Undecidability of States and Epistemic Horizons as Quantum Gravity

Lawrence Crowell

[1]AIAS, Budapest, Hu/Albuquerque, NM/ Jefferson, TX 75657
goldenfieldquaternions@gmail.com

Abstract

Quantum mechanics places limits on what can be observed. Further, while it is a deterministic wave theory, outcomes of specific measurements are purely stochastic. The geometry of quantum entanglements, or quantum phases in general, describes a topological obstruction to unitary transformations between GHZ and W states or bipartite and tripartite entanglements. The momentum map defined on Kirwan polytopes is in the framework of a fractal sets in state space has p-adic measure. Axiomatic incompleteness of any p-adic algorithm illustrates how these define obstructions. This is similar to how the Euclid 5^{th} axiom is undecidable, and geometry has different model systems. This is then argued to connect with quantum gravitation in how spacetime is an epiphenomenology from entanglement.

1 Introduction

Quantum mechanics requires an extension of real variables of classical mechanics into complex variables. This may be seen with the replacement of the Poisson bracket with the commutator $\{p,\ q\}\ \rightarrow\ \frac{i}{\hbar}[\hat{p},\hat{q}]$ [1]. This extension is related to forcing in axiomatic set theory. This an extension of an axiomatic set theoretic model to include additional axioms, often themselves undecidable statements. The extension of any mathematics from strictly real variables to complex variables is related to forcing, where the $i\ =\ \sqrt{-1}$ is a general form of multiplication of a pair of real numbers. The axiomatic nature of numbers is more general and additional structure arises. Forcing is often done with the inclusion of some self-referential proposition in a set theory, such as the Bernays-Cantor consistency of the continuum hypothesis with Zermelo-Fraenkel set theory, to extend a model [2]. This gives rise to some suggestion of a relationship between quantum mechanics and Gödel's theorem

The relationship between quantum mechanics (QM) and Gödel's theorem (GT) has occurred to others [3] [4]. There is the famous story of Gödel throwing John Wheeler out of his office for suggesting such a connection. Yet there are some curious parallels between QM and GT, such as how QM is most difficult to understand with measurements. Niels Bohr made an ancillary postulate that the world is both quantum and classical, and measurements of a quantum system are done by a classical measurement apparatus. Heisenberg pointed out an issue with any cut between the quantum and classical worlds is not sharp. A measurement of any quantum system is where the quantum phase or superposition of states of a system is transferred to an entanglement with some needle state. With a general apparatus we then have a needle state that measures the needle state, so to speak, which involves larger mass or action. Photomultiplier tubes used in nuclear and particle physics operate by generating a growing cascade of electrons between grids in response to a single measurement. In this way the classical system is a large number of quantum states that encode other quantum states, in particular the system detected, so the measurement is a sort of Gödel number [5] encoded by quantum bits.

The role of entanglement is important. Feynman said much of QM can be understood with the 2-slit experiment, now considered. A quantum wave upon passing through the two slits, with state vectors $|1\rangle$ and $|2\rangle$ a distance d apart has the wave amplitudes $exp(i\vec{k}\cdot\vec{x})$ and $exp(i\vec{k}'\cdot(\vec{x}+\vec{d}))$ at the point x and the state

$$|\psi\rangle \ = \ e^{-f(x)}(exp(i\vec{k}\cdot\vec{x})|1\rangle \ + \ exp(i\vec{k}'\cdot(\vec{x}+\vec{d}))|1\rangle),$$

For $e^{f(x)}$ a real valued envelope function. The modulus square of the wave is then

$$|\psi|^2 \ = \ e^{-f(x)}\{2 \ + \ cos[(\vec{k}-\vec{k}')\cdot\vec{x}+\vec{k}\cdot\vec{d}]Re(\langle 1|2\rangle) \ + \ i \ sin[(\vec{k}-\vec{k}')\cdot\vec{x}+\vec{k}\cdot\vec{d}]Im(\langle 1|2\rangle).$$

The resulting wave pattern is the interference observed by many instances on the screen. Now consider the case where we place a spin at one of the slits to detect if a spin passed through. This means we have spin states $|+\rangle$, $|-\rangle$ entangled with the slit states such that $\langle\pm|\pm\rangle \ = \ 1$ and $\langle\pm|\mp\rangle \ = \ 0$. The orthogonality of the spin states, which serve as needle states, means the modulus square of the entangled state no longer exhibits an interference pattern.

The needle state is coupled to another state with a larger mass or action. This results in principle in a quantization of some massive needle state. However, such quantization on the large does not ordinarily exist, and the observed needle is in one classical-like state. This classical output is an einselected state that is stable against quantum noise or environmental decoherence. The "avalanche," similar to a photomultiplier cascade, requires an open world system that permits a quantum state to encoded as a type of Godel number. The reservoir of instrument or environmental states to record an outcome require an energetic process, whether that be a chemical change in a photoplate emulsion, a flip of a solid state device state or the photomultiplier cascade of electrons. This is somewhat different from the standard view of quantum mechanics that views the evolution of a quantum state as a closed system. Decoherence of a quantum state requires some reservoir of states that are not treated as the same closed system. This means an additional requirement for nature to present a quantum outcome or so-called collapse. Since the establishment of a classical-like needle state is associated with entanglement, the encoding of a Godel number of a qubit is a physical analogue of incompleteness.

