Why We Still Don't Have Quantum Nucleodynamics

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Abstract

Quantum electrodynamics (QED) is called the "jewel of atomic theory" because it allows for quantitative predictions of a huge number of atomic states using quantum mechanics. Although the QED techniques were adapted to the problems of nuclear theory in the 1950s, they did not lead to a rigorous quantum nucleodynamics (QND). The core problem has been the *assumption* of a central nuclear potential-well to bind nucleons together, in analogy with the Coulomb force that binds electrons to the nucleus. By replacing that fictitious long-range nuclear potential-well with the experimentally-known, short-range nuclear force, QND becomes possible.

I. Introduction

"Quantum nucleodynamics" (QND) is a phrase that was used sporadically in the 1950s to describe the intended quantum mechanical formalization of nuclear structure theory along the lines of quantum electrodynamics (QED). Unfortunately, despite the development of a quantum mechanical foundation for modern nuclear theory, the nuclear version of QED turned out to be "so difficult that no one has ever been able to figure out what the consequences of the theory are" (Feynman, 1963, p. 39) and the promise of a unified, quantitative explanation of the atomic nucleus has not been realized. Already by the late-1950s, most theorists had turned their attention to high-energy *particle physics* and, skipping over the chronic problems of nuclear structure theory, engaged in the development of quantum chromodynamics (QCD). Meanwhile, the enticing QND phrase has been effectively abandoned and was in fact last used in a physics publication by Feynman in 1963.

In the present essay, I argue that the early demise of QND can be attributed directly to theoretical assumptions concerning the nuclear force. Specifically, the nuclear force in both the independent-particle model (IPM) and the shell model (and their later variants) was postulated in the 1940s to be a long-range and *centrally-located* potential-well, in analogy with the central force that binds electrons to their nuclei. That assumption was made despite the fact that the already well-established liquid-drop model (LDM) had successfully described many nuclear properties based upon the diametrically-opposite idea, i.e., a strong and short-range nuclear force that acted only among nearest-neighbor nucleons. In other words, it was argued that, in order to use the Schrodinger equation and quantum mechanical techniques at the nuclear level, the nucleus itself must be considered to be a tiny gas of "point-like" protons and neutrons that freely orbit within the nuclear interior. Although the analogy with atomic structure was admittedly dubious, it produced theoretical predictions that were in spectacular agreement with experimental facts, and the IPM soon became the central paradigm of nuclear structure theory.

Whatever the historical reasons for making experimentally counterfactual assumptions about the nuclear force, the first indications that the predictive successes of the IPM could be maintained *without* assuming a central nuclear potential-well did not emerge until the 1970s. Unfortunately, already by the early 1960s a huge amount of theorizing based on the idea of a nuclear "gas" had accumulated, more PhDs had been awarded in nuclear physics than in any other scientific field in history, and the real-world politics of academia made skepticism concerning the nuclear force appear to be crack-pot hallucinations. Had not nuclear physicists harnessed nuclear power? Had they not effectively won the Second World War and given unlimited cheap energy to the world?

In hindsight, answers to those questions have become complex, but it is a historical fact that the "effective" nuclear potential well used in the shell model (ca. 1949) played *no role* in the development of nuclear bombs (ca. 1942) or in the design of the first nuclear reactors (ca. 1947). On the contrary, it was the realistic, liquid-phase LDM that was used by Bohr and Wheeler (1939) to predict the huge release of energy in nuclear fission and it is the LDM that is employed in modern-day fission technology. In contrast, the "effective" nuclear force remains a *theoretical toy*, elaborated on in the massively higher-dimensional parameter space that electronic computing has made possible, but with no direct contacts with experimental reality. It is worth emphasizing that, unlike the short-range "realistic" nuclear force that is known experimentally, the "effective" nuclear force is a purely theoretical construct: it is surmised to be the "mean field," time-averaged, net result of many local nucleon-nucleon interactions, but it cannot be directly measured. For this reason, the "effective" force is used primarily in an after-the-fact fashion to explain experimental findings, but has been notably unsuccessful in predicting new phenomena (e.g., predicting the existence of *stable* or *long-life* superheavy nuclei with Z>112, Kumar, 1989).

