exceptional algebra, are shown to exhibit properties similar to graphene and systems with quantum criticality. This permits an estimate of a renormalized cosmological constant due to quantum criticality.

Introduction

The classical black hole is the ultimate gate to doom. Death by black hole is infinite death, for anyone who falls in can never be revived or reconstructed. Tidal forces exerted on any system grow utterly extreme so that elementary particles and quarks in hadrons become disassociated as $r \rightarrow 0$. From the perspective of an external observer any object which approaches the horizon slows down and the radiation it emits becomes enormously redshifted so it effectively disappears. The external observer witnesses the object become irreversibly absorbed by the event horizon. It can never be reconstituted. The exterior observer is unable to access information from within the black hole. Yet the Bekenstein and Hawking results indicate the mass-energy, charge and angular momentum interior to a black hole may quantum tunnel to the outside as radiation.

Susskind, Thorlasicus, and Ulgum demonstrated how transverse modes of a string are preserved on the stretched horizon[1], and this defines a world hologram [2]. The string becomes distended across the event horizon of the black hole as high frequency modes become observable to the exterior observer. The black hole in an anti de Sitter spacetime is perfectly stable and the field theoretic information held by the black hole is preserved on the horizon. This indicates that quantum mechanics obtains in black hole physics and that information which enters a black hole is preserved and converted into another form in Hawking radiation. Hawking radiation manifests itself away from the event horizon, but information preservation indicates the horizon is not a perfect classical membrane. The horizon is a null surface with a quantum uncertainty which permits information interior to a black hole to escape.

The black hole interior is then in some form accessible to exterior observers. A distant observer sees quantum oscillations of objects approaching the black hole become extremely red shifted as objects are absorbed onto the horizon. Since the event horizon exhibits uncertainty fluctuations the quantum modes tied to the event horizon are correlated with the field theoretic content in the interior. This will become increasingly the case as the black hole becomes smaller and approaches the Planck mass. Hence the event horizon is not a complete black membrane of ignorance to the outside. The field theoretic information on the event horizon is correlated quantum mechanically with quantum information interior to the black hole. The mysterious singularity of a black hole accessible to an observer who falls inward is related by some quantum complementarity to information absorbed on the event horizon as detected by the exterior observer. A string which approaches singularity will become distended by the Weyl curvature terms and increase the string tension. The change in the string dynamics then has some relationship to the string dynamics as observed by an external observer on the stretched horizon. The critical difference between the two cases is that the string approaching the singularity has a rapidly changing tension, and this tension t_1 associated with the $D1$-brane shares a duality with the $NS5$-brane, or so called black brane [3]. This physics is then explored to examine whether there exist duality principles for quantum information and black holes.

As a closedthe string approaches the singularity it is transformed into an open string. The string is then dual to a $D5$-brane, and associated with a rank two tensor tensor for a two-form. With the disappearance of the closed string the open strings assume tachyonic properties which form a condensate. The gravitational modes which vanish transform into a condensate of tachyons that define an $M2$-brane. The $M2$-brane obeys anyonic statistics and exhibits quantum criticality similar to graphene. In the duality principle advanced dual fields which obey $E_8 \times E_8$ also exhibit a quantum phase transition. It is then advanced that this transition is what einselected some properties of the observable universe from the string landscape.

This black hole duality is between an event horizon of spacetime and a type of quantum horizon, the $M2$-brane identified with the interior singularity. A complementarity principle between these two means some limited information is available concerning the interior of a quantum black hole. This limited information available defines the total quantum information of a black hole, as well as a cosmology, and the complementarity principle conserves quantum information.

Gravity and strings

The string in a gravity field in a gravity field will have its dynamics influenced by the curvature of spacetime across the string. A string parameterized according to its length sweeps out a world sheet of evolution. The string has a classical description, the motion of a body through this parameterized space, plus quantum modes which pertain to particular quantum modes. These quantum modes are similar to the vibrational harmonics on a piano or guitar string. It is not difficult to determine the kinetic energy of the string, which is a summation of the square of the vibrational velocities for each mode. For a string in a gravity field the vibrational modes are perturbed. If the curvature of spacetime is significant over the length of the string the modes of the string are adjusted. The curvature might be thought of as increasing the tension along the string which raises the frequency of the vibrational modes. This is analogous to tuning the pitch of a musical instrument to a sharper note by increasing the tension on a string. The motion of the string will contain a first order term, which is a frame dependent connection term that may be eliminated, and a second order term depending upon the curvature. The string world sheet of the string is similarly adjusted by gravity.