Szangolies presents another perspective on incompleteness in QM with a general framework on how any observer acquires information, but in so doing must sometime sacrifice information acquired in the past [6]. This is a sort of horizon perspective of physics that all observers are local, all physics observed is local, and this locality is bounded by an epistemic horizon. It is possible to show that for programs that compute states there is a Cantor diagonalization [6] [7]. The j^{th} program that k^{th} state is a tuple that a function f acts on $f(j,k):\mathbb{N}\times\mathbb{N}\rightarrow \ \{0, \ 1\}$ formed as a diagonalization map $D:\mathbb{N} \ \rightarrow \ \mathbb{N}\times\mathbb{N}$. There is then the commutative map corresponding to the Cantor diagonalization of programs and states. It is then possible to illustrate how there can never be a complete listing of all such programs. With Spekkens toy model some aspects of QM are demonstrated.

This epistemic horizon has a similar construction in general relativity. Event horizon enforce limitations on any observer's ability to account for states. In the case of a black hole the quantum states of the material that fell behind the horizon are hidden from view. Two black holes composed from entirely different materials, one from dark matter and the other from luminous matter, will appear identical. The quantum states of the material are hidden by the event horizon. The tortoise coordinate does mean some aspect of any material is accessible, but this is so red shifted that only quantum gravitation oscillators contribute to the exterior. Included with this are angular momentum and charge. This the no-hair theorem result that short ranged charges and quantum states are not accessible. Yet if qubits are conserved this quantum information is then not destroyed. It is simply converted into forms that local observers can't access.

2 Observables and Causal Limitations in QM

Quantum mechanical theorems, such as Bell's theorem and CSCH, invoke the concept of statistical independence [8]. This states that given a hidden variable λ and devices, either state preparation systems or detectors $\mathbf{a}$ and $\mathbf{b}$ that probability obeys $p(\mathbf{a},\mathbf{b}|\lambda)~=~p(\lambda)$. The probabilities are determined by the density ρ as $p~=~\langle\rho\rangle$, and for quantum mechanics the implication is that quantum states have state preparations independent of detection or similarly that multiple detectors have independent outcomes.

Superdeterminism is a proposal by 't Hooft [9] that quantum mechanics has some underlying structure. The underlying structure is a form of cellular automata, there the assumption of statistical independence no longer holds and $p(\mathbf{a},\mathbf{b}|\lambda)~\neq~p(\lambda)$. This is one reason superdeterminism is largely dismissed by physicists. However, this has useful structure. The existence of an underlying "circuit board" behind QM runs into questions of causality, such as faster than light propagation and retrocausality. Superdeterminism may though be used to examine epistemic horizons and undecidability.

Invariant set theory (IST) was advanced by Palmer to resolve paradoxes with measurement of quantum systems. This is a postulate on a relationship between fractal geometry and quantum mechanics. A state space with a metric defines states that are ontological and those that are unreal. IST is a deterministic theory and for some nonlinear and chaotic dynamics principles of this system are on a fractal set I_U. Quantum systems are strictly linear, where while an internal space can permit nonlinear dynamics the exterior geometry or symmetries must be completely linear. Hence, this defines a measure of a state space that has some nonlinearity. The dynamics of this fractal set are loops, or helixes lifted by a fibre, where the dynamics are those of $U(1)$, such as a line bundle, and where there is a Sierpinski-like self-similar set of loops on loops, or that a helix has a path that on a smaller scale is a helix. This loop-helical structure is tied to the Argand plane of complex numbers and is the ingredient that gives $\{p,~q\}~\rightarrow~\frac{i}{\hbar}[\hat p,\hat q]$. This is similae to epicycles in the Ptolemaic world. This self-similar structure has fractal content for a nonlinear process. For many nonlinear systems there is a violation of unitarity as well. Zurek makes this point in his paper [10].

This fractal-like structure is associated with counterfactual measurements. While IST is Bell nonlocal, it violates the factorization assumption of Bell's theorem, it also does not obey statistical independence. There can be correlations between apparatuses in an experimental set up. A detector outcome may be established by an action at a distance interaction from the state preparation or by retrocausal means. The self-similar structure of these loops and helices means the state space is best described by a p-adic metric. This space has distance defined by a p-adic norm. A p-adic metric $d(x,~y)$ is a quadratic function and its square is a quadratic equation. The metric is a form of Diophantine equation with a p-adic solution.

The standard metric of quantum mechanics is the Fubini-Study metric [11] of $\mathbb{C}P^n$ that obtains from a line bundle fibration on Hilbert space $\pi:\mathcal{H}~\rightarrow~P\mathcal{H}$,

$$ds^2~=~\frac{1}{2}\partial_i\partial_{\bar j}(1~+~|z|^2)dz^idz^{\bar j}.$$

Entanglements have polytope realizations of states [12]. For tripartite entanglements these polytopes are representations of the concurrences $\tau_i|(\psi_{ABC}\rangle)$ with

$$\tau_1(|\psi_{ABC}\rangle)~=~\frac{1}{3}(\tau_{A|BC}~+~\tau_{B|CA}~+~\tau_{C|AB})$$

$$\tau_2(|\psi_{ABC}\rangle)~=~\frac{1}{3}(\tau_{A|B}~+~\tau_{B|C}~+~\tau_{C|A}).$$

The first of these is a representation of entanglements with three states according to how each state is entangled with the other two in an entanglement. The second describes three estates in entanglements purely according to bipartite entanglements. The difference $\tau_1(|\psi_{ABC}\rangle) \; - \; 2\tau_2(|\psi_{ABC}\rangle) = \; \tau_3(|\psi_{ABC}\rangle)$ with

$$\tau_3(|\psi_{ABC}\rangle) \; = \; \tau_{A|BC} \; - \; \tau_{A|B} \; - \; \tau_{A|C}.$$

is an obstruction called the monogomy principle $\tau_{A|BC} \;\; \geq \;\; \tau_{A|B} \; + \; \tau_{A|C}$ [13]. This is a residue of entanglement called the 3-tangle that has no description in a 2-body system. $\tau_1(|\psi_{ABC}\rangle) \; \leq \; \tau_1(|GHZ\rangle)$ and $\tau_2(|\psi_{ABC}\rangle) \;\; \leq \;\; \tau_2(|W\rangle)$ are bounded above by the GHZ and W states respectively. This is an obstruction to the holographic conservation of information in quantum gravitation, which leads to a firewall.