Although the debate concerning the nuclear force itself has never been satisfactorily resolved, the IPM and shell model descriptions of nuclear spins, magnetic moments, shells, subshells and parity states were simply too overwhelming to ignore. Without the independent-particle description of individual *nucleon* states and their simple summation to describe *nuclear* states, how can the two million-plus data points summarized in the Firestone *Table of Isotopes* (1996) be systematically understood? If a central potential well and a gaseous nuclear interior are incorrect starting assumptions, how can quantum mechanics be applied at the nuclear level? And if the IPM and shell model are discarded, which of the other nuclear models can better explain the empirical data of nuclear physics? Good questions and, until recently, there were no answers.

Despite those seemingly decisive obstacles to a theoretical reconstruction of nuclear theory in the 21st century, it is surprisingly easy to show how the QED "jewel of atomic theory" can indeed be replicated at the nuclear level. First of all, the fiction of the mean-field nuclear force must be rejected in favor of the realistic, strong and short-range nuclear force that has already been well-studied experimentally. In abandoning the gaseous-phase model of nuclear structure theory, we are, however, not forced to retreat to earlier, less rigorous, non-quantum mechanical models of nuclear structure, but rather can proceed directly to QND. The seemingly-paradoxical key to the reconstruction of nuclear theory is to retain the IPM description of nucleon quantum states without insisting on the fiction of a nuclear "gas". Let us begin the reconfiguration of nuclear theory with a brief review of the application of quantum theory to the problems of atomic structure.

II. Quantum Electrodynamics

Since the beginnings of quantum theory, many conceptual insights, countless verifications and – not to be overlooked – several profound philosophical debates concerning its interpretation have been initiated. Controversial interpretations of quantum theory continue to be the source of interesting speculations (parallel universes, time travel, parapsychology, and so on), but, as a matter of fact, practicing physicists can rely on the mathematical formalism developed over the past century to predict nuclear, atomic, molecular and solid-state phenomena. In that respect, there is no doubt that quantum mechanics is correct, and has had its widest practical applications in the form of QED. Notably, unlike the many debates concerning the interpretation of quantum phenomena (the collapse of the wave function, the interpretation of the uncertainty principle, the wave-particle duality, the stochastic nature of reality, etc.), there are today few dissenting opinions on the amazing precision of QED. As a quantitative theory that allows for an understanding of the absorption or emission of photons in terms of the transitions of electrons from one quantal state to another, QED remains unchallenged.

The quantum mechanics of the atom is technically complex, but its conceptual simplicity can be illustrated as in Figure 1. As first understood by Bohr in the 1920s, for a hydrogen-like atom in which there is one electron orbiting around a central nucleus containing Z-charges, the entire set of excited states, their transitions and light spectra can be calculated on the basis of certain quantal assumptions (Figure 1A). Adding a second electron introduces electron-electron effects that can be computed, and further electrons introduce screening effects that must be handled on an ad hoc basis, but the fully developed theory of QED remains qualitatively accurate and, with suitable parameter adjustments, quantitatively meaningful (Figure 1B).