A string approaching a black hole is observed by a distant observer to exhibit a time dilation. The periodicity of quantum oscillations is observed to increase to infinity as the string asymptotically reaches the horizon. Just as a longer string has a lower pitch, the string as observed by the distant observer becomes elongated and winds around the event horizon of the black hole [1]. This fact also indicates how the black hole exhibits a holographic effect with quantum field theory [2]. According to an observer who falls in with the string, on a commoving reference frame, the string behaves in a completely different manner. For the spacetime curvature near the black hole much smaller than the reciprocal of the length of the string squared, $R~<<~1/L^2$ the string changes little. The commoving observer detects no change in the string dynamics until the interior singularity is approached.

The string is observed to have entirely different dynamics according to the wise observer who remains outside the black hole and the foolish observer who falls in with the string. The infalling observer will detect the elongation of the string due to tidal acceleration. Tidal accelerations are due to the existence of the Weyl curvature. In four dimensions a spacetime may be flat according to the standard Ricci curvature, but may exhibit an obstruction to conformal flatness, which is the source of tidal forces. The Weyl curvature is a tensor C_{abcd} which classifies spacetimes according to eigenvectors which satisfy the eigenvalued equation $C_{abcd}V^cV^d~=~\lambda V^aV^b$ [3]. A black hole has two such eigenvalues, and each of these is contracted, or in a dot product, with the velocity term ∂_aX^μ in the string kinetic energy $K~=~\frac{1}{2}V_\mu\partial_aX^\mu V_\nu\partial^aX^\nu$. This modifies the string world sheet as the string approaches the singularity of the black hole. A closed string is elongated from a circular to an elliptical shape under the tidal acceleration. As the closed string approaches the black hole the elliptical shape collapses into a line. The closed string is converted into an open string.

For the 26 dimensional bosonic string, two dimensions are vacuum tachyon modes and the remaining 24 dimensions define an $SO(24)$ group. The graviton is determined by the the string mode operators α^a_{-1} so that $\alpha^a_{-1}\alpha^b_{-1}|0\rangle$ projects the vacuum $|0\rangle$ to $|\Omega^{ab}\rangle$. The traceless symmetric portion defines the spin-2 graviton, the trace term is a dilaton field, and the remainder is a gauge-like field. The closed string is tidally distended into an open string the graviton modes in the two directions cancel each other and result in a tachyonic state.

The open string has distinct clockwise and counter clockwise wave modes, called left and right, which travel around the string. These modes are independent. The spectrum of the string has a Regge trajectory $\alpha' M^2 = 4(n - 1)$, which relates the mass of the string with its eigen-mode number [4]. As the string approaches the singularity the circular shape collapses and the string is an open string with end points. There is then a distinct Regge trajectory for this type of string $\alpha' M^2 = n - 1$, and thus carries different particle physic data than the closed string. The closed string carries graviton modes which vanish near the singularity, and are replaced by an antisymmetric field tensor plus tachyon modes. The independent modes of the closed string constructively and destructively interfere with each other in specific ways so the graviton modes are lost, and are replaced by tachyon and gauge-like field modes. An important field which remains is the dilaton field, which means this transformation is conformally invarant.

The closed string is transformed into an open string, where that open string interacts with a $D2$-brane. The $D2$-brane holds a condensate of tachyon valued strings. The quantum information in the closed string is transferred to a brane, the $NS5$-brane, sometimes called the black brane in 10 dimensions. The five dimensional supermembrane sweeps out a six dimensional volume in time. There exists a charge on this brane world sheet in six dimensions dual to a charge on a boundary term in $10 - 6 - 1 = 3$ dimensions. One of those three dimensions is dual to the $NS5$ brane, and there is a remaining two dimensions which defines an $M2$-brane. The one dimension dual to the 5-brane is a $D1$-brane, a one dimensional brane or (type IIA) string, which is attached by Dirichlet boundary conditions to the $M2$-brane. The tensions of the $NS5$-brane and the open string satisfy a duality condition $t_1 t_5 = \pi/k$ [5]. The charges are associated with the BPS mass of a black hole [6], with particle physics data.