The 3-tangle is a topological obstruction for a pure state $|\psi_{ABC}\rangle$ written according to the matrix G^{ijk}. This matrix, an element of the unitary local group G_L, defines the SLOCC that transforms an initial state into a final state. This group, unlike unitary groups, is not necessarily compact and its orbits may not be closed or obey Cauchy sequence ordering. One orbit can sit within the closure of another in a complex and recursive topology. This is the sort of topology that is indicated by the loop-helix orbit structure of IST. The concurrence or 3-tangle is proportional to the hyperdeterminant of this matrix as

$$\tau_3(|\psi_{ABC}\rangle) \; = \; 4|Det_3(G)|.$$

This is an invariant under the local unitary operators and SLOCC $G_L \; = \; GL(2, \; \mathbb{C})^{\otimes 3}$. This conserves the distinguishable nature of GHZ and W states with $\tau_3|GHZ \rightarrow = \; 1$ the upper bound. For $SL(2, \; \mathbb{C})$ on the magic square of Freudenthal and Tits this is the 3-matrix of elements of the $\mathbb{C} \times \mathbb{C}$ entry. The octonions or octooctonians are the $\mathbb{O} \times \mathbb{O}$ entry. This cubic form is the Jordan matrix $J^3(\mathbb{O})$ with the Freudenthal determinant of eigenvalues [15].

The transformation of states for m systems of size n are given by momentum maps with the Kirwan polytope in $\mathbb{R}^{m(n-1)}$ as a map $\mathbb{C}P^{n^m-1‘} \rightarrow \; \mathbb{R}^{m(n-1)}$ [14]. For a given qubit $n \; = \; 2$ we have $\mathbb{C}P^1$ with the line bundle fibration with $\mathbb{C}$. The dynamics is $U(1)$, with $z \; = \; (q, \; p)$ variable(s). We then have for z^i a function $J_i \; = \; J_i(p, \; q)$ that defines the transformation of coordinates by

$$\delta z^i \; = \; \{z^i, \; \omega^k J_k\}, \; \delta J_i \; = \; \{J_i, \; \omega^k J_k\} \; = \; \epsilon_{ijk}\omega^k J^j,$$

where the last term is $\omega_{ij}J^j$. Here ω_{ij} is the transformation of an element ω_j by the action $ad_g\gamma, \omega^i$. Here $\gamma \; \in \; g^*$ a vector space dual to g.

$$\langle ad_g\gamma, \omega^i\rangle \; = \; \langle\gamma, \; g^{-1}\omega^1 g\rangle \; = \; \langle\gamma, \; g^{-1}\omega^i g\rangle$$

For $g \; = \; e^{i\omega}$ is then equal to

$$\langle ad_g\gamma, \omega^i\rangle \; = \; \langle\gamma, \; \omega^i\rangle \; + \; i\langle\gamma, \; [\omega, \; \omega^i]\rangle,$$

With an implicit summation over indices with ω. This defines a Hamitlonian vector field or symplectic form, and is a commutator element of a Lie algebra. For a single qubit this is $SU(2) \; \simeq \; SO(3)$, and the orbit is generated by $U(1)$. These are the loops and helices of IST.

The momentum map acts in a symplectic manner on elements x in a manifold $\mathcal{M}$ in a dynamics $x \; \rightarrow \; \Phi_g(x)$. The momentum map $\mu : \mathcal{M} \; \rightarrow \; g^*$ with

$$\mu(\Phi_g(x)) \; = \; ad_g(\mu(x)),$$

where the vector field of the Lie algebra is

$$\omega(x) \; = \; \frac{d}{dt}\Phi_{e^i\omega t}(x)|_{t\rightarrow 0}.$$

This Hamiltonian vector field describes the evolution of $\mu_\omega = \langle \mu(x),\ \omega\rangle$ which conserves the Kirwan polytope, or upholds the monogamy principle, and physically defines the evolution of entanglement.

These dynamics have interpretations according to Morse theory, Floer cohomology and the dynamics of gauge fields. It suggests a duality or complementary principle between entanglement and gauge fields. This fits in well with the Jacobson and Raamsdonk thesis on the emergence of spacetime from entanglements of states. Entanglements and gauge symmetries are dual, complements or equivalent aspects to physics. This is similar to the relationship between topological ordered states, that have long range entanglements, and symmetry protected states with local entanglements. There is a prospect for a duality between local physics of gauge interactions and nonlocal physics of entanglements. The connection to the $J^3(\mathbb{O})$ means entanglement and the Chern-Simons Lagrangian over $\mathbb{O}$ are connected in some duality with supergravity. See supplementary material for more.