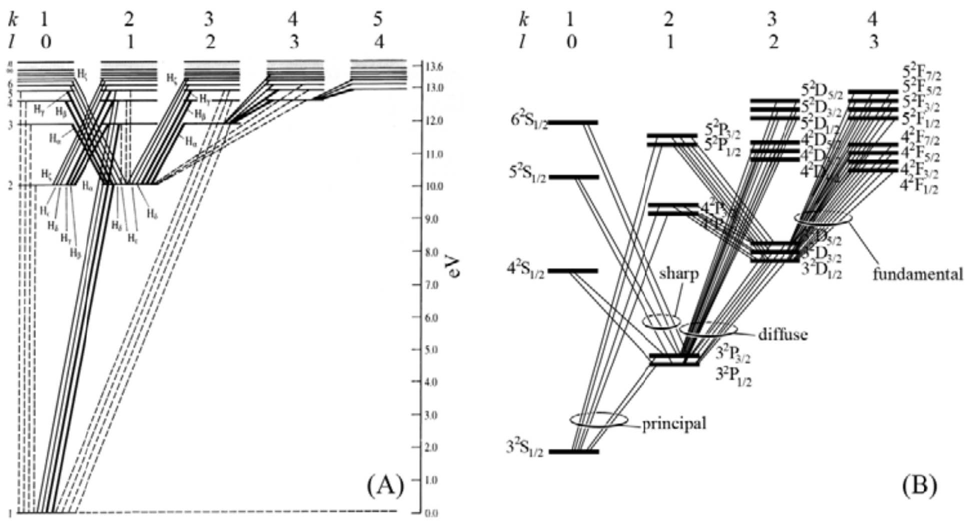

Figure 1: (A) The energy states of Hydrogen – all of which can be calculated in quantum mechanics. (B) Energy levels and allowed one-electron transitions of the sodium atom.

The most impressive results concern the light spectra, but, from a theoretical perspective, the underlying quantal "texture" of the electron states is also significant. As illustrated in Figure 1, each electron state is a specific configuration of ***n***, ***l*** and ***m*** quantum numbers – that are used in the calculation of the photon energies and of the allowed and forbidden transitions. The Schrodinger equation that embodies the relationships among ***n***, ***l*** and ***m*** is:

$$\Psi_{n,l,m} = R_{n,l}(r)\, Y_{m,l}(\theta, \phi) \qquad \text{Eq. 1}$$

The permutations of ***n***, ***l*** and ***m*** – and their dual occupancy with spin-up and spin-down electrons (***s***) provides the entire theoretical framework for determining the energy states of electrons (Eqs. 2-5, Table 1). As stated in all textbooks on atomic theory, quantum numbers, ***n***, ***l***, m_l and m_s can take certain integer or half-integer values:

$n = 1, 2, 3, 4, \ldots$ Eq. 2
$l = 0, 1, 2, \ldots, n-1$ Eq. 3
$m_l = -l, \ldots, -2, -1, 0, 1, 2, \ldots, l$ Eq. 4
$m_s = s = 1/2, -1/2$ Eq. 5

Based on the regularities of electron occupancy in the shells of Eqs. 1-5, it became possible to explain the length of the periods in the Periodic Table of the elements, and that theoretical achievement was a decisive factor in establishing the quantum theory of the atom. Say what one will about notions concerning the philosophical implications of quantum theory, the pattern of electron states (Table 1) and its implications for light spectra are the bedrock of atomic theory, and the foundation upon which the Periodic Table – and essentially all of chemistry – is now understood.

		K	L				M								
Quantum Numbers	n	1	2				3								
	l	0	0	1			0	1			2				
		s	s	p			s	p			d				
	m_l	0	0	-1	0	1	0	-1	0	1	-2	-1	0	1	2
	m_s	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩
Closed (sub)Shells		K	(L1)	L2			(M1)	M2			(M3)				
Number of States		2	2	6			2	6			10				
Total Electrons		2	10				18				36				

		N									O			
Quantum Numbers	n	4									5			
	l	0	1			2					0	1		
		s	p			d					s	p		
	m_l	0	-1	0	1	-2	-1	0	1	2	0	-1	0	1
	m_s	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩
Closed (sub)Shells		(N1)	N2			(N3)					(O1)	O2		
Number of States		2	6			10					2	6		
Total Electrons						54								

		N							O					P			
Quantum Numbers	n	4							5					6			
	l	3							2					0	1		
		f							d					s	p		
	m_l	-3	-2	-1	0	1	2	3	-2	-1	0	1	2	0	-1	0	1
	m_s	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩
Closed (sub)Shells		(N4)							(O3)					(P1)	P2		
Number of States		14							10					2	6		
Total Electrons		86															

Table 1: The full set of ***n***-, ***l***-, $\boldsymbol{m_l}$- and $\boldsymbol{m_s}$-quantal states of the first 86 electrons. The structural complexity of the electron orbitals makes quantum mechanics mathematically difficult, but its conceptual simplicity lies in the integer relationships among the quantum numbers.