Exceptional nature

The physics has two components to it. In 11 dimensional supergravity, which embeds the above 10 dimensions, the dynamics has a reduced $2 + 1$ dimensional physics plus an 8 dimensional extended field dynamics. The low dimensional physics is analogous to the physics of graphene, or other two dimensional systems. The statistics of bosons and fermions involves the interchange of the two particles. For fermionic or spinor systems there exists a double covering with a 2π and 4π sign convention. However, in two dimensions this covering information does not exist. As a result there is a phase which interchanges statistics between fermions and bosons [7]. The IIA string exists on the $M2$ brane, and exhibits anyonic behavior. The dynamics is determined by a Chern-Simons (CS) Lagrangian,

$$\mathcal{L} = \epsilon^{abc}\big(A_a \partial_b A_c + \frac{2}{3} A_a A_b A_c\big)$$

which has certain topological properties. These topological indices correspond to charges, or quantum numbers, which determine a phase of the system or a quantum Hall effect. These quantum phase transitions occur in the heterotic $E_8 \times E_8$ physics by duality. .

The exceptional group E_8 plays an important role in string theory. The group has 240 roots which correspond to elementary particles. The closed heterotic string $E_8 \times E_8$ contains the graviton field with spin $= 2$. Closed strings contain the physics of gravity, and are unattached to branes. They move in 11 dimension and are not limited to just four dimensional spacetime. The heterotic supersymmetric is governed by a product of two $256 = 2^8$ dimensional $CL(8)$ Clifford algebras. The 120 dimensional grade $= 2$ portion of this group is the 120 dimensional $spin(16)$, which with a half spinor representation determines the E_8.

The E_8 is an exceptional group, which with three additional dimensions specify the 11 dimensions of supergravity. The optimal system for supergravity is the Jordan $J^3(\mathcal{O})$ algebra. The Jordan algebra exists over a field with certain multiplication rules. The basic Jordan multiplication between two elements is denoted by

$$x \odot y = \frac{xy + yx}{2}.$$

A special case of Jordan algebras are exceptional Jordan algebras. The Jordan algebra over the octonions, or the E_8 group, as a 3×3 matrix of octonions and diagonal scalars of the form [8],

$$\begin{pmatrix} z_1 & \mathcal{O}_0 & \bar{\mathcal{O}}_2 \\ \bar{\mathcal{O}}_0 & z_2 & \mathcal{O}_1 \\ \mathcal{O}_2 & \bar{\mathcal{O}}_1 & z_0 \end{pmatrix}.$$

The off diagonal terms are three copies of the octonions, or elements of E_8 algebra, and the diagonal terms are scalars. Since each of the octonions $\mathcal{O}$ in this structure spans an independent 8 dimensional space there are then $24 + 3 = 27$ dimensions in total. The underlying $SO(8)$ induces a triality condition extended to $SO(8) \times S^3$, which permits a permutation of the octonions and scalars according to a discrete map. The action for the system is then given by the determinant of this matrix under triality maps.

To start we look at the scalars z_i. The action term contains cubic products in these scalars, which are antisymmetric on their index values. These scalars become gauge covariant momentum-like quantities $z_i \rightarrow \partial_i + A_i$ and are restricted to a light cone coordinate condition. The field A_i is an $SO(2,1)$ gauge potential and obeys anyonic statistics. This dynamics is identified with the tachyon dynamics on the $M2$-brane. The remaining exceptional valued fields obey a mirror dynamics according to a Lagrangian quadratic in octonionic fields, such as $\mathcal{O}\nabla\mathcal{O}'$, and cubic in these field $\mathcal{O}\mathcal{O}'\mathcal{O}$". The brane tensions associated with the exceptional valued fields have a duality with the brane tension of the $D1$-string which obeys the CS Lagrangian.

