The Four Pillars of Fundamentality

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A four-point heuristic is presented for evaluating the fundamentality of any physical theory. A theory considered "fundamental" has four features: It is general, parsimonious, relational, and mechanism-suggestive. These qualities are examined in the context of general relativity, Darwinian evolution, and their historical antecedents, as well as a hypothetical case in which inhabitants of a 1+1-dimensional world seek a theory of an observed phenomenon and an explanation for the value of a mysterious constant of nature. This analysis reveals the role that fundamentality plays in the progression of science, suggesting pathways toward a fundamental theoretic structure that describes the mechanisms of the world.

With apologies to Edwin Abbot [1], consider a world called Oscillatorland, with one dimension of space and one dimension of time. A distinguishing feature of this world is a point–object that oscillates continuously in space. The intelligent inhabitants — who define their units of length and time by this oscillation [see Endnote] — seek a mathematical description of the point–object's motion, such that its position at any time can be predicted. This proves elusive; the motion being curiously nonlinear, inhabitants initially can predict future positions only by extrapolating records of past positions, with limited accuracy. Meanwhile, the common folk imagine that there must be rival twin Oscillation-Gods, ever-pulling at the point–object, in an eternal struggle for control over this magical object at the center of the world.

Then, the one-dimensional physicist Erwin Rowdinger discovers that the position at any time can be calculated using a Taylor series [see Endnote]. The famous "Rowdinger equation" involves a mysterious constant, a number a bit larger than 3, which he calls *I*. (Remember, this is a one-dimensional world — they can't write the symbol π.) No one in Oscillatorland knows the provinance of *I*, and the number seems unrelated to anything else in their world. But soon, the mathematician Leonhard Reuler* discovers a continued fraction [see Endnote] that can calculate *I* to as many digits as necessary. At last, Oscillatorlanders have a seemingly complete theory of the point–object's motion, which allows them to predict its position at any time, to arbitrary precision.

It isn't long before the theory, centered on the time-symmetric Rowdinger equation, is called the most successful theory in the history of physics: It allows the point–object's position evolution to be calculated with astonishing accuracy. But some aren't satisfied; the theory doesn't explain what's really going on with the point–object. What's making it undulate in such a peculiar manner? The theory doesn't seem to be *fundamental*, although no one can explain why, or what that word even means. Rowdinger's followers develop a response to these critics: "Shut up and calculate!"

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* Oscillatorlanders deliberately mispronounce their own names for humorous effect.

Finally, a lowly bureaucrat named Albert Linestein makes a discovery: He wonders what it would be like to transition out of Oscillatorland altogether, with help from a second dimension of space. In doing so, he discovers the concept of *curvature* (which eludes the understanding of most lay Oscillatorlanders). This leads him to a thought experiment: He imagines traveling along a 1-manifold of constant curvature, at a constant speed, and eventually returning to his starting point in one unit of time — all while creating a projection onto Oscillatorland that reaches a maximum excursion of one unit of length. He publishes a paper (well, more like a ticker-tape) demonstrating that this projection would coincide *exactly* with the point–object's position (see **Fig. 1**). This explains the curious, start/stop/reverse oscillations, which now can be understood in terms of continuous motion along a manifold. Genius! At the same time, Linestein's discovery also explains the mysterious constant of nature *I*: It's simply the length of the manifold, in Oscillatorland units. *I* is just the circumference!

As if that weren't enough, Linestein even shows that if the motion along the manifold is projected onto a surface normal to Oscillatorland, the resulting value indicates the point–object's *velocity* in Oscillatorland at that instant. One-dimensional minds are blown, indeed.