III. Quantum Nucleodynamics

Nuclear structure is both similar to atomic structure (in terms of quantal states) and different (in terms of the forces holding these systems together), but the conventional view of the nuclear IPM since the 1950s has been that the two systems are analogous even in terms of the underlying forces. That is, by assuming a time-averaged nuclear potential-well that mimics a long-range force and, moreover, by invoking the Pauli exclusion principle (that is presumed to "block" local nucleon-nucleon interactions in the high-density nuclear interior), a theoretical model similar to that in atomic theory was developed for use in nuclear theory. The theoretical contortions that have been devised to maintain this low-density/high-density story for the nucleus are outlined in the textbooks, but there is an alternative view that has been some decades in the making.

It began with Wigner's Nobel Prize winning publications from the 1930s, and was developed by Everling in the 1950s, Lezuo in the 1970s, and by Cook, Dallacasa, DasGupta, Musulmanbekov and various others ever since. The key insight, stated by Wigner in 1937, is that the quantal symmetries of nucleon eigenvalues correspond to the symmetries of a face-centered cubic (fcc) lattice. Wigner himself was a mathematician and his discussion of nuclear states was in terms of an abstract, multidimensional "momentum space," but all subsequent developments of the lattice model of nuclear structure have been in terms of coordinate space, i.e., 3D geometry. In retrospect, the early emphasis on the common-sense geometry of the lattice model was perhaps a tactical mistake, because the nucleus, whether a lattice or a diffuse gas, is a quantum mechanical object that defies common sense in many respects. Moreover, the unfortunate, but inevitable first impression of (pre-)classical physics and platonic solids made the lattice representation of nuclear symmetries appear to be wrong-headed attempts to return to pre-modern ideas. Nonetheless, as demonstrated in dozens of publications in the physics literature, there is a remarkable mathematical identity between nuclear quantal states and the symmetries of an antiferromagnetic fcc lattice with alternating isospin layers.

From the perspective of the gaseous IPM, the lattice representation of nuclear symmetries might be dismissed as a "lucky coincidence" without physical meaning, but the contrary view is worth considering: Could it be that the gaseous-phase IPM fortuitously mimics the symmetries of the lattice, rather than vice versa? In terms of the known dimensions of the nucleus, is the lattice not a far more realistic (LDM-like) model of the nuclear texture than a Fermi gas? And, most pointedly, is it not more reasonable to construct a nuclear theory on the basis of the *known* short-range nuclear force, rather than construct *de novo* a theoretical long-range force in order to justify a gaseous model?

A. Theoretical framework

The significance of the identity between the IPM and the lattice is that *every* known nuclear state in the IPM has a specific analog in 3D coordinate space. *Every* transition of nucleons from one quantal state to another – explicable in terms of integral changes in the quantum numbers of the Schrodinger equation – necessarily corresponds to a specific vector in the nuclear lattice space. As a consequence, without resorting to the fiction of a nuclear "mean-field," the quantum mechanics of the gaseous-phase IPM can be reconstructed within the lattice. From a computational perspective, the most interesting aspect of the lattice is that its inherent geometry leads to a fine-grained, realistic, local-interaction version of the IPM, i.e., what might be considered to be the structural foundations of QND.

In comparison to atomic theory, there are two factors that increase the complexity of the nuclear version of the Schrodinger wave equation. The first is that the nucleus contains two types of nucleon, protons and neutrons, that are distinguished in terms of the so-called isospin quantum number, ***i***. The second is the notion of the coupling of orbital angular momentum (***l***) with intrinsic angular momentum (***s***) – giving each nucleon a total angular momentum quantum value (***j***=***l***+***s***). As a consequence, the nuclear version of the wave-equation has two additional subscripts (Eq. 6) and a slightly more complex pattern of shell/subshell occupancy (Table 2).

$$\Psi_{n,j\,(l+s),m,i} = R_{n,j\,(l+s),i}\,(r)\; Y_{m,j\,(l+s),i}\,(\theta, \phi) \qquad \text{Eq. 6}$$

Despite those additional quantum numbers, the nuclear wave-equation holds the same promise of QED in being a finite set of explicit quantal states into which nucleons can come and go with the release or absorption of photons. The universally-acknowledged strength of the IPM (ca. 1950) lay in the fact that each nucleon in the model has a unique set of quantum numbers, as specified in Eq. 6 and Table 2. Using that foundation for describing individual nucleons, the IPM makes it possible to explain *nuclear* states as the simple summation of the properties of its "independent" *nucleons* (Figure 2) and to compare theoretical predictions with experimental data. Those predictions were great successes in the early 1950s and led to optimistic predictions about the impending development of a rigorous, quantitative QND theory.