The structure of this theory involves the conversion of a closed string to an open string. This is an artifact of M-theory. The exponentiation of the CS Lagrangian, results in an elementary form of the Born-Infeld Lagrangian, when the field terms are written explicitly according to string modes. This Lagrangian, in it most elementary form $\mathcal{L} = \sqrt{1 + F}$, for F the field tensor, is a cornerstone of string-brane dynamics. This is one indication of connections with M-theory and "braney" transformations of string types.

Quantum criticality and topology of Fermi surfaces

The essential physics we concentrate on now is the anyonic physics of the $M2$-brane and its analogue with quantum criticality in graphene. Graphene renormalizes the mass of an electron to a very small value. The fermion is a relativistic quasi-particle at low energy [12], with Dirac kinetic energy $H = v_F\sigma\cdot\mathbf{p}$, where $v_F \simeq c$. The charges on the $M2$-brane are analogous to electric charges on graphene. The quantum critical point occurs when the magnetization of the system vanishes at $B = 0$ due to the breakdown of a Fermi-Landau electron fluid [9]. The Fermi-Landau liquid theory describes the overlap between electron states $|e\rangle = b^\dagger_k|0\rangle$ and spinon states which depend on magnetization $|q\rangle = \sigma M_q b^\dagger_{k-q}|0\rangle$. This overlap determines the renormalized fermion mass

$$\frac{m}{m^*} \simeq |\langle e|q\rangle|^2$$

The criticality associated with the divergence of the effective electron mass, $m^* \rightarrow \infty$ as the overlap vanishes. This correlates with the vanishing of the magnetic susceptibility at the quantum critical point. There is further a fractal-like scale physics with respect to a temperature $T \simeq \hbar/kt$, for t a Euclideanized fluctuation time. The scaling principles obey conformal symmetries and are related to the quantum Hall effect [10].

An outstanding problem in cosmology is the value of the cosmological constant. The cosmological constant, Einstein's "greatest blunder," as he called it, has returned to cosmology and physics with inflationary cosmology and the observation of the accelerated motion of galaxies. The cosmological constant Λ enters into the Einstein field equation,

$$R_{ab} - \frac{1}{2}RG_{ab} + \Lambda g_{ab} = 8\pi GT_{ab},$$

as some cosmic gravity field which accelerates galaxies outwards. The observational data puts this small curvature term at $\Lambda \simeq 10^{-52}m^{-2}$. A naive calculation of the cosmological constant indicates it is $\Lambda \simeq M^4_{pl}$, which is 120 orders of magnitude larger than the observed estimate of $10^{-47}GeV^4$. The cosmological constant is proportional to the vacuum energy and pressure $\Lambda = 8\pi G(\rho_{vac} + 3p_{vac})$. Currently theory and data suggests that $p = -\rho$, or an equation of states with $w = -1$. For the vacuum energy density computed from a Planck scale, this result in M_{pl}^4 an absurdly high value. Raphael Bousso and Joseph Polchinski demonstrated how four fluxes incident on a $D7$-brane determined the cosmological constant. It is a device which renormalizes the huge Planckian value for the cosmological constant according to how this field is incident on the $D7$-brane. This is determined by how the brane is folded or wrapped in a low energy compactification. The field can have multiple flux incidences depending upon how the brane is wrapped. It turns out that the conditions for such compactifications and correct fluxes are very highly specialized. This has been cited as some sort of anthropic principle or coincidence.

The fluxes incident on the brane are dual to a field effect on the $M2$-brane, with which the IIA string interacts. The physics is analogous to currents on a material approaching a quantum critical point where the magnetic susceptibility approaches is $\chi \sim log(T/T_0)$ and approaches zero as $T \rightarrow T_0$. This is a quantum phase transition with a complete invariance of scale. A quantum phase transition occurs by quantum fluctuations which drive a transition of phase at zero temperature, or near zero temperature. This requires organizational principles according to boson and fermion quantum statistics. This problem confronts the fact that Fermi-Dirac physics is the "square root" of bosonic physics, which implies negative probabilities. However correspondences between the quantum critical point and the anti-de Sitter/conformal field theory duality [12] relate the fermionic problem to a gravitational one. Fermion states away from the quantum critical point are seen to exhibit properties of a Fermi-Landau liquid state [13]. This lends results on the nonprobabilistic aspects of fermion fields and the sign problem. The breakdown of the Fermi-Landau fluid occurs when the electron or associated quasi-particle assumes an infinite mass. Physically this mass should not completely diverge, but reach some very high value. In the case corresponding to the cosmological constant this is the nave cosmological constant value $\Lambda \sim M_p^4$. The physical cosmological constant is adjusted from this value by the variation of the system Hamiltonian or Lagrangian on the Fermi surface.

