

# Unitarity, locality and spacetime geometry: Foundations that are not foundations

Lawrence B. Crowell

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Alpha Institute of Advanced Study 10600 Cibola Lp 311 NW Albuquerque, NM 87114 also 11 Rutafa Street, H-1165 Budapest, Hungary, email: [lcrowell@swcp.com](mailto:lcrowell@swcp.com);

This essay discusses a possible route towards the removal or deformation of existing physical postulates. This is looked at in light of history. The foundations of interest are unitarity and locality. How these principles are changed or abandoned is first examined in light of previous changes in the understanding of physical foundations. These foundations are examined to question their firmness, and if they give way then unto what do they submit to as emergent properties. Suggested approaches are then proposed within light of the AdS/CFT correspondence, nonlocal BCFW amplitudes in QCD, cosmological quantum phase structure and ultimately the replacement of unitarity by deeper principles of modularity.

## 1 Foundations and Principles

Newton laid down his laws of physics as axiom of natural philosophy. It was long held that these laws were carved into existence, often thought by God. Newton was thought to have found the most fundamental secrets to the universe from 1683 up to Einstein's Annus Mirabilis in 1905. However, prior to then other physics were found, in particular the field theory of electromagnetism and thermodynamics that augmented Newton's laws. In the early 20<sup>th</sup> century changes in the foundations of physics became sufficiently profound that Newton's laws were found to be restricted cases of far more general principles.

Newton's laws of motion were laid down in the third book of the *Principia* called *De Mundi Systemate (On the system of the world)*[1]. These are codified in the three laws of motion.

Newton's first law of motion: A body in a state of motion will remain in that state of motion unless acted upon by a force. This introduces the inertial

reference frame where no external forces act upon it, from which one observes a body at rest there remaining at rest, or that an accelerating body is subject to a force.

Newton's second law of motion: The acceleration is directly proportional to a force applied to it. The momentum of a body is its mass times its velocity  $p = mv$  as determined by an inertial observer. The time rate of change of the momentum is given by calculus as  $dp/dt = ma$ .

Newton's third law of mechanics: Whenever one body exerts a force on a second body, the second body exerts a force of equal magnitude on the first in the opposite direction. Consequently physics operates in homogeneous and isotropic space with translation and rotation symmetry, and forces between bodies are independent of position and orientation in space.

Einstein changed Newton's laws by adjusting the first and third laws, motivated by the locality of electromagnetic fields predicted by Maxwell's equations. Einstein recognized field locality mandated a change in the interpretation of Newton's laws, though not their abandonment! The third law is a symmetry principle of space, which Einstein changed to include time in spacetime. The symmetry is four dimensions are rotations with Lorentz boosts. General relativity changed the meaning of the first law according to the equivalence principle.

The far bigger change was quantum mechanics. Newtonian mechanics was found by Maupertuis and Lagrange in the middle 18<sup>th</sup> century to be equivalent to the minimal variation of  $L = T - U$ , for  $T$  the kinetic energy and  $U$  potential energy. The variation of the action  $S = \delta \int L dt$  is zero, called the principle of least action, gives the Euler-Lagrange equation of motion. The motion of a particle executes little variations to find the proper path. The principle is not deterministic, but determinism emerges because there is only one path which obeys Newton's laws. This was seen as strange, similar to teleology; a moving particle or system does some sort of sampling of paths and ends up in the one correct path that is deterministic. Quantum mechanics introduced a wave mechanics for particles on a small scale. A profound development in quantum mechanics was the path integral for a quantum field  $\phi$

$$Z[\phi] = \int \mathcal{D}[\phi] e^{-iS(\phi)},$$

where computations require functional derivatives similar to calculus of variations. A particle does indeed sample paths in a quantum superpositions of different paths [2].

Often considerable changes in physics are not a radical abandonment of foundations, but rather reinterpretations of them. These principles may assume very different forms, such as the symmetries implied by Newton's third law, and the corresponding Lorentz symmetry of spacetime. The most radical shift is from the variation of the action in classical physics and the path integral in the quantum mechanics of fields.