3 Morse indices and entanglement

Exceptional eigenvalues and curvatures can derive topological indices and critical sets for the occurrence of entanglements. The value of an entanglement is given by the extremal condition on a quantum overlap. A basic quantum mechanical derivation is with the uncertainty spread. Given the quantum state $|\psi(t)\rangle$ its evolute overlaps with this by

$$\langle\psi(t)|\psi(t\ +\ \delta t)\rangle\ =\ \langle\psi(t)|\psi(t)\rangle\ +\ \langle\psi(t)|\frac{\partial}{\partial t}\psi(t)\rangle\delta t\ +\ \frac{1}{2}\langle\psi(t)|\frac{\partial^2}{\partial t^2}\psi(t)\rangle\delta$$

$$=\ \langle\psi(t)|\psi(t)\rangle\ -\ i\langle\psi(t)|H\psi(t)\rangle\delta t\ +\ \frac{1}{2}\langle\psi(t)|H^2\psi(t)\rangle\delta t^2,$$

so that $|\langle\psi(t)|\psi(t\ +\ \delta t)\rangle|^2\ =\ \sqrt{\langle H^2\rangle\ -\ \langle|H|\rangle^2}\delta t^2$. This is the uncertainty spread in energy ΔE a quantum system in a pure state, multiplied by the spread in time δt. This is invariant for a pure state as the Fubini-Study metric. This is a measure of a quantum phase or entanglement. The momentum map derives a form of this overlap as a variance. The momentum map is the coadjoint action $\mu([gv])\ =\ Ad^*_g\mu([v])$ for $g\ \in\ K\ =\ exp(\mathfrak{k})$ that maps the symplectic manifold $\mathcal{M}\ =\ (M,\ \omega)$ into the Cartan subalgebra, which is the K coadjoint orbit in $\mathcal{M}$ or $\mu : \mathcal{M}\ \rightarrow\ \mathfrak{k}^*$. These orbits intersect the positive Weyl chamber $\mathfrak{t}_+$ so that the orbit space there is $\phi : \mathcal{M}\ \rightarrow\ \mathfrak{t}_+$ such that $\phi(x)\ =\ \mu(K\cdot x)\cap\mathfrak{t}^*_+$. The reflection points define a Kiewan polytope, with vertices corresponding to the entanglement group.

The lie algebra of K has the adjoint and coadjoint representations on $i\mathfrak{k}$ and $i\mathfrak{k}^*$ corresponding to the elements $\langle x|\ \xi\rangle\ =\ x^\dagger\cdot\xi$ as the invariant inner product. The momentum map is then

$$\mu^*(x)\ =\ \sum_{n=1}^{dim\ K}(\mu^*(x))^\dagger\cdot\xi_n\xi_n\ =\ \sum_{n=1}^{dim\ K}\mu^*_{\xi_n}(x)\xi_n$$

The modulus square of the momentum map is

$$||\mu(x)||^2\ =\ (\mu^*(x))^\dagger\cdot\mu^*(x)\ =\ \frac{1}{2}\sum_{n=1}^{dim\ K}\left(\frac{v^\dagger\cdot\xi v}{v^\dagger\cdot v}\right)^2$$

The physical meaning of $||\mu(x)||^2$ is equivalent to $\langle|H|\rangle^2$ in the overlap of a wave function. With respect to the group K the overlap is a variance $\sigma(v)$

$$\sigma(v)\ =\ \frac{\sum_n v^\dagger\cdot\xi_n^2 v}{|v|}\ -\ \frac{\sum_n(v^\dagger\cdot\xi_n v)^2}{|v|^2},$$

which is identical in form of the quantum uncertainty definition $\Delta E \Delta t \ = \ \hbar$. This is expressed according to the momentum map as

$$\sigma(v) \ = \ \frac{\sum_n v^\dagger \cdot \xi_n^2 v}{|v|} \ - \ 4||\mu(x)||^2(v)$$

where the first term is the evaluation of the Casimir operator. The meaning of this operator with respect to the reflection on the Weyl chamber then determines the SLOCC group for the entanglement.

The reflections correspond to the extremal condition on $\Sigma(v)$. This is defined according to a Morse index. The extremal condition is then computed by looking at the differential of $||\mu(x)||^2$

$$d||\mu(x)||^2 \ = \ 2\sum_{n=1}^{dimK} \mu^\dagger_{\xi_n}(x) \cdot d\mu_{\xi_n}(x).$$

Since $d\mu_{\xi_n} \ = \ \omega(-i\xi_n, \ \cdot \ )$, The evaluation of this on a vector $i\xi^{'}$ gives $d\mu_{\xi_n} i\xi' \ = \ i\omega(-i\xi_n, \ \xi')$. This is then by $tr(\mathbf{A} \circ \mathcal{P}_\lambda) \ = \ 2\omega(\xi, \ \xi')$ for this case with a sum on $n \ = \ 1, \ \ldots, \ dim\ K$ is a set $Ric(\xi_n, \ \xi^{')} = \lambda g(\xi_n, \ \xi^{')}$. The Hessian of $||\mu(x)||^2$ is then

$$H||\mu(x)||^2 \ = \ 2\sum_{n=1}^{dimK} d\mu^\dagger_{\xi_n}(x) \otimes d\mu_{\xi_n}(x) \ + \ 2\sum_{n=1}^{dimK} \mu^\dagger_{\xi_n}(x) \cdot H\mu_{\xi_n}(x).$$

A minimal critical point is evaluated on $\mu_{\xi_n}(x) \ = \ 0$, which means that for all n the second term vanishes. The general result is that

$$H||\mu(x)||^2 \ = \ 2\sum_{n=1}^{dimK} d\mu^\dagger_{\xi_n}(x) \otimes d\mu_{\xi_n}(x) \ + \ 2||\mu(x)||H\mu_\xi$$

The eigenvalues are further realized from $\mu^*(v)v \ = \ \lambda v$, which are associated with the Ricci curvature and symplectic form. See supplementary material for more.