The Fermi surface is a surface of constant energy E_F in the momentum space, or $\mathbf{k}$ recirpocal wave vector space. The Fermi energy in any system is that occupied by the highest orbital or band eigenstate. A universal structure for Fermi surfaces is they are classified by K-theory, or the homotopy value of an associated general linear group. This is a generalization of the connection between Fermi liquids and p-branes of string theory. The duality above between the $M2$-brane dynamics and the p-brane dynamics according to exceptional groups permits a tie between the quantum phase dynamics and the fine tuning or "selection mechanism" for the cosmological constant.

The "$\Gamma\cdot p$" theory and its K-theory [14] is made explicit for the fermion fields with the action

$$S = \frac{1}{2k_4^2}\int d^4x\sqrt{-g}\big(R + \lambda_0 - \frac{Z}{4}F_4^2 - \bar\Psi\Gamma^M D_M\Psi - m|\Psi|^2\big),$$

for $\Gamma^M = E_A^N\Gamma^A$, and $\{\Gamma^A,\ \Gamma^B\} = 2g^{AB}$. Here E_A^N is a vielbein and Γ^A are spacetime gamma matrices. The derivative D_M is covariantly gauged. This action must be extended to the full 11-dimensional bulk with one sector that is $N =$ SUSY in $AdS_4\times S^7$ dual to the conformal theory in $d = 3$ $N = 8$ SUSY YM theory for a conformal fixed point. The Dirac equation for the fermionic field obeys a spectral function determined by the Greens function [13]

$$A(\omega\ k) = \frac{1}{\pi}Im\big(Tr\ i\sigma^0 G_R(\omega,\ k)\big),$$

which has AdS/CFT content on a quantum critical point. The duality above then determines the value of the supersymmetric Yang-Mills gauge and fermion fields. This means that the cosmological constant term $\Lambda = \lambda_0 - ZF_4^2/48 + \bar\Psi\Gamma\cdot p\Psi$ is fixed by the breakdown of the Landau Fermi fluid on the quantum critical point of the $d = 3$ or $SO(2,1)$ theory. The lattice spaces of the four-field fluxes adjust with the quantum critical point, becoming large in correlation with the fermionic fluid breakdown. This adjusts the flux to an optimal value to cancel most of the bare cosmological constant. The cosmological constant is adjusted to a value in response to a quantum critical point on the $M2$-brane.

Cosmological constant and fine tuning

The bare cosmological constant is not completely cancelled. The cosmological constant is associated with the QFT through the vacuum, and is adjustable by inflationary fields. The value of the cosmological parameter $\Lambda = \Lambda(\phi,\ \dot\phi)$ has adjusted lower since the early inflationary pressure, where ϕ is interpreted as the inflaton. The adjustment of the parameter to its current low value is the onset of the quantum critical phase in the universe. The value for the cosmological constant is bounded above zero because the effective mass of the fermionic states can't exceed the Planck scale. This may in fact surprisingly be the solution to the same divergence in standard solid state physics. The quantum critical point refuses any scaling structure which forbids the occurrence of a Fermi energy. Yet for a very large m^*, or equivalently a large four field on the 7-brane in 11 dimensional space, there is a cut off which breaks the scale invariance. Hence as the mass m^* becomes very large the quantum system exhibits a near perfect scaling principle which appears to preclude any scale, such as a Fermi energy. Yet ultimately the electron mass does not diverge to infinity, but rather terminates at some value, which means the system is perturbed away from a pure quantum critical point into some other state of matter. This cuts off the renormalization group (RG) flow near the Planck scale. With a "strong-weak" S-duality argument a low energy RG cut off is the onset of the Higgs mechanism around the $1TeV \simeq 10^{-16}E_{pl}$.