## 2 Modern foundations and their deformations

The coupling constant  $G$  of general relativity (GR) with units of area, or  $G^{1/2}$  with units of inverse mass, while quantum field theories (QFT's) are unitless coupling constants in naturalized units. Further, general relativity is nonlinear with a Lorentzian metric signature. The moduli space of all possible frames, or “gauge-like conditions,” is not Hausdorff. This means solutions do not obey nice regular conditions. In addition the theory is non-renormalizable. The gravity field is spacetime, where the quantization of gravity implies the quantization of spacetime. QFTs are constructed in order to compute propagators of fields in spacetime. However, constructing a propagator for a field on that very same field makes quantum gravity peculiar and intractable. These are difficulties with the locality of fields and their causal structure.

GR and QFT have different definitions of time. The invariance of the four dimensional path length defines time in GR. This is the proper time a clock on the path ticks off. Coordinates may change by a change of frame, say by a Lorentz boost; standard coordinates are chosen by an observer or analyst and are artifacts. Quantum field theory works with differential wave equations, which establish at every point on a spacetime a harmonic oscillator solution to that wave equation [3]. To solve these equations one must fix field amplitudes on a spatial surface in spacetime. The amplitudes at each point are independent and thus commute, or  $[\phi(x), \phi(x')] = \delta^3(x - x')$ . This fixes the initial data of a quantum field on a spatial slice in spacetime. At each point on that spatial surface is a time direction pointing out of the spatial surface, which gives the evolution of the spatial surface in coordinate time. These data and wave equation describe the evolution of the quantum field.

The quantization of spacetime means observables are no longer local. The diffeomorphism of quantized spacetime fields are not invariant according to commutative local structures. The light cone at any point is subject to quantum fluctuations. Consequently the point where all null rays pass through is indeterminate; null rays in the region are not connected to a unique point. The light cone in effect disappears as the scale approaches the Planck scale  $L_p = \sqrt{G\hbar/c^3}$ .

The central issues that need consideration are the nature of time, locality and the nature of causality. It would be preferable if any of these must be changed there be no or minimal change in the symmetries of spacetime, or those of supergravity. This happened with the transition from the principle of least action in classical mechanics to the quantum path integral. If a 19<sup>th</sup> century physicist had conjectured there was a statistical aspect to the principle of least action it is likely this person would have worked up something similar to the Langevin theory of Brownian motion. In this case there would likely have been small random impulses proposed that randomly moves a particle on a small scale. This would not quantum mechanics in any natural or realistic way. We do not want to follow this same mundane path. However, we do have potentially a bird in hand, so to speak, just as the classical physicists had the principle of least action.

### 3 What we cannot speak we must learn to talk

This is a counter-statement to Wittgenstein's [4] "Wovon man nicht sprechen kann, darüber muß man schweigen." If spacetime turns out to obstruct further developments this is forced upon us. If our postulates obstruct us we do not necessarily have the machinery to think, work and talk about these problems. However, just as with the transition from the least action principle to quantum physics it turns out the seeds were planted in the ground. The question is what indicators are there this is happening?

It is likely that spacetime is not a complete concept. The "fuzzing out" of light cones as the Planck scale is approached is a convenient heuristic. To probe nature on smaller scales requires larger energy  $\Delta p \simeq \hbar/\Delta x$  or  $\Delta E \simeq \hbar/\Delta t$ , which happens until you reach  $L_{pl} = \sqrt{G\hbar/c^3}$ . The Planck scale is where the quantum wavelength equals the Schwarzschild radius and there is a quantum of a black hole. Interaction energy or transverse momentum corresponding to a wave length comparable to the Planck length produces a quantum of black hole with Planck mass  $M_{pl} = \sqrt{\hbar c/G}$ . If you increase energy further you just create a black hole made of many Planck masses. It is not possible to probe any observable to arbitrary degree of accuracy, for at some point the measurements only produce black holes which conceal any wanted information. Observables cannot be detected to arbitrary accuracy, and further the observable to such a sufficiently high degree of accuracy simply does not exist. A measuring apparatus meant to measure an observable to great accuracy, or to a small scale, must be very massive, such as the LHC. If the apparatus is made of  $N$  atoms, or degrees of freedom, the size of the apparatus increases proportionately and the error in measurement decreases as  $e^{-N}$ . As this happens the number of degrees of freedom is proportional to  $N = M/M_{pl}$ , and for the mass contained in a region of dimension  $r$  once  $M \sim rc^2/2G$  the system can transition into a black hole. Spacetime itself is a barrier to the complete specification of an observable, which includes observables associated with spacetime itself. There are then no precise local observables, but information conservation demands an  $e^N$  exists beyond the observer's reach.