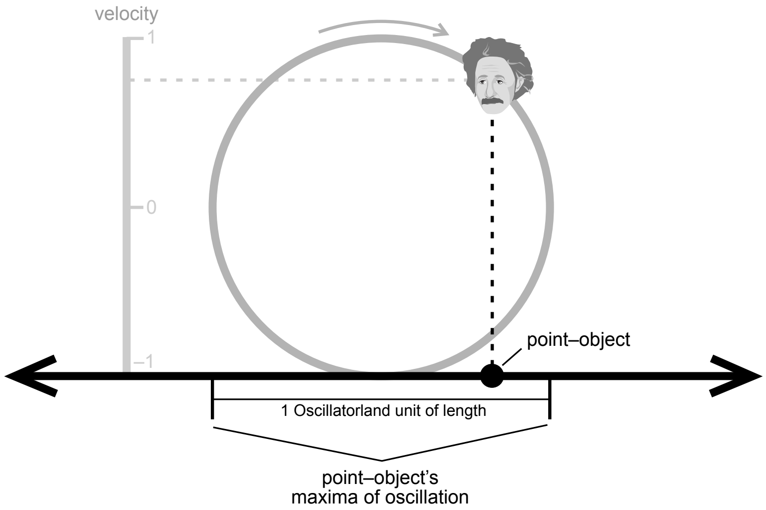

Fig. 1. In his thought experiment, Albert Linestein imagines traveling along a manifold of constant curvature (gray circle) and projecting his position onto Oscillatorland — and he realizes that the projection's apparent motion coincides exactly with the point–object's motion. As a bonus, he discovers the cosine function as the derivative, with a projection onto the gray vertical line indicating the rate of change of the point–object's position.

Linestein hasn't written a theory of everything, just a theory of the point–object's observed motion. But Oscillatorland already had such a theory before Linestein came along — and it worked incredibly well. Even though both theories produce the same numerical results, many claim that Linestein's theory is a *fundamental* description, while Rowdinger's is not.

What does *fundamental* mean? Is it possible to assess the fundamentality of competing theories? Can such an analysis steer us toward a fundamental theory of the world?

In the quest for a "final theory" [2] or a "theory of everything," that word comes up again and again, in different contexts: there are fundamental particles, fundamental interactions, and fundamental physical constants. In these usages, fundamentality implies an irreducible or indivisible quality. A *fundamental theory*, then, is something like an irreducible, indivisible framework of knowledge that describes some physical mechanism. This essay argues that in order for any theory (including the long-sought theory of everything) to be recognized as fundamental, it must fulfill four requirements. The *four pillars of fundamentality* can serve as a heuristic to compare competing theories, similar to Occam's razor, and evaluate the fundamentality of each. To present the case for each pillar, we will examine two highly successful theories, general relativity and Darwinian evolution, and their historical antecedents. Afterward, we'll come back to Oscillatorland and compare Linestein's and Rowdinger's theories in light of the four pillars. Finally, we'll see what this analysis tells us about the progress of science and the search for an ultimate theoretic structure or "theory of everything."

Pillar #1: A Fundamental Theory Is General

We expect that a fundamental theory, describing any physical mechanism, should apply to everything associated with the mechanism, including all phenomena emerging directly from the mechanism, under all possible conditions and in all possible reference frames, possibly even in other worlds. A fundamental theory may transcend its associated phenomena in terms of scale, dimension, geometry, etc. Phenomena themselves may be unified by a general theory.

After Albert Einstein conceived special relativity [3], he went to work generalizing SR for accelerated reference frames. The general theory [4] satisfies the first pillar of fundamentality: The observed path of any free-moving object, including massless particles, in any reference frame (including rectilinear and rotational accelerated frames), can be predicted with GR. The same is true of the observed rate of any clock, or the observed length of any object. GR's explanatory power derives, in part, from the theory transcending the boundaries of its mechanism's phenomena: Although we observe three dimensions of space and one of time, and the geometry of trivial spatial measurements is Euclidean, GR not only unifies space and time, it gives the resulting spacetime a non-Euclidean geometry, often visualized as an embedding in a higher dimension. GR-like phenomena emerge from non-Euclidean geometry in a 1+1-D demonstration [5]. None of the above can be said about its antecedent, Newtonian gravitation. General relativity is a general description of the mechanism that produces all relativistic and gravitational phenomena.

Similarly, Darwinian evolution theory [6] is a general description of the mechanism responsible for the diversity of life. One of its theoretic antecedents, Lamarckian evolution, attributed adaptive traits to the heritability of traits acquired during an organism's lifetime: Giraffes have long necks, for example, because giraffes stretch upward for food, and thus the neck gradually lengthens across generations. One is hard pressed, though, to imagine how this mechanism would lead to thorns on rosebushes, or cell capsules on bacteria. Darwin successfully formulated a theory that applied to all life forms in all environments. Its explanatory power derives, in part, by transcending the ontology of an individual organism or a reproductive lineage: It incorporates competition within groups, and variation within a group. Darwin's theory transcends the individual organism in other ways: It applies to the adaptation of isolated groups (which produces speciation [6]), and Dawkins showed how "survival of the fittest" applies even at the gene level [7]. Darwin's theory is so general it has been applied to fitness within a society ("social Darwinism") and even to the einselection of quantum pointer states ("quantum Darwinism," [8]) and the development of the brain ("neural Darwinism," [9]).

Pillar #2: A Fundamental Theory Is Parsimonious

This is the Occam's razor aspect of fundamentality: a fundamental theory describes a mechanism with a minimum number of entities in causal or static relations. Put in a more intuitive way, a natural–physical "machine" has as few parts (either moving or fixed) as necessary to do what it does. The parts, of course, can be anything: objects, interactions/forces, environmental conditions and other free parameters, and including entities assumed to exist but which are unknown or not well understood.

The historical progression from Ptolemaic astronomy to Newtonian mechanics to general relativity demonstrates a progression toward parsimony. The Ptolemaic system assumed the individual mechanisms of epicycles, which determine the paths of bodies across the sky. As observations improved, the epicycles had to be multiplied — not good from a parsimony standpoint. By Newton's time, the mysterious epicycle mechanisms had been vanquished and replaced by a heliocentric model operating on one mechanism, a (mysterious) gravitational force between masses that operates at a distance. However, and this is a subtle point, Newtonian mechanics assumed that the interaction was *between bodies*, and that each individual body (or pair of bodies) generates an attractive force. While his theory reduced the mechanism to one *type* of force, it assumed that each massive body exuded this force — which reached out, across space, to affect other massive bodies, and vice versa. The mysterious mechanism of force-generation (or perhaps force-interaction) is what Newtonian gravitation describes; without it, the theory would tell us nothing. Einstein provided another leap in parsimony by assuming one spacetime fabric with a geometry. This vanquished the attraction force apparently exuded by masses. Although we don't know the mechanism by which mass–energy alters spacetime geometry, that mechanism has been removed from GR, to be described by some other, future theory. GR states that a particular distribution of mass–energy corresponds to a particular predictable geometry, with all phenomena of observed trajectories, lengths, and durations emerging from that geometry and that geometry alone. A more parsimonious description of the mechanism that produces gravitational (and relativistic) phenomena likely could not be given.

Traditionally, the diversity of life is attributed to supernatural creation, producing the various "kinds" of life in separate creative acts. Darwin reduced the mechanism to one governing principle, arriving at his theory via plain observations (the Earth appears to be old, offspring vary, differential fitness exists, unfit organisms die). Although many pressures are involved, they all contribute to the single irreducible quality of reproductive fitness. As with GR, a more parsimonious description of the mechanism producing the diversity of life cannot be imagined.

Just as Einstein removed force-generation-by-mass from the gravitation mechanism, Darwin removed abiogenesis from the mechanism from which the diversity of life emerges. Einstein and Darwin both realized that these are separate mechanisms, which need to be described by their own theories. This gets at what fundamentality is all about: Progress in science involves teasing apart previously tangled theories, such that each new theory is the most simple, irreducible description of the most simple, irreducible mechanism. Science is a progression toward fundamentality.

Pillar #3: A Fundamental Theory Is Relational

In his 2003 TED Talk, "Science and Democracy" [10], Lee Smolin discussed three stages through which science has progressed: the hierarchical universe of antiquity, with Earth at the center and bodies moving on concentric spheres; the Newtonian universe, where objects can move freely but everything happens within a rigid framework of space and time; and the relational universe initiated by Einstein, which features neither an absolute center nor any absolute framework; the universe is described, instead, as a network of relationships. Given this progression, we expect that any fundamental theory should be relational.

Smolin points out that in the Ptolemaic system, heavenly bodies' motion was constrained by their distance from Earth, an absolute reference point marking the center of the universe. That gave way to the more liberal Newtonian universe, which had no absolute center. In that view, bodies' locations have to be described in relation to each other; however, these distances can be thought of as marked off by an absolute framework of space, where one mile is measured the same everywhere and under all conditions, and a universal clock ticks the same time everywhere, at the same rate. It was as if the divine creator had paced off distance units and ticked duration units that were as fixed and absolute as today's electron rest mass. That all went away with Einstein and the notion that time and space can trade off for each other, that any distance or duration measurement must be described relative to a frame of reference, and that no preferred reference frame exists. GR is *purely relational* — it has no room for absolute quantities or entities of any kind. No theory of gravity could be more relational than GR.

In contrast, no theory for the diversity of life could be *less* relational than the creationist explanation, where an absolute causal (but itself uncaused) agent creates "kinds" in mysterious, unknown ways. Darwin eliminated the need for an absolute creative agent by explaining diversity purely in terms of relations: relations between organisms of the same species, between organisms of different classes/phyla/kingdoms (competing for food, for example), and between organisms and their non-biologic environments. As Smolin points out in his TED Talk, this relational view reflects self-organizational principles at work, as opposed to the traditional explanation, where organization and complexity are introduced by a causal agent external to the world. Although Lamarck also vanquished the need for such an absolute creative agent, and his mechanism involved relations between organisms and their environment (and perhaps organisms that could serve as food or breeding partners), there's no place in Lamarckism for the relations between similar organisms competing for resources. Differential fitness is an essential feature in such competition, and differential fitness is relational. Darwinian evolution is therefore more relational than Lamarckian evolution; like GR, it is a purely relational theory. “Natural selection would only make sense in a relational universe,” says Smolin in his talk.

Pillar #4: A Fundamental Theory Is Mechanism-Suggestive

So far, this essay has used the word *mechanism* without a definition. A mechanism can be considered a set of causal interactions and/or static relations, from which emerge an observable phenomenon or group of phenomena. The mechanism of an air conditioner produces the phenomenon of colder air; the Standard Model’s mechanisms are responsible for (among other things) the stability and instability of the atomic nucleus, from which emerge alpha particles and other phenomena. Given the previous sections and their examples, we expect a fundamental theory to describe a natural mechanism or mechanisms in terms that are maximally general, parsimonious, and relational. But we also expect a kind of “least action” relation between a theory and the mechanism(s) that it describes. A mechanism or mechanisms should fall out of a fundamental theory like gumballs falling out of a candy machine. This may include the prediction of sub-mechanisms not yet described by their own sub-theories, and even phenomena that have not yet been directly observed.

GR is a perfect example of mechanism suggestivity. The theory describes the mechanism as a geometry; there could be no more straightforward description of the gravitational mechanism than GR’s. Likewise, its resultant phenomena, including the observed curvature of geodesics in certain reference frames, and related phenomena like gravitational time dilation, fall right out of the mechanism of non-Euclidean spacetime. The force-at-a-distance mechanism of Newtonian gravitation, in contrast, was mysterious and problematic, to say nothing of the Ptolemaic system and its cycle/epicycle mechanisms. In neither case did the theory directly suggest the mechanism. Not only did GR directly explain how gravitation works, it predicted the sub-mechanisms of (1) black holes, whose phenomena were initially unobserved; (2) gravitational waves, whose existence was considered so likely that hundreds of millions of dollars were spent on the LIGO experiment to detect them, which it did; and (3) wormholes, which remain unobserved. GR is astonishingly mechanism-suggestive.

In a similar manner, the mechanism of natural selection trivially falls out of the theory of Darwinian evolution: the theory just describes the mechanism in words. Darwin also described the sub-mechanism of sexual selection, a special case of natural selection, as early as *The Origin of Species* [6]. And, famously, Darwin’s theory predicted that there had to be a biochemical mechanism of heritability, which was confirmed many decades later with the discovery of DNA. It can be argued that a heritability mechanism was also predicted by Lamarckian theory, but only as it relates to highly limited cases such as the giraffe’s neck or the ant’s mechanical strength. Mechanism-suggestivity may have been the only pillar of fundamentality that Lamarck fulfilled. Creationism, meanwhile, was more tautological than anything, describing the creation mechanism as just the unknown/unknowable mechanism of the creator, without offering more nuanced explanatory power.

Oscillatorland Revisited

Let's return to the Oscillatorland story and examine Albert Linestein's theory, and its historical antecedent, in light of the four pillars of fundamentality.

Linestein's description transcends Oscillatorland's 1+1 dimensions, deriving sinusoidal motion along one spatial dimension as a special, projected case of the more general constant motion around a circle. The theory could be applied to describe any sinusoidal motion that occurs in Oscillatorland (perhaps compression waves of sound), and further, that more-complex oscillations can be decomposed in terms of projections of circular motions on top of circular motions. Rowdinger's equation — already cumbersome with its infinite series — becomes unworkable when applied to oscillations superimposed on oscillations. But Linestein could also demonstrate that non sinusoidal oscillations can be described by altering the geometry of the 1-manifold: If its curvature changes as a function of phase, he might travel along an ellipse, and the projected motion becomes increasingly triangular as the ellipse "flattens." Other geometries produce other projected motions. Rowdinger's theory is arguably generalizable for all sinusoidal motions, but Linestein's theory is generalizable to other motions, and more conducive to generalizing.

Although Linestein's theory requires a projection from a curved manifold, the point–object's sinusoidal motion in this context becomes a function of simpler, constant circular motion. Like Darwin removing abiogenesis from the theory of life, and Einstein removing the causal mechanism of mass–energy from the theory of gravity, Linestein has removed motion-creation from the theory of oscillation, to be explained separately by another theory. Given the apparent existence of a circular-motion mechanism, Linestein's projection-mechanism theory explains how that simple motion becomes peculiar, start-and-stop motion in a world of one-dimensional observers. (One imagines Rowdinger-theory holdouts refusing to accept Linestein's projection theory, and instead theorizing that Oscillatorland's point–object is actually a *computer* that's constantly calculating infinite series!) The projection theory is a parsimonious description of that particular, specific mechanism — a good candidate to be part of a fundamental structure of fundamental theories, when the work of Oscillatorland's physicists is done.

Before Linestein, many Oscillatorlanders imagined the point–object's sinusoidal motion as arising out of whole cloth, created by their pushing-and-pulling Oscillation-Gods. Linestein discovered a relation to constant motion — abandoning the idea that start-and-stop motion is of absolute provenance. Also, Linestein's derivation of the value of *I* is a ratio, representing a basic geometrical relation from which the oscillatory motion emerges: It's just the relation between the circumference (the length of the manifold) and the diameter (the point–object's excursion). Where no one could find how the start-and-stop motion, or *I*, relates to anything except for a Taylor series and a continued fraction, Linestein found both of those relations by looking beyond the trivially observable. He also found an unexpected relation, between the point–object's position and its velocity. That last one suggested he was really on to something!

Finally, where Rowdinger's mechanism is a black box in which sinusoidal motion somehow emerges out of pure mathematics, Linestein's theory suggests the mechanism for the sinusoidal motion, as emerging from continuous motion embedded in 2D space. The mechanism falls right out of the theory, as a matter of geometry — not unlike in GR.

Since Linestein's theory satisfies all four pillars of fundamentality, we can consider it a fundamental theory. However, it only describes the mechanism that produces sinusoidal motion; Oscillatorland does not yet have a fundamental theory of everything. Higher theories would need to describe the mechanism of the circular motion, the mechanism that creates the projection, and the provenance of the circle and Oscillatorland's dimensions of space and time (as well as explaining why there's an Oscillatorland at all, rather than nothing!).

The Progression of Science

This analysis clarifies an often-confusing aspect of scientific theory, the nature and role of parsimony. In these examples of life, gravity, and oscillatory motion, we've seen a teasing apart of tangled theories attempting to describe complex mechanisms, into simpler theories directly describing irreducible mechanisms. The number of theories/mechanisms goes up, a seemingly less-parsimonious situation, as the theories themselves become *more* parsimonious. Consider the progression of theories concerning the nature of stars, from medieval times to the Enlightenment. Originally, there was one celestial sphere upon which the stars were fixed — a theory that may appear to be parsimonious. Later, the stars were free to move with respect to each other — so in going from one entity to countless entities, the later theory may appear to be vastly more profligate. But, given a universe in which extremely complex phenomena are observed, parsimony must apply only to each individual theory/mechanism, not to the *number* of individual theories/mechanisms, nor the number of entities observed to emerge from such mechanisms. A theoretic structure comprising many theories/mechanisms — each of them *fundamental*, according to the four pillars of fundamentality — is more fundamental overall than a structure with only a few tangled theories and their complex, mysterious mechanisms, trying to explain the same phenomena. In a world sufficiently complex to produce galaxies and uranium atoms and DNA molecules and brains, science becomes a progression from a few non-fundamental theories, describing mysterious mechanisms, to many fundamental theories, out of which their irreducible mechanisms fall.

Endnote: The Mathematics of Oscillatorland

Life is tough in Oscillatorland. Everyone is *so* one-dimensional. If you want to communicate a message, it has to go through lots of other people. And you literally cannot get ahead in life. But, somehow, they've developed mathematics, or at least elementary algebra.

Oscillatorland's measurements of length and time are based on the point–object's oscillations: One unit of time is one full oscillation cycle, and one unit of length is the width of the excursion zone, i.e., the distance the point–object travels in one half-cycle. For millions of oscillations, the inhabitants of Oscillatorland kept track of the motion of their point–object, in one-dimensional tables (which are really cumbersome to use). Then Erwin Rowdinger discovered a Taylor series which, with the help of a constant, allowed them to approximate the point–object's position at nearby times. We Earthlings recognize the "Rowdinger equation" as a bad Taylor-series expansion of the sine function:

$$s = \frac{1}{2}\left(2I\mathrm{t} - \frac{\left(2I\mathrm{t}\right)^3}{3!} + \frac{\left(2I\mathrm{t}\right)^5}{5!} - \frac{\left(2I\mathrm{t}\right)^7}{7!} \cdots \right)$$

where s is the location of the point–object at time t, as measured from the excursion zone's midpoint, and I is the constant that Rowdinger discovered.

At the legendary Solray Conference, when asked why he arbitrarily included a coefficient in his terms rather than simply doubling his constant, I, Erwin Rowdinger had a stroke and died. In his memory, it was decided that the Rowdinger equation would be left untouched forever.*

The continued fraction discovered by Leonhard Reuler is, of course, just the continued fraction for π:

$$I = \cfrac{4}{1 + \cfrac{1^2}{3 + \cfrac{2^2}{5 + \cfrac{3^2}{7 + \cfrac{4^2}{\ddots}}}}}$$

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* Please accept this *deus ex machina* as a way of satirizing Earthlings' usage of π. Using τ (2π) instead, along with a radius-based unit of length, would have made Rowdinger's equation simpler — I daresay more fundamental.

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