Quantum Numbers	n	0	1			2						3										4					
	l	0	1		0	2			1		0	3				2			1		0	4					…
	j	1/2	3/2		1/2	5/2			3/2		1/2	7/2				5/2			3/2		1/2	9/2					…
	$\|m\|$	1/2	3/2	1/2	1/2	5/2	3/2	1/2	3/2	1/2	1/2	7/2	5/2	3/2	1/2	5/2	3/2	1/2	3/2	1/2	1/2	9/2	7/2	5/2	3/2	1/2	7/2
	s	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩	⇧⇩
Number of States		2	4		2	6			4		2	8				6			4		2	10					…
(Semi)magic Numbers		**2**	(6)		**8**	(14)			(18)		**20**	**28**				(34)			(38)		(40)	**50**					…
Total Nucleons	i	4	12		16	28			36		40	56				68			76		80	100					…

Table 2: The quantum states of nucleons in the IPM. As in atomic physics the theoretical shells and subshells can be adjusted to explain the existence of closed shells at the "magic" numbers.

Unfortunately, the IPM was based on the dubious assumption of a gaseous nuclear interior with "point" nucleons orbiting unimpeded inside the nucleus. Although the central attractive force in *atomic* physics – where the nucleus itself attracts the orbiting electrons – was well-founded and the electron is *small* relative to the atomic volume, similar assumptions in nuclear theory have turned out to be incorrect. Although not yet known in the 1930s, when the Fermi gas model was first considered, the experimental work of Hofstadter in the early 1950s (Nobel Prize in 1961) showed that both the proton and the neutron have hard-core particle structure and diameters of ~1.8 fm. Since a center-to-center nearest-neighbor internucleon distance of 2.0 fm reproduces the known nuclear density (0.17 nucleons/fm^3), it is neither true that nucleons can be thought of as "points" nor true that they are free to "orbit" in the nuclear interior. To deal with those inconvenient facts, a huge industry of theoretical developments ensued to explain the surprising successes of the IPM, but that effort has not led to clarity concerning either the nuclear force or the multitude of known excited states (e.g., Figure 2).

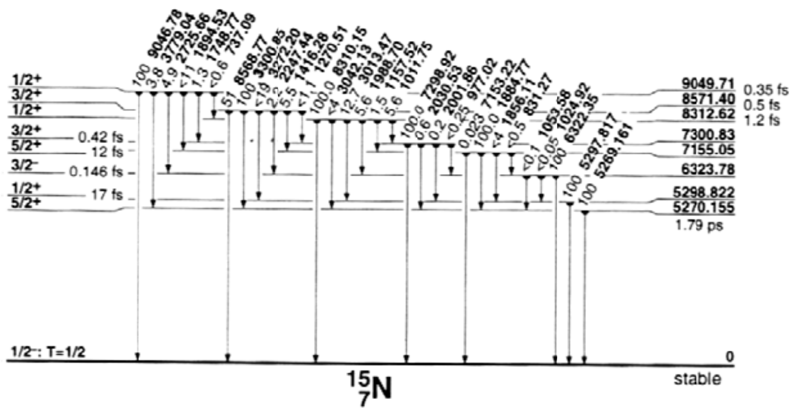

Figure 2: An example of the level of experimental detail in nuclear spectroscopy. The ***J***-values, parities, lifetimes, relative transition probabilities and energies of low-lying excited states of ^{15}N are known. Total angular momentum ***J***-values and parities are consistent with the IPM.