We often perform experiments from asymptotic infinity, just as S-matrix theory is based on sending something in from infinity and measuring things at infinity. The structure of quantum gravity has this property with the AdS/CFT correspondence[5]. Our world is on the boundary of an anti-de Sitter (AdS) spacetime, where the interior is quantum gravity and on the boundary is conformal quantum field data. This is the closest thing to a quantum theory of gravity, which admits  $\mathcal{N} = 4$  supersymmetry for the  $AdS_5$  in five dimensions. However, this invokes a standard notion of spacetime. The  $AdS_4$  spacetime is a solution to the Einstein field equations. So the geometric concept of spacetime remains in this picture. However, the imprecision in a measurement means an observer on the boundary is unable to make a precise measurement of the quantum gravity configuration of the interior. The  $e^{-N}$  error is in terms of the area of the boundary of the  $AdS_n$  spacetime of  $n$  dimensions  $A = \partial AdS_n$  defines  $N = A/G_n$ , for  $G_n$  the Newton gravity constant in  $n$  dimensions. The

number of degrees of freedom  $N$  or information bits is proportional to entropy  $S = kN$ . This equals the entropy of the universe as determined on the boundary. This implies a de Sitter horizon, as a de Sitter spacetime is a conformally flat spacetime which can exist on the  $AdS_n$  boundary. This loss of information by holography is countered by an  $e^S$  which accounts for the information which is not available to this observer. Thus the observer on the boundary with partial information is a holographic "plate" which contains all the information in the interior, which is this  $e^{+S}$ , with  $S > 0$  state for emphasis. This means that we are limited in the amount of information we can learn about the universe at large, which is due to quantum gravity.

This breakdown in the ability to measure everything about the universe means spacetime is not something which can be assigned to a perfect observable. Spacetime is due to the assignments we make to certain coarse grained observations of the world, but which are approximate and dependent upon something deeper with the symmetries of spacetime. This is maybe a subgroup, but where the geometric assignments we impose on these symmetries of spacetime are emergent. We must now identify what exist in our tool box of physics that might be emergent. These are likely then to be time, unitarity, locality and causality.

There is now an enormous amount of QCD data being compiled by the LHC detectors. Much of this data must be benchmarked against enormously complex Feynman diagrams of perturbative quantum calculations of scattering amplitudes. These calculations of gluon amplitudes have resulted in curious results that are simple and which do not have explicit reference to spacetime constructions. In 2005 the Britto Cachazo Feng Witten (BCFW) [6][7] recursion relationship was discovered for gluon amplitudes. Some Feynman diagrams which turn out to have remarkable properties, in particular they are not local in spacetime[8]. In addition there are infinite dimensional symmetries in these amplitudes, with conformal invariance. A process with two particles in and four out is a very complicated amplitude to compute using Feynman's rules, however for a process with gluons with  $(+, -)$  helicities in and  $(+, +, +, -)$  gluons out that there is an amplitude

$$\langle 1^+ 2^- 3^+ 4^- 5^+ 6^+ \rangle = \frac{\langle 2^- 4^- \rangle}{\langle 1^+ 2^- \rangle \langle 2^- 3^+ \rangle \langle 3^+ 4^- \rangle \langle 4^- 5^+ \rangle \langle 5^+ 6^+ \rangle},$$

which has no local content. The calculation is reduced to a quotient of simple processes. More general amplitudes may be calculated with the BCFW recursion. These amplitudes are invariant under conformal transformations where momenta are differences of positions. This is completely nonlocal. The QFT approach of imposing "arrows of time" on a spatial slice is no longer necessary. By the same reason the relativistic notion of a proper time is no longer fundamental.