4 p-adic locality and undecidability of IST quantum fractal

The role of loops and helicies in entanglement is laid bare, where these are also an aspect of IST. If these orbits are recursive they may have a fractal type of geometry. This geometry in IST leads to a nonlinearity, which QM is not well suited to. There is then some correspondence between quantum states and a nonlinear system. This is like the complementarity of Bohr, which correlates with the concept of spacetime physics, can be nonlinear and built up from entanglements. This fractal dynamics is best worked with p-adic number theory, which has a way of "flipping" any series that is very divergent or where a metric separation approaches "∞" to a form that is finite. It is a way of working with metric spaces and corresponds to solutions to polynomials or equivalently Diophantine equations.

The Cantor set, such as the standard derived with the removal of the middle third of an interval, and then the iteration with the remaining intervals leads to a set that is "dust." This splitting of the interval is a $\mathfrak{C}_2$ set, which can be studied with 2-adic numbers. The Cantor set could describe a hopping system, where a particle can hop between the first two intervals or probabilities $(1/3, \ 2/3)$, then with the next iteration hopping between intervals $1/9, \ 3/9, \ 6/9, \ 8/9)$ and so forth. A system with a set of bifurcating frequencies might also suffice. This is physically what happens with these cycles and helicies, where there is a branching bifurcation of frequencies or frequency multiplications in a system. Given any $x, \ x' \in \ \mathfrak{C}_2$ elementary operations on these do not lie in the $\mathfrak{C}_2$, and any operations between these two elements is impossible to define in a standard manner. However, in a 2-adic setting this set can be closed under operations of $+,\ \times$ and their inverses as a proper mathematical field. For any periodicity or equivalently $\mathfrak{C}_n$ then for a prime decomposition of the Cantor set a p-adic number system may be used.

The momentum map with an action K, or group action with a lattice space, on a space s with symplectic form ω gives a set of Kähler forms ω_i define the curvature $-2\pi i\omega_i$ on a line bundle L_i. Let σ be a holomporphic section of a line bundle, n_i a positive integer then for $s_i\ \in\ \sigma(s, L_i)^K$ the fibre metric is $\langle s_i,\ s_j\rangle\ =\ |s_i|^2\delta_{ij}$. The Lie derivative on this metric with V is then

$$\mathcal{L}_V\langle s_i,\ s_j\rangle\ =\ -4\pi n_i\Phi^V_i\langle s_i,\ s_j\rangle,$$

with $V\ =\ \nabla\Phi^V$. This holds for each Kähler form ω_i for ω then $s\ =\ s_1^{n/n_1}\otimes s_2^{n/n_2}\otimes\cdots\otimes s^n/n_m$, $n\ =\ n_1n_2\dots n_m$ and define a function $f=\ \langle s_1,\ s_1\rangle^{x_1/n_1}\langle s_2,\ s_2\rangle^{x_2/n_2}\dots\langle s_m,\ s_m\rangle^{x_m/n_m}$. The general Lie derivative is then

$$\mathcal{L}_V(f)\ =\ -4\pi\sum_{i=1}^m a_i\Phi^V_i$$

Kirwan found that for the Fubini-Study metric semi-stability is equivalent to semi-stability for integer Käler metric [14].

The integers define the lattice space for a Weyl chamber or Kirwan polytope. The solution to a polynomial equation on K then defines the Kirwan polytope or the roots of a Weyl chamber. However, for the fractal case here, this polygon can only be solved in a p-adic number system. In general, with the equation above we have a summation of eigenvalued-like equations, which hold for n a prime decomposition. The Riemann ζ-function by hypothesis defines the distribution of prime numbers, and so the fundamental quantum numbers of the universe are defined accordingly.

This is where undecidability enters the picture. Hilbert's 10^{th} problem was to find a global solution to all possible Diophantine equations. These polynomials have their equivalence with Diophantine equations. The $MDPR$ theorem illustrates how the set of prime numbers is a Diaphantime set, and Matiyasevich proved in general there is no global solution method for Diophatine equations [16]. Consequently, such solutions can only be locally defined or by specific instances.

Consequently, it is not physically possible to extend the locality of any fractal set to another in some global manner, This connects us with the nature of the 3-tangle; $\tau_1(|\psi_{ABC}\rangle$ and $\tau_2(|\psi_{ABV}\rangle)$ have independent polytopes and thus separate primes or Diophantine sets. The two in general are not computed in equivalent ways, and any nonlinear fractal set for measurements with a GHZ state is different from a W state. This topological obstruction that prevents a bipartite state from evolving into a tripartite state is a consequence of incompleteness of Diophantine sets to any global solution. This may be generalized for a quadpartite entanglements as well[citekey-17, which may be generalized further.

5 Undecidability at the heart of the quantum

This started with the two-slit experiment. Feynman once said that all of quantum physics might be found there. The paths through the two slits from the source to a spot on the screen form a loop with a nontrival homology and homotopy. The two slits form a topological obstruction that defines the superposition. Then with a needle state this is converted to entanglement; the quantum phase for the superposition is converted to entanglement. The 3-tangle is an inherent property of what is quantum, what separates one type of entanglement from another and is tied to undecidability of propositions associated with fractal sets of paths. Solutions to Diophantine equations, which are associated with the nested frequencies or periodicities of orbits, can be solved locally, but there is no global solution method. This is a form of Szangolies' epistemic horizon [6]. The Cantor set here then has some undecidable properties in the p-adic setting, where any global field of numerical operations in a p-adic setting is incomplete.