The bare cosmological constant corresponds to the effective mass of the quasi-particle, which becomes enormous at the quantum critical point. The Hamiltonian on the Fermi surface is $H = v_F\sigma\cdot p_f + V(p)$ and the effective mass is

$$m^* = \frac{p_F}{u_f} = \frac{p_F}{(\partial E/\partial p)_{p=p_F}},$$

where $E = \langle H\rangle$. We may differentiate the expectation of $H = v_F\sigma\cdot p_f + V(p)$ by p and expressed according to m is

$$\frac{1}{m^*} = \frac{1}{m} + \frac{1}{p_F}\Big(\frac{\partial V(p)}{\partial p}\Big)_{p=p_F}.$$

The estimate of $\partial V(p)/\partial p$ on the Fermi surface by the flux terms of the four field tensor is

$$\Big(\frac{\partial V(p)}{\partial p}\Big)_{p\,=\,p_F} \sim \frac{d\lambda}{dn} = \frac{2nT^2}{Z} = 2nT\sqrt{\lambda_0 Z},$$

for T the $M2$-brane tension and Z determined by a quantization condition with the $D8$-brane dual in 10 dimension to the $M2$-brane. The $M2$-brane tension is $T = 2\pi M_{11}^3$ which gives the value $\lambda_0\kappa^4 10^{-75}$ [11]. However, the rapidity factor γ for the $M2$-brane adjusts this to

$$\Big(\frac{\partial V(p)}{\partial p}\Big)_{p\,=\,p_F} \simeq \lambda_0\kappa^4 10^{-75}exp(-2\gamma).$$

A physical assumption is made $exp(-\gamma)$ is a rapidity for black holes from their "soft" domain of $\simeq 1TeV$ to the Planck scale, $exp(-\gamma) = L_p/L_{TeV} \simeq 10^{-16}$. There is a 10^{-97} factor in the reduction of λ_0. If further elementary particles (quarks and leptons), with masses $\simeq 100MeV$ are considered to be some form of BPS black hole and the rapidity is set to the Compton wavelength $\sim 10^{-13}cm$ of such particles. The reduction factor is the 10^{-117}, which is close to what is expected for a physical cosmological constant.

The scaling physics with the "stretching" of the brane from low energy dynamics to extremely high energy physics at a black hole core, or $M2$-brane, is an area in need of continued research. The employment of elementary particle masses or with a scale of high energy physics possibly connects the physics of the Higgs mechanism to quantum gravity. Some physics accessible today may solve issues of fine tuning and the cosmological constant.

The occurrence of the cosmological constant near a quantum critical point for the bare cosmological constant indicates that the RG flow is established for orbifold or Calabi-Yau configurations which permit an extremization of complexity. The scale invariance associated with quantum criticality manifests itself in the structure of the universe at all scales, with a cut-off due to the finitude of the m^*, or equivalently λ_0. The brane analogue of the Fermi-Landau liquid near criticality means the low energy configurations of branes, orbifolds and wrappings, is such that there is a fractal-like scaling of structures in the universe. Quantum criticality on p-branes and their parallel criticality with such quantum phase transitions of Fermi liquids suggests the universe is fine tuned according to physical principles. This removes the need for anthropic principle or multiverse arguments in establishing the fine tuning of the universe. The universe may be fine tuned as it is because it was constrained by quantum phase transitions.

The occurrence of this scaling invariance according to a quantum critical point means the universe is set by a quantum fluctuation with an imaginary time proportional to $1/T$. For low enough temperature the quantum fluctuations are sufficiently large to induce a phase change. This suggests the enormous temperature at the Planck scale is renormalized downwards according to quantum scaling of fluctuations. The universe is potentially one quantum fluctuation, where the scale invariance of the quantum phase is a conformal structure of the universe. A time dependent conformal factor Ω with $g_{\mu\nu} \rightarrow \Omega g_{\mu\nu}$ can easily reproduce the de Sitter cosmology, which is the physical "twin" to the anti-de Sitter spacetime. The RG group of particle flows in the AdS and the conformal structure of the dS spacetime are cases of a universal scale invariance established by a quantum critical point.