Bern et al have found similar relationships with gluon amplitudes which are unitary. In addition their analysis considers the graviton as a form of gluon pair, or a quantum entanglement of gluons, or QCD gluon-like fields, into a

$spin = 2$  quantum state corresponding to a graviton [9]. This relies upon an identity called the Bern, Carrasco, Johansson (BCJ) duality, where gauge theory amplitudes may be arranged into a form where diagrammatic numerators obey identities isomorphic to Jacobi identities obeyed by color charges.

## 4 Possible strategies with physical postulates.

There are several postulates to examine. If this were strictly Boolean, where we turn certain postulates “off” and see what happens, this would be similar to turning on or off mathematical axioms, such as turning off Euclid’s fifth axioms to work up nonEuclidean geometries consistent with the other four axioms. However, physics is not so simple. The transition from Galilean relativity meant replacing the acceleration of a body as seen from an inertial frame as an invariant with the invariance of light speed. It is often that a restrictive physical postulate is replaced by one which has more universal content. This is often forced upon us by circumstances with known physics, such as Maxwell’s equations and the speed of light, which conflicts with other physics, such as classical mechanics.

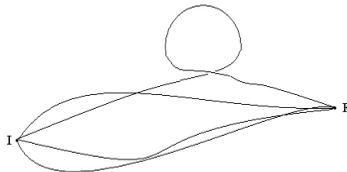
Cardenas proposed inflation preserves the holographic principle. In [10] it is demonstrated how a closed FLRW spacetime cosmology with  $k = 1$  upon turn around to recollapse exceeds the entropy bound of Bekenstein and the holographic principle. Crowell [11] proposed from this quantum cosmologies with  $k = 1$  exhibit a quantum phase transition. This transition changes the topology of the evolving space from a sphere to a flat space in a de Sitter vacuum with exponential expansion, or equivalently inflation. Quantum fluctuations have a scale of disorder in a system given by the Euclideanized time  $\tau = it$  equivalent to a disorder of a system due to thermal fluctuations at temperature  $T$ . Equating the quantum phase and the Boltzmann factor  $exp(-E\tau/\hbar) \simeq exp(-E/kT)$ , results in the scale  $\tau = \hbar/kT$  equivalent to the reciprocal of the temperature. There may be a quantum critical point with these fluctuations [12]. When the holographic bound is exceeded a phase transition occurs which shifts the universe into an inflation state. The nature of the quantum phase transition is a Landau tricritical point, where the second phase transition occurs at the end of inflation, which is the slow accelerated expansion with a small cosmological constant.

A possible mechanism for the phase transition from a spherical spacetime  $S^4$  into a flat space with time  $\mathbb{R}^3 \times \mathbb{R}^+$  is with the splitting of open strings attached to D3-branes. The endpoints of open strings are similar to quarks in a meson, where the string is analogous to a QCD gluon flux tube connecting the quarks. The two D3-branes are then carriers of the color charge at the string end points. If two D3-branes separate with sufficient velocity or force between them the open string attaching them will break under the increased tension. The separation of two D3-branes may be facilitated by an inflaton or dilaton field that expands the string bulk of 10 or 11 dimensions and repels the respective D3-branes. The dynamics of D-branes is a subject of considerable research. If two D3-branes are subject to a large mutual acceleration the string

will break and a new D3-brane forms.