Wheeler with his question, "Why the quantum," pondered whether quantum mechanics was built from more elementary nuts and bolts. In the old Oxford symposium book on quantum gravitation Wheeler speculated quanta were built from some undecidability of an elementary system [18]. Wheeler was thrown out of Gödel's office for asking this question, and for anyone who has pondered this and deigned to mention this, it is often greeted with disapproval. Szangolies presents arguments for undecidability by considering an elementary model and a Cantor diagonalization [6]. This leads to a form of information barrier or epistemic horizon. In this paper it is argued that different entanglements are obstructed away from each other. This obstruction is fundamentally the same as the epistemic horizon is what keeps two entanglement types topologically separate. In what is presented here the door is further opened into not just the underlying structure of quantum mechanics, but possibly quantum gravitation as well.

With a measurement there is a growth of entanglement, which accompanies a process such as the avalanche of electrons in a photoelectric tube. This results in a large N-tanglement, and the above result illustrates there is no comprehensive system for a solution set. This illustrates there are limits to what can be predicted and information stored. The holographic black hole is also a large N-qubit system that has entanglement, and this too has limits. This limit entails these topological obstructions, such as the 3-tangle. This means the conservation of quantum information and the equivalence principle should be seen in a different light. Spacetime built from entanglements or QM equivalent to GR means conservation of quantum information and the equivalence principle are either equivalent themselves or are in some duality with each other.

6 Supplementary Material

Consider the exceptional Jordan eigenvalue problem $\mathbf{A}v ~=~ \lambda v${key-19. This is evaluated with the Jordan product $\mathbf{A}\circ\mathbf{B} ~=~ \frac{1}{2}\mathbf{AB} ~+~ \mathbf{BA}$, so that for $\mathbf{V} ~=~ vv^\dagger$ $\mathbf{A}\circ\mathbf{V} ~=~ \lambda\mathbf{V}$. From the Jordan product the Freudenthal product is defined

$$\mathbf{A}*\mathbf{B} ~=~ \mathbf{A}\circ\mathbf{B} ~-~ \frac{1}{2}\left(\mathbf{A}Tr(\mathbf{B}) ~+~ \mathbf{B}tr(\mathbf{A})\right) ~+~ \frac{1}{2}\left(tr(\mathbf{A})tr(\mathbf{B}) ~-~ tr(\mathbf{A}\circ\mathbf{B}).\right)$$

Now build up the characteristic equation

$$\mathbf{A}^3 ~-~ (tr\mathbf{A})\mathbf{A}^2 ~+~ \sigma(\mathbf{A})\mathbf{A} ~-~ (det\mathbf{A})\mathbf{I},$$

for $\sigma(\mathbf{A}) ~=~ tr(\mathbf{A}*\mathbf{A})$. This cubic matrix equation for the matrix $\mathbf{A}v ~-~ \lambda v$ gives

$$-det(\mathbf{A} ~-~ \lambda\mathbf{I}) ~=~ \lambda^3 ~-~ (tr\mathbf{A})\lambda^2 ~+~ \sigma(\mathbf{A}) ~-~ (det\mathbf{A})\mathbf{I}$$

which vanishes for λ eigenvalues of the matrix $\mathbf{A}$. For real eigenvalues this equation is not zero, but rather $det(\mathbf{A} - \lambda\mathbf{I}) ~+~ z ~=~ 0$.

The matrix $\mathbf{A} ~=~ (u,~ v,~ w)$, composed of the three column vectors, transformations of a vector $u ~\mapsto~ u ~+~ \delta u$ are given by

$$\xi ~=~ \frac{d}{dt}X(t)\big|_{t=0} ~=~ i\frac{de^{i\xi t}}{dt}\big|_{t=0}$$

where the map is $v ~\rightarrow~ v ~+~ \xi v\delta t$. The variation in the vector $v^{'} ~=~ v ~+~ \xi v$ means the operator X on $V ~=~ v^\dagger v$ is

$$V^{'} ~=~ Xvv^\dagger X ~\simeq~ (1 ~-~ i\xi\delta t)vv^\dagger(1 ~+~ \xi\delta t)$$

for δt small. The variation in the matrix $\delta V ~=~ V^{'} ~-~ V$ gives

$$\frac{\delta V}{\delta t} ~=~ -i[\xi,~ V],$$

which in the limit $\delta t \;\to\; 0$ gives

$$\frac{\partial V}{\partial t} \;=\; i[V,\; \xi],$$

The trace of this derivative $tr(\frac{\partial V}{\partial t}) \;= v(\frac{\partial V}{\partial t})v^\dagger$ vansishes

$$tr\left(\frac{\partial V}{\partial t}\right) \;=\; i(v^\dagger \xi v \;-\; v^\dagger \xi v)v^\dagger v \;=\; 0$$

With two variations X_{ξ_1} and X_{ξ_2} it is easy to compute a second order equation based on variations of the vector v along ξ_1 and ξ_2

$$tr\left(\frac{\partial_{\xi_1}\partial_{\xi_2}}{\partial t}V\right) \;=\; [[\xi_2,\; V],\; \xi_1].$$

The Jacobi identity shows this vanishes. The annulment of $[[\xi_1,\; \xi_2],\; V] \;=\; 0$ illustrates V is in involution with the symplectic matrix

$$\omega([\xi_1,\; \xi_2]) \;=\; \frac{iv^\dagger[\xi_1,\; \xi_2]v}{2v^\dagger v}.$$

and constant with respect to variations with respect to V, or $\partial_v\omega \;=\; 0$. Thus the phase space volume is invariant, and in a quantum mechanical setting it is equivalent to unitarity.

