When physics is geometry: a new proof for general relativity through geometric interpretation of Mössbauer rotor experiment. Celebration of the 100th anniversary of general relativity

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Abstract

General relativity is not only one of the greatest and most elegant scientific theories of all (perhaps **the** greatest and **the** most elegant), but also the best example showing that Mathematics is Truth instead of Trick. It is indeed well known that Einstein's vision of gravity is *pure geometry*. In this Essay, we celebrate the centennial of this intriguing pre-established harmony between geometry and physics, marked by the year 2015, giving a correct interpretation of a historical experiment by Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect (Mössbauer rotor experiment). This experiment has been recently first reanalyzed and then replied by an experimental research group. The results of reanalyzing the experiment have shown that a correct re-processing of Kündig's experimental data gives an interesting deviation of a relative redshift between emission and absorption resonant lines from the standard prediction based on the relativistic dilatation of time. That prediction gives a redshift $\frac{\nabla E}{E} \simeq -\frac{1}{2}\frac{v^2}{c^2}$ where v is the tangential velocity of the absorber of resonant radiation, c is the velocity of light in vacuum and the result is given to the accuracy of first-order in $\frac{v^2}{c^2}$. Data re-processing gave $\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2}$ with $k = 0.596 \pm 0.006$. Subsequent new experimental results by the reply of Kündig experiment have shown a redshift with $k = 0.68 \pm 0.03$ instead. By using Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference), in this Essay we reanalyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference by using a *completely geometrical* general relativistic treatment. It will be shown that previous analyses missed an important effect of clock synchronization and that the correct, purely geometric, general relativistic prevision in the rotating frame gives $k \simeq \frac{2}{3}$ in perfect agreement with the new experimental results. Such an effect of clock synchronization has been missed in various papers in the literature with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects. Our geometric general relativistic interpretation shows, instead, that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent, proof of Einstein's elegant, purely geometric, vision of gravity.

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1. Introduction

The year 2015 marks the 100th anniversary of Einstein's general relativity. This majestic theory is not only considered, together with quantum field theory, the best scientific theory of all [15], but represents also the most elegant example that Mathematic is Truth instead of Trick. In fact, Einstein's vision of gravity is *pure geometry* [10]. In that beautiful tapestry, gravity is **not** a force, but spacetime curvature instead [10, 15]. A little test mass m within a gravitational field generated by a big mass M, with $m \ll M$, is forced to go through a *geodesic*, i.e. the generalization of the notion of a "straight line", which works in a flat spacetime, to a spacetime which has been curved by the source M. In other words, in general relativity the "natural" motion of a test mass in a gravitational field is the "free falling" motion, where for "free falling" we mean the behavior of the test mass to move along a geodesic "world line", see [10, 15] and the technical endnotes of this Essay for details. Here, we further celebrate this intriguing pre-established harmony between geometry and physics giving a correct interpretation of a historical experiment by Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect (Mössbauer rotor experiment) [3]. The Mössbauer effect (discovered by R. Mössbauer in 1958 [14]) consists in resonant and recoil-free emission and absorption of gamma rays, without loss of energy, by atomic nuclei bound in a solid. It resulted and currently results very important for basic research in physics and chemistry. In this Essay we will focus on the so called Mössbauer rotor experiment. In this particular experiment, the Mössbauer effect works through an absorber orbited around a source of resonant radiation (or vice versa). The aim is to verify the relativistic time dilation for a moving resonant absorber (the source) inducing a relative energy shift between emission and absorption lines.

In a couple of recent papers [1, 2], the authors first reanalyzed in [1] the data of a known experiment of Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect [3], and second, they carried out their own experiment on the time dilation effect in a rotating system [2]. In [1] it has been found that the original experiment by Kündig [3] contained errors in the data processing. A puzzling fact was that, after correction of the errors of Kündig, the experimental data gave the value [1]