D-branes are composed of strings in a way similar to a Fermi-electron surface in a crystal. A close examination of a D-brane would reveal a “gas” of strings. On larger scales the D-brane appears classical-like. The QCD-like nature of strings is expressed according to  $SU(2N)$  Lie groups which have matrices  $U, V$  that obey a noncommutative rule  $UV = VUe^{2\pi i/n}$ , where  $n$  is the number of degrees of freedom or string modes on the D3-brane [13]. The Lie groups are also the isometry groups for anti-de Sitter spacetime  $AdS_4$  for  $SU(4) \rightarrow SU(2, 2)$  corresponding to the metric signature of the embedding space of the hyperboloid  $AdS_4$ . The D-brane is composed of cells with minimal uncertainty  $[p, x] = \hbar$ . Assume the momentum-energy has an uncertainty  $\Delta E = \hbar/\Delta t$ . The uncertainty in time is a resolution for a “picture,” just as the duration a shutter remains open. If the two branes separate away at enormous longitudinal momentum this serves much as a relativistic time dilation that slows down the apparent motions in the transverse direction. The longitudinal momentum is less than the uncertainty in momentum. If the brane is boosted further, this increases the longitudinal momentum and narrows the resolution time. This then increases the number of degrees of freedom on the D-brane. In particular for an observer on one of the original D-branes it means there is the generation of more gluon-like-parton modes on the newly created brane. This increases the number of gravitons on the brane, which assumes AdS properties. This process then increases the number of degrees of freedom in the theory, just as with generating partons, but fundamentally it is expected the number of degrees of freedom for a fundamental theory should be constant.

This large number of degrees of freedom are a gauge redundancy parameterized by the boost parameter. Nature does not admit such variable degrees of freedom. The solution to this problem is nonlocality of field information. The paper [14] is a start at addressing this problem. This pertains to nonlocality of field in the sense of the BCFW recursion. This can be seen by looking at conformal flows. The Cauchy-Riemann conditions on the complex plane are generalized to something called holomorphy in four dimensions. In QCD the rescaling of the theory at high energy, so called asymptotic freedom is an RG flow. The conformal field  $CFT_3$  exists on the  $2 + 1$  spacetime boundary of an  $AdS_4$  spacetime. The paths on this boundary space have a knot theory description by the Chern-Simons action. A path integral for the Chern-Simon Lagrangian is depicted as



The equation is the Jones’ polynomial and a skein relationship for a knot. The function  $W(C) = exp(i \int_c A \cdot dx)$  is the Wilson line or loop integral for the valuation of a gauge connection. The expectation value is the path integral

$$\langle W(C) \rangle = \int D[g, A] W(C) e^{-iS}$$

The Jones polynomial is a Skein relationship

$$\alpha \langle W(L^+) \rangle - \alpha^{-1} \langle W(L^-) \rangle = z \langle W(L^0) \rangle.$$

between the over and undercrossing elements. The loop in the path integral is diagrammatically

$$\alpha \langle W(L^+) \rangle - \alpha^{-1} \langle W(L^-) \rangle = z \langle W(L^0) \rangle$$

The element  $\alpha = 1 - 2\pi i/k$ , for  $k =$  momentum vector, and  $z = -2\pi i/k$ . For  $k$  very large  $\alpha^{-1} = 1 + 2\pi i/k$  so at high high momentum  $\langle W(L^+) \rangle + \langle W(L^-) \rangle \simeq \langle W(L^0) \rangle$ . For  $k \rightarrow 0$  to  $O(k^{-1})$  we have  $\langle W(L^+) \rangle \simeq \langle W(L^0) \rangle$ . The UV and IR physics differ with their respective presence and absence of  $\langle W(L^-) \rangle$ . The high energy physics is  $k \simeq M_{pl}$ , and the low energy physics is  $k \rightarrow 1/k \simeq M_\Lambda = 10^{-18} GeV$ . The ultrahigh energy physics with connected paths that cover space is replaced by simple paths with quantum loops defining the vacuum zero point energy. The cosmological constant  $\Lambda = 8\pi G\rho/3$ , and the energy density  $\rho$  corresponds to an energy in a unit volume of  $M_\Lambda = 10^{-18} GeV$ . These two limits define an intermediary momentum or mass  $K = \sqrt{M_\Lambda M_{pl}}$ , so the dual when written according to proper units is  $k \rightarrow K^2/k$  which  $\simeq M_\Lambda$  for  $k \simeq M_{pl}$ . This is the value of the vacuum energy for the universe and the cosmological constant.