It is possible string theory and loop quantum gravity (LQG) are ultimately two aspects of nature. Now we take an unexpected turn to LQG to see if quantum criticality provides connections between string theory and LQG. ADM relativity defines a set of constraints that are Lagrange multipliers that are annulled on the solution in phase space. The first constraint is given by the Einstein field equation, which defines a Hamiltonian density

$$H' \;=\; -G_{ijkl}\pi^{ij}\pi^{kl} \;-\; g^{1/2}R^{(3)}(g)$$

for $\pi^{ij} \;=\; \sqrt{g}(g^{ij}Tr(K) \;-\; K^{ij})$ and K^{ij} the Gauss second fundamental form or extrinsic curvature term and $R^{(3)}$ the curvature of the 3 dimensional spatial surface. G_{ijkl} the (mini) superspace metric, which is the phase space configuration for a classical gravitational system. This is the Hamiltonian constraint $NH \;=\; 0$ for N the lapse function, a Lagrange multiplier. There is also a momentum constraint N_iH^i for $H_i \;=\; 2\pi^{ij}_{ij}$. The lapse and shift functions $N,\ N_i$ are constraints on how a spatial surface moves in a time evolution and how it is elastically stretched with the general metric

$$ds^2 \;=\; -N^2dt^2 \;+\; g_{ij}(dx^i \;+\; N^idt)(dx^j \;+\; N^jdt).$$

The quantized form leads to the Wheeler DeWitt equation for $\pi_{ij} \;=\; -i\hbar\delta/\delta g^{ij}$. The classical constraint equations

$$\mathcal{H} \;=\; NH \;=\; 0 \;and\; \mathcal{G} \;=\; N_iH^i \;=\; 0$$

become the quantum equations

$$\mathcal{H}\Psi[g] \;=\; 0,\ \mathcal{G}\Psi[g] \;=\; 0.$$

The classical constraints annul the quantum state of gravity.

However, there are ambiguities from matter of functional analysis. To remove these ambiguities we impose a spinor gauge condition on the momentum variables

$$\pi_{ij} \;\rightarrow\; \pi_{ij} \;+\; ig\sigma^aA^a_{ij},$$

where σ^a is a Pauli matrix and A^a_{ij} is a quaternionic gauge variable that satisfies certain Kaufmann conditions for knot equation. The gauge variable defines the field terms (analogous to electric and magnetic fields) E^a_j which satisfies the equation

$$Q^a \;=\; \nabla^iE^a_i,$$

which is Gauss' law.The Q^a is the total "charge" associated with this gravitational analogue of the electric field. If one were to imagine a closed three dimensional space with a single charge, the field lines would wrap around and "drive" the system wild. Thus we demand the constraint $B_aQ^a \;=\; 0$, which annihilates fields on the solution space. The total Hamiltonian

$$\mathcal{H} \;=\; \int d^3r(NH \;+\; N^iH_i \;+\; B^aQ_a)$$

vanishes under all the constraints.

The Landau-Fermi liquid theory defines the renormalized mass by

$$\frac{m}{m*} \;=\; |\langle e|q\rangle|^2,\; |e\rangle \;=\; b^\dagger_k|0\rangle,\; |q\rangle \;=\; \sigma M(q)b^\dagger_{k-q}|0\rangle.$$

Translated to the B_aQ^a constraints the magnetization define by Gauss' law $Q_a \;=\; \nabla_i\sum_q M(q)^i_a$. This constraint variable is connected to the quantum critical point, and is annulled on the entire manifold. LQG is background independent, but string theory is not. String background is required to fix the compactification of extra dimensions in a stable manner. Yet a quantum critical phase which sets a contraint in LQG and string-brane compatification might bridge the two theories.

This paper title asks the question, "Can we see inside a black hole?" Clearly for a large astrophysical black hole the answer is no. This would require a complete accounting of quantal data which composes the black hole. For a quantum black hole it is in principle possible to observe the interior. The event horizon is sufficiently quantum mechanically uncertain that fields associated with it and fields in the interior are interchangeable or entangled directly, and observably so. Such black holes would be produced by extremely high energy scattering of particles, and possibly have been observed by RHIC [15]. An indirect way of observing a black hole interior is to look out in the universe. The cosmological constant as detected by SN1 data and accelerated galaxies is an indirect observation of the physics of black hole interiors. Looking at the interior of a black hole is to examine the world around us. The scaling of physics and the complexity of the world reflects some elementary quantum critical phenomena at the core of black holes.

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