The replacement of the vector $\xi_2 \;\to\; i\xi_2$ leads to

$$\omega([\xi_1,\; i\xi_2]) \;=\; \frac{v^\dagger\xi_2 v v^\dagger \xi_1 v}{v^\dagger v} \;=\; g(\xi_1,\; \xi_2)$$

which is the metric of the gauge-like space that parameterizes these motions. Also the variation in the symplectic form translates into $dg \;=\; 0$ so the metric is covariantly constant.

This machinery is established to look at the dynamics of quantum entanglements. A quantum entanglement group is defined at a certain minimum region that has a Morse index. The above dynamics permits us to look at the quantum entanglements according to topological indices or think of $\xi \;\in\; \mathfrak{k}$. The momentum map and this vector projected onto each other is

$$\langle dp_v^\dagger|\xi\rangle \;=\; \frac{\langle e^{-i\xi t}v|e^{i\xi t}\rangle}{\langle v^\dagger v\rangle}\bigg|_{t\to 0} \;=\; i\langle v|\xi v\rangle \;+\; -i\langle \xi v|v\rangle \;=\; 0$$

and for $\xi \;\to\; i\xi$

$$\langle dp_v^\dagger|i\xi\rangle \;=\; 2i\langle v|\xi v\rangle$$

which the momentum map [14] redefines as

$$\mu(v) \;=\; \frac{1}{2}\frac{i\langle v|\xi v\rangle}{\langle v|v\rangle} \;=\; \frac{1}{2}\frac{iv^\dagger\xi v}{v^\dagger v} \;=\; \frac{1}{4}\frac{dp_v}{v^\dagger v}.$$

A similar calculation for $\pi_v \;=\; (gv)(gv)^\dagger$, a matrix and for $g \;=\; e^{i\xi t}$, leads to a similar result that

$$\frac{d\pi_v}{dt}\bigg|_{t\to 0} \;=\; 2i\left(\xi v v^\dagger \;+\; v(v\xi)^\dagger\right) \;=\; \frac{vv^\dagger \circ (\xi v)(\xi v)^\dagger(\xi v)^\dagger}{v^\dagger \cdot (\xi v)}$$

For $u \;=\; \xi v$ this is the identity for the Jordan product $2uu^\dagger \circ vv^\dagger \;= (u^\dagger \cdot v)(uv^\dagger \;+\; vu^\dagger)$

This matrix is a projector operator and the eigenvalues for exceptional algebras can be constructed. The Freudenthal product between two projectors is

$$uu^\dagger * vv^\dagger \;=\; uu^\dagger \circ vv^\dagger \;-\; \frac{1}{2}\left(uu^\dagger\; tr(vv^\dagger) \;+\; vv^\dagger\; tr(uu^\dagger)\right) \;+\; \frac{1}{2}\left(tr(uu^\dagger)tr(vv^\dagger) \;-\; tr(uu^\dagger \circ vv^\dagger)\right)\mathbf{I}$$

where $tr(uu^\dagger \circ vv^\dagger) \;=\; (u^\dagger \cdot v)^2$. This is equivalent to $(uu^\dagger * vv^\dagger) \;=\; (u \times v)(u \times v)^\dagger$. With the transformation matrix ξ this is also $(uu^\dagger * vv^\dagger) \;=\; (u \times \xi u)(u \times \xi u)^\dagger$. The Jordan product $\mathbf{A} \circ \mathbf{V}$ gives the eigenvalue equation

$$\mathbf{A}vv^\dagger \;+\; vv^\dagger\mathbf{A} \;=\; 2\lambda vv^\dagger$$

where we consider the matrix $\mathbf{A} \;=\; d\xi \;+\; [\xi,\; \xi^{'}]$. The trace of this equiaton then results in

$$\frac{v^\dagger d\xi v}{v^\dagger v} \;+\; \frac{v^\dagger[\xi,\; \xi^{'}]v}{v^\dagger v} \;=\; \lambda$$

where the right hand side gives the symplectic form $\omega(\xi,\; \xi^{'})$. For the replacement $\xi^{'} \;\rightarrow\; i\xi^{'}$ this defines the metric $g(\xi,\; \xi^{'}) \;=\; \lambda$, and the Ricci tensor is $Ric(\xi.\; \xi^{'}) \;=\; \lambda g(\xi,\; \xi^{'})$.