$$\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2}, \tag{1}$$

where $k = 0.596 \pm 0.006$, instead of the standard relativistic prediction $k = 0.5$ due to time dilatation. The authors of [1] stressed that the deviation of the coefficient k in equation (1) from 0.5 exceeds by almost 20 times the measuring error and that the revealed deviation cannot be attributed to the influence of rotor vibrations and other disturbing factors. All these potential disturbing factors have been indeed excluded by a perfect methodological trick applied by Kündig [3], i.e. a first-order Doppler modulation of the energy of $\gamma-$quanta on a rotor at each fixed rotation frequency. In that way, Kündig's experiment can be considered as the most precise among other experiments of the same kind [4–8], where the experimenters measured only the count rate of detected $\gamma-$quanta as a function of rotation frequency. The authors of [1] have also shown that the experiment [8], which contains much more data than the ones in [4–7], also confirms the supposition $k > 0.5$. Motivated by their results in [1], the authors carried out their own experiment [2]. They decided to repeat neither the scheme of the Kündig experiment [3] nor the schemes of other known experiments on the subject previously mentioned above [4–8]. In that way, they got independent information on the value of k in equation (1). In particular, they refrained from the first-order Doppler modulation of the energy of $\gamma-$quanta, in order to exclude the uncertainties in the realization of this method [2]. They followed the standard scheme [4–8], where the count rate of detected $\gamma-$quanta N as a function of the rotation frequency ν is measured. On the other hand, differently from the experiments [4–8], they evaluated the influence of chaotic vibrations on the measured value of k [2]. Their developed method involved a joint processing of the data collected for two selected resonant absorbers with the specified difference of resonant line positions in the Mössbauer spectra [2]. The result obtained in [2] is $k = 0.68 \pm 0.03$, confirming that the coefficient k in equation (1) substantially exceeds 0.5. The scheme of the new Mössbauer rotor experiment is in Figure 1, while technical details on it can be found in [2].

In this Essay, Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of refer-

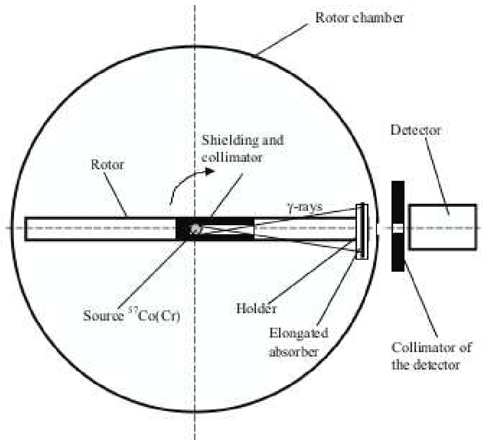

Figure 1: Scheme of the new Mössbauer rotor experiment, adapted from ref. [2]

ence) will be used to reanalyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference by using a full geometric general relativistic treatment [16]. The results will show that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ [16] in perfect agreement with the new experimental results of [2]. In that way, the geometric general relativistic interpretation of this Essay shows that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of the correctness of Einstein's vision of gravity, further celebrating that gravity is geometry and, in turn, Mathematics is Truth instead of Trick. We also stress that various papers in the literature missed the effect of clock synchronization [1]-[8], [11]-[13] with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13].

2. Geometric interpretation of time dilatation

Following [9, 16] let us consider a transformation from an inertial frame, in which the space-time is Minkowskian, to a rotating frame of reference. Using cylindrical coordinates, the line element in the starting inertial frame is [9, 16]

$$ds^2 = c^2dt^2 - dr^2 - r^2d\phi^2 - dz^2. \tag{2}$$

The transformation to a frame of reference $\{t', r', \phi' z'\}$ rotating at the uniform angular rate ω with respect to the starting inertial frame is given by [9, 16]

$$t = t' \quad r = r' \quad \phi = \phi' + \omega t' \qquad z = z' \ . \tag{3}$$

Thus, eq. (2) becomes the following well-known line element (Langevin metric) in the rotating frame [9, 16]

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - 2\omega r'^2 d\phi' dt' - dr'^2 - r'^2 d\phi'^2 - dz'^2. \qquad (4)$$

The transformation (3) is both simple to grasp and highly illustrative of the general covariance of GR as it shows that one can work first in a "simpler" frame and then transforming to a more "complex" one [16, 17]. As we consider light propagating in the radial direction ($d\phi' = dz' = 0$), the line element (4) reduces to [16]

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - dr'^2. \qquad (5)$$

Einstein Equivalence Principle permits to interpret the line element (5) in terms of a curved spacetime in presence of a static gravitational field [10, 15, 16]. In that way, we obtain a purely geometric interpretation of the pseudo-force experienced by an observer in a rotating, non-inertial frame of reference [16]. Setting the origin of the rotating frame in the source of the emitting radiation, we have a first contribution which arises from the "gravitational redshift" that can be directly computed using eq. (25.26) in [10] , which in the twentieth printing 1997 of [10] is written as

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{received} - \lambda_{emitted}}{\lambda_{emitted}} = |g_{00}(r'_1)|^{-\frac{1}{2}} - 1 \qquad (6)$$

and represents the redshift of a photon emitted by an atom at rest in a gravitational field and received by an observer at rest at infinity. Here we use a slightly different equation with respect to eq. (25.26) in [10] because here we are considering a gravitational field which increases with increasing radial coordinate r' while eq. (25.26) in [10] concerns a gravitational field which decreases with increasing radial coordinate [16]. Also, we set the zero potential in $r' = 0$ instead of at infinity and we use the proper time instead of the wavelength λ [16]. Thus, combining eq. (5), we get [16]