By the mid-1960s, nuclear structure theory had ossified into an utter paradox – an insoluble enigma where the nucleus is said to be both a dense-liquid and a diffuse-gas and punctuated with alpha-particle clusters. Paradoxes unlike anything in atomic theory remained. Considerations of nuclear size, density and binding energies clearly demonstrated a high-density LDM-like nuclear texture; considerations of alpha-decay and the binding energies of the small 4n-nuclei indicated the presence of alpha particles in both stable and unstable nuclei; and considerations of nuclear spins, magnetic moments and parities suggested the reality of a nuclear gas with each independent nucleon having its own unique set of quantized characteristics. Although it was a convoluted theoretical story (that is still reiterated in the textbooks), it was also true that the pace of developments in nuclear weaponry, nuclear power and applications of nuclear isotopes made skepticism about nuclear theory appear nonsensical. As a consequence of its quantum mechanical foundation, the IPM, rather than the LDM or cluster models, became the centerpiece of nuclear structure theory and, ever since, theorists have struggled to justify the assumption of a central nuclear potential-well in a substance that appears to be a dense, chunky liquid.

So, what are the overwhelming strengths of the IPM that make it so important? To begin with, the known range of nucleon quantum numbers can be explained in close analogy with the quantal characteristics of electrons:

$\boldsymbol{n} = 0, 1, 2, \ldots$ Eq. 7

$\boldsymbol{j} = 1/2, 3/2, 5/2, \ldots, (2\boldsymbol{n}+1)/2$ Eq. 8

$\boldsymbol{m} = -\boldsymbol{j}, \ldots, -5/2, -3/2, -1/2, 1/2, 3/2, 5/2, \ldots, \boldsymbol{j}$ Eq. 9

$\boldsymbol{s} = 1/2, -1/2$ Eq. 10

$\boldsymbol{i} = 1, -1$ Eq. 11

Together with the Schrodinger equation itself, Eqs. 7-11 are essentially a concise statement of the established quantum mechanical structure of the nucleus. Both its IPM character and the “magic” numbers of the shell model can then be obtained by manipulations of the nuclear shells and subshells (Table 2). Historically, this was interpreted as “proof” of the gaseous nature of the nucleus, but it was later found that the entire pattern of quantal states of the nucleus can also be stated in terms of the lattice coordinates (x, y, z) for each nucleon (Eqs. 12-14):

$$x = |2\boldsymbol{m}|(-1)^{(\boldsymbol{m}-1/2)} \qquad \text{Eq. 12}$$
$$y = (2\boldsymbol{j}+1-|x|)(-1)^{(\boldsymbol{i}/2+\boldsymbol{j}+\boldsymbol{m}+1/2)} \qquad \text{Eq. 13}$$
$$z = (2\boldsymbol{n}+3-|x|-|y|)(-1)^{(\boldsymbol{i}/2+\boldsymbol{n}-\boldsymbol{j}+1)} \qquad \text{Eq. 14}$$

And the Cartesian coordinates of the nucleons can then be used to define their quantal characteristics (Eqs. 15-19):

$$\boldsymbol{n} = (|x| + |y| + |z| - 3) / 2 \qquad \text{Eq. 15}$$
$$\boldsymbol{j} = (|x| + |y| - 1) / 2 \qquad \text{Eq. 16}$$
$$\boldsymbol{m} = |x| * (-1)^{((x-1)/2)} / 2 \qquad \text{Eq. 17}$$
$$\boldsymbol{s} = (-1)^{((x-1)/2)} / 2 \qquad \text{Eq. 18}$$
$$\boldsymbol{i} = (-1)^{((z-1)/2)} \qquad \text{Eq. 19}$$

The significance of Eqs. 12~19 lies in the fact that, if we know the IPM (i.e., quantum mechanical) structure of a nucleus, then we also know its lattice structure, and vice versa. The known pattern of quantum numbers and the occupancy of protons and neutrons in the ***n***-shells and ***j***- and ***m***-subshells are *identical* in both descriptions, but, in coordinate space, the abstract symmetries of the Schrodinger equation exhibit familiar geometrical symmetries, as well. The ***n***-shells and, ***j***- and ***m***-subshells have spherical, cylindrical and conical symmetries, respectively, while ***s***- and ***i***-values produce orthogonal layering. Examination of the symmetries in relation to the Cartesian coordinates shows the validity of Eqs. 12-19 (see the Appendix) and the quantal structure of even the large nuclei can be easily analyzed