At the Hagedorn energy/temperature near the Planck energy path integrations “go all over the place.” At a phase transition temperature, this quantum tricritical point, path integrals assume a dual form at low energy where the path integrations carry different orientation quantum numbers. A path integral consists of paths which cover all of what becomes the emergent spacetime, but at low energy loops that time reverse or that cross event horizons are disconnected from simple paths as vacuum fluctuations. A path integral at high energy with topologically nontrivial paths is broken into a set of topologically trivial paths with vacuum loops. A path integral which winds paths across what emerges as event horizons is then partitioned into what appears as decoherent histories at low energy. What becomes a nontrivial set of paths for one particle becomes a collection of paths for an ensemble of particles at low energy and with local field description. These are emergent or redundant degrees of freedom associated with the emergence of spacetime. Thus individual particles we observe in the universe are an illusion. Only one of each of the basic elementary particles exists, but appears in multiple places with different configuration variables. There is then only one electron, one up quark, one photon and so forth, but

which appear in a multiplicity of configuration variables as approximately local observables. This is the source for the apparent increase in the number of degrees of freedom. Hence we preserve holography with inflation, inflation is due to a quantum critical phase, and this ultimately requires that locality not be fundamental.

In a related development in [14] a holographic induced nonlocal approach to string theory is advanced. The holographic  $AdS_3/CFT_2$  correspondence is examined. The boundary of the  $AdS_3$  is  $S^2 \times \mathbb{R}^1$  with the conformal quantum field with the group  $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ . This theory of operators is over quantum states equivalent to the Hartle-Hawking vacuum, and the cylinder boundary of the  $AdS_3$  is a string world sheet. It is equivalent to a Euclidean form of QCD with two colors, which gives the BCFW recursion relationship above. The gravity theory in the interior is described by a Chern-Simons Lagrangian in the  $2 + 1$  space plus time. The Chern-Simons Lagrangian gives the knot Skein relationship, which defines the BCFW recursion relationship. This means field observables are fundamentally nonlocal and string theory is based on nonlocal fields. This is a parallel development with the cosmological result above. How the two fit into a single theory is not clear at this time. It suggests some form of inverse relationship between the de Sitter spacetime and the anti de Sitter spacetime. The underlying structure is the abandonment of locality and unity.

## 5 Further implications

The loss of locality and unitarity carries deeper implications. It may imply that the universe is not only noncommutative, but nonassociative. From the noncommutative product rule  $e^q XY = e^{-q} YX$ ,  $q = e^{\pi i/n}$  we define the product  $X \hat{\star} Y = e^q XY + e^{-q} YX$ , which for  $n$  large is  $X \hat{\star} Y \simeq XY + YX + q[X, Y]$  and is the commutative Jordan product for  $q = 0$ . The product  $\hat{\star}$  means this is a variant of the star product for noncommutative geometry. The product of this with another observable  $Z$  or  $(X \hat{\star} Y) \hat{\star} Z$  and the associated  $X \hat{\star} (Y \hat{\star} Z)$  have the difference

$$(X \hat{\star} Y) \hat{\star} Z - X \hat{\star} (Y \hat{\star} Z) = (1 + q)[Y, XZ] + q^2[[Z, X], Y]$$

where the  $O(q^2)$  term is evaluated using the Bianchi rule. This is nonzero in general and this is a nonassociative product rule. This is related to the weak associativity rule of Landsman [15], where commutative geometry is recovered for  $q = 0$ .

This opens the door to a range of structures from exceptional algebras, octonions and modular forms. This may then mean the most fundamental principles of physics and cosmology are a quantum error correction code. The most general of these is the Mathieu codes, or equivalently the Leech lattice. This quantum error correction code is a part of the automorphism group for the Fischer-Greiss group, sometimes called the monster group. This approach to revisioning the foundations of physics may lead underlying principles based on quantum error correction codes which conserve information.

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