$$\begin{aligned} z_1 &\equiv \tfrac{\nabla\tau_{10} - \nabla\tau_{11}}{\tau} = 1 - |g_{00}(r'_1)|^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{1 - \frac{\left(r'_1\right)^2\omega^2}{c^2}}} \\ &= 1 - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \simeq -\frac{1}{2}\frac{v^2}{c^2}, \end{aligned} \qquad (7)$$

where $\nabla\tau_{10}$ is the delay of the emitted radiation, $\nabla\tau_{11}$ is the delay of the received radiation, $r'_1 \simeq c\tau$ is the radial distance between the source and the detector and $v = r'_1\omega$ is the tangential velocity of the detector [16]. Hence, we find a first contribution, say $k_1 = \frac{1}{2}$, to k [16]. We stress again that the power of Einstein Equivalence Principle enabled us to use a pure geometric treatment of physics in the above discussion [16].

3. Geometric interpretation of clock synchronization

Notice that we calculated the variations of proper time $\nabla\tau_{10}$ and $\nabla\tau_{11}$ in the origin of the rotating frame which is located in the source of the radiation [16]. But the detector is moving with respect to the origin in the rotating frame [16]. Thus, the clock in the detector must be synchronized with the clock in the origin, and this gives a second contribution to the redshift [16]. To compute this second contribution we use eq. (10) of [9] which represents the proper time increment $d\tau$ on the moving clock having radial coordinate r' for values $v \ll c$

$$d\tau = dt'\left(1 - \frac{r'^2\omega^2}{c^2}\right). \tag{8}$$

Inserting the condition of null geodesics $ds = 0$ in eq. (5) one gets [16]

$$cdt' = \frac{dr'}{\sqrt{1 - \frac{r'^2\omega^2}{c^2}}}, \tag{9}$$

where we take the positive sign in the square root because the radiation is propagating in the positive r direction [16]. Combining eqs. (8) and (9) one obtains [16]

$$cd\tau = \sqrt{1 - \frac{r'^2\omega^2}{c^2}}dr'. \tag{10}$$

Eq. (10) is well approximated by [16]

$$cd\tau \simeq \left(1 - \frac{1}{2}\frac{r'^2\omega^2}{c^2} + ...\right)dr', \tag{11}$$

which permits to find the second contribution of order $\frac{v^2}{c^2}$ to the variation of proper time as [16]

$$c\nabla\tau_2 = \int_0^{r'_1}\left(1 - \frac{1}{2}\frac{(r'_1)^2\omega^2}{c^2}\right)dr' - r'_1 = -\frac{1}{6}\frac{(r'_1)^3\omega^2}{c^2} = -\frac{1}{6}r'_1\frac{v^2}{c^2}. \tag{12}$$

Thus, as $r'_1 \simeq c\tau$ is the radial distance between the source and the detector, we get the second contribution of order $\frac{v^2}{c^2}$ to the redshift as [16]

$$z_2 \equiv \frac{\nabla\tau_2}{\tau} = -k_2\frac{v}{c^2}^2 = -\frac{1}{6}\frac{v^2}{c^2}. \tag{13}$$

Then, we obtain $k_2 = \frac{1}{6}$ and using eqs. (7) and (13) the total redshift is [16]

$$\begin{aligned} z \equiv z_1 + z_2 &= \tfrac{\nabla\tau_{10} - \nabla\tau_{11} + \nabla\tau_2}{\tau} = -\left(k_1 + k_2\right)\tfrac{v^2}{c^2} \\ &= -\left(\tfrac{1}{2} + \tfrac{1}{6}\right)\tfrac{v^2}{c^2} = -k\tfrac{v^2}{c^2} = -\tfrac{2}{3}\tfrac{v^2}{c^2} = 0.\bar{6}\tfrac{v^2}{c^2}, \end{aligned} \tag{14}$$

which is completely consistent with the result $k = 0.68 \pm 0.03$ in [2].

We stress that the additional factor $-\frac{1}{6}$ in eq. (13) comes from clock synchronization [16, 17]. In other words, its theoretical absence in the works [1]-[8], [11]-[13] reflected the incorrect comparison of clock rates between a clock at the origin and one at the detector [16, 17]. This generated wrong claims of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13] which, instead, must be rejected. Notice that, even in discussing the effect of clock synchronization, we performed a pure geometric treatment of physics.