The determinant of the eigenvalue equation $\mathbf{A} \;-\; \lambda\mathbf{I}$

$$det\,(\mathbf{A} \;-\; \lambda\mathbf{I}) \;=\; (\mathbf{A} \;-\; \lambda\mathbf{I}) \circ ((\mathbf{A} \;-\; \lambda\mathbf{I}) * (\mathbf{A} \;-\; \lambda\mathbf{I}))$$

is expressed according to the projector $\mathcal{P}_\lambda \;=\; (\mathbf{A} \;-\; \lambda\mathbf{I}) * (\mathbf{A} \;-\; \lambda\mathbf{I}) = \; v^\dagger v/(v^\dagger v)$ as

$$det\,(\mathbf{A} \;-\; \lambda\mathbf{I}) \;=\; (\mathbf{A} \;-\; \lambda\mathbf{I}) \circ \mathcal{P}_\lambda$$

The trace $tr(\mathbf{A} \circ \mathcal{P}_\lambda) \;= 2\omega(\xi,\; \xi^{'})$ and for $\xi^{'} \;\rightarrow\; i\xi^{'}$ this is the Ricci scalar curvature $tr(\mathbf{A} \circ \mathcal{P}_\lambda) \;= R(\xi,\; \xi^{'})$ The projectors $\mathcal{P}_\lambda$ and $\mathcal{P}_{\lambda^{'}}$ are orthogonal. We have then that

$$\mathcal{P}_\lambda \circ (\mathbf{A} \circ \mathcal{P}_{\lambda^{'}}) \;=\; (\mathcal{P}_\lambda \circ \mathbf{A}) \circ \mathcal{P}_{\lambda^{'}} \;\rightarrow\; \lambda\mathcal{P}_\lambda \circ \mathcal{P}_{\lambda^{'}} \;=\; \lambda^{'}\mathcal{P}_\lambda \circ \mathcal{P}_{\lambda^{'}}.$$

However, $\lambda \;\neq\; \lambda^{'}$ which leads to $\mathcal{P}_\lambda \circ \mathcal{P}_{\lambda^{'}} \;=\; 0$. This is the orthogonality condition on the projector operators. It is now possible to compute $tr\,((\mathbf{A} \;-\; \lambda\mathbf{I}) \circ \mathbf{P}_\lambda)$,

$$tr\,((\mathbf{A} \;-\; \lambda\mathbf{I}) \circ \mathbf{P}_\lambda) \;=\; R(\xi,\; \xi^{'}) \;-\; 2\lambda tr(\mathbf{A}) \;+\; 3\lambda^2.$$

This is the 2×2 eigenvalued equation. The Ricci scalar is equal to $\frac{1}{2}\left(tr(\mathbf{A})^2 \;-\; tr(\mathbf{A})^2\right)$, which is the same form as the Ricci curvature scalar for the extrinsic tensor $K \;=\; dn$ in ADM general relativity. This equation leads to the final Freudenthal determinant or characteristic equation for the 3×3 octonionic eigenvalued problem

$$-det\,((\mathbf{A} \;-\; \lambda\mathbf{I})) \;=\; \lambda^3 \;-\; tr(\mathbf{A})\lambda^2 \;+\; R(\xi,\; \xi^{'})\lambda \;-\; det(\mathbf{A})$$

The 3×3 matrix $\mathbf{A}$

$$\mathbf{A} \;=\; \begin{pmatrix} x_1 & \mathbb{O}_1 & \mathbb{O}_2^* \\ \mathbb{O}_1^* & x_2 & O_3 \\ O_2 & \mathbb{O}_3^* & x_3 \end{pmatrix},$$

where $\mathbb{O}_1,\; \mathbb{O}_2,\; \mathbb{O}_3 \;\in\; J^3(\mathbb{O})$. The trace and determinant conditions are

$$tr(\mathbf{A}) \;=\; x_1 \;+\; x_2 \;+\; x_3$$

$$\sigma(\mathbf{A}) \;=\; \frac{1}{2}\left((tr(\mathbf{A})^2 \;-\; tr(\mathbf{A}^2)\right) \;=\; x_1x_2 \;+\; x_2x_3 \;+\; x_3x_1 \;-\; |\mathbb{O}_1|^2 \;-\; |\mathbb{O}_2|^2 \;-\; |\mathbb{O}_3|^2$$

$$det(\mathbf{A}) \;=\; x_1x_2x_3 \;+\; \mathbb{O}_1(\mathbb{O}_3\mathbb{O}_2) \;+\; \mathbb{O}_2^*(\mathbb{O}_1^*\mathbb{O}_3^*) \;-\; x_1|\mathbb{O}_3|^2 \;-\; x_2|\mathbb{O}_2|^2 \;-\; x_3|O_1|^2$$

The determinant characteristic equation is then in general given by $det(\mathbf{A} \;-\; \lambda\mathbf{I}) \;=\; -x$, where x is one of two roots of the equation

$$x^2 \;+\; 2Re\,([O_1,\; \mathbb{O}_2]\mathbb{O}_3) \;-\; \left|[\mathbb{O}_1,\; \mathbb{O}_2,\; \mathbb{O}_3]\right|^2$$

where $[\mathbb{O}_1,\ \mathbb{O}_2,\ \mathbb{O}_3] \ = \ (\mathbb{O}_1\mathbb{O}_2)\mathbb{O}_3 \ - \ \mathbb{O}_1(\mathbb{O}_2\mathbb{O}_3)$. The roots are

$$x \ = \ Re\,([\mathbb{O}_1,\ \mathbb{O}_2]\mathbb{O}_3) \ \pm \ \sqrt{Re\,([\mathbb{O}_1,\ \mathbb{O}_2^*]O_3)^2 \ - \ \big|[\mathbb{O}_1,\ \mathbb{O}_2,\ \mathbb{O}_3]\big|^2}$$

which are zero only if $Re\,([\mathbb{O}_1,\ \mathbb{O}_2^*]\mathbb{O}_3)^2 \ = \big|[\mathbb{O}_1,\ \mathbb{O}_2,\ \mathbb{O}_3]\big|^2 \ = \ 0$. The real valued x determines real eigenvalues for the 3×3 octononic matrix. This matrix with the three 8×8 E_8 matrices is then a 24×24 matrix with real eigenvalues.

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