We evoked the appropriate reference [9] for a discussion of the Langevin metric. This is dedicated to the use of general relativity in Global Positioning Systems (GPS), which leads to the following interesting realization [16, 17]: the correction of $-\frac{1}{6}$ in eq. (13) is analog to the correction that one must consider in GPS when accounting for the difference between the time measured in a frame co-rotating with the Earth geoid and the time measured in a non-rotating (locally inertial) Earth centered frame (and also the difference between the proper time of an observer at the surface of the Earth and at infinity). Indeed, if one simply considers the gravitational redshift due to the Earth's gravitational field, but neglects the effect of the Earth's rotation, GPS would not work [16, 17]! The key point is that the proper time elapsing on the orbiting GPS clocks cannot be simply used to transfer time from one transmission event to another because path-dependent effects must be taken into due account, exactly like in the above discussion of clock synchronization [16]. In other words, the obtained correction $-\frac{1}{6}$ in eq. (13) is not an obscure mathematical or physical detail, but a fundamental ingredient that must be taken into due account [16, 17]. Further details on the analogy between the results of this Essay and the use of general relativity in Global Positioning Systems have been highlight in [16].

4. Conclusion remarks

We used the power of Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) to reanalyze from a pure geometric point of view the theoretical framework of the new Mössbauer rotor experiment in [2] directly in the rotating frame of reference. The results have shown that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ in perfect agreement with the new experimental results in [2]. Thus, in this Essay we have shown that the geometric interpretation of the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of Einstein general relativity. The importance of our results is stressed by the issue that various papers in the literature missed the effect of clock synchronization [1]-[8], [11]-[13] with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13]. Thus, our results are a celebration of the 100th anniversary of Albert Einstein's presentation of the complete theory of general relativity to the Prussian Academy as intriguing pre-established harmony between geometry and physics and a strong endorsement to the statement that Mathematics is Truth instead of Trick.

5. Acknowledgements

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Technical endnotes

1. *Geodesic*: in differential geometry a geodesic is the generalization to a curved space of the notion of a straight line in a flat space. In the framework of the Riemannian geometry, which is the "arena" of general relativity, a geodesic is, locally, the shortest path between points in the space. In that case, geodesics describe the motion of inertial test masses in a gravitational field.

2. *Equivalence principle*: in the framework of general relativity, the equivalence principle is any of the various connected concepts involving, on one hand, the equivalence of inertial and gravitational mass, and on the other hand the observation by Einstein that the gravitational "force" that an observer experiences locally when standing in the gravitational field generated by a massive body (the Earth) is equivalent to the pseudo-force experienced in an accelerated (non-inertial) reference frame.

3. *World line*: in physics a world line is the unique path of a test mass as it moves through the 4-dimensional spacetime. In other words, it is the sequence of spacetime events which correspond to the history of the test mass. The world line of a test mass free from all external, non-gravitational forces, is a geodesic.

4. *Time dilation in the theory of relativity:* it is a difference of elapsed time between two events as measured by observers either moving relative to each other or differently situated in a gravitational field. In the special theory (absence of gravitational field), clocks which move with respect to an inertial reference frame run more slowly. The effect is taken into account through the Lorentz transformation. In the general theory, clocks at a position where the gravitational potential is lower (in closer proximity to the mass which is source of the gravitational field) run more slowly. The two (special and general relativistic) effects can combine (as in the case of GPS). In this Essay we used the equivalence principle to show that time dilatation due to observers moving relative to each other with respect to a non inertial, rotating, reference frame is equivalent to gravitational time dilatation.

5. *Doppler effect*: the Doppler shift is the change in frequency of a wave as measured by an observer which moves relative to the source of the wave.

If the wave propagates in a medium the velocity of the source and of the observer result relative to the medium. In that case, the total Doppler effect can result from motion of the source, motion of the observer and motion of the medium. If the wave propagates in vacuum only the relative difference in velocity between the source the observer must be taken into account. Doppler effect can be both classical and relativistic. In the latter, effects described by the special theory of relativity have to be taken into account.

6. *Redshift*: in physics redshift happens when electromagnetic radiation from a source is decreased in frequency (shifted to the red end of the frequency spectrum). Independent on the issue that the radiation is within or without the visible spectrum, "redder" means a lower frequency and a lower photon energy. Some redshifts are examples of the Doppler effect discussed in point 5. Another famous redshift is the *cosmological redshift* that arises from the expansion of the universe. For sufficiently distant light sources the cosmological redshift corresponds to the rate of increase in their distance from the solar system. The gravitational redshift is a relativistic effect observed in electromagnetic waves which move out of a gravitational field. In this Essay we used the equivalence principle to discuss the transverse Doppler shift in a rotating system as equivalent to a gravitational redshift.

7. *Line element*: in general relativity, the line element, also called metric tensor (or metric to further simplify) is, perhaps, the most important object of study. It can be intuitively thought as the generalization of the gravitational potential of the Newtonian theory of gravity. The metric governs both the geometric and causal structures of spacetime, and enables to introduce notions such as curvature, distance, volume, angles, past and future.

8. *Null geodesics*: they are the geodesics of massless particles, like photons and gravitons. The condition of null geodesics in a particular geometry of spacetime is determined by imposing equal to zero the line element of the spacetime ($ds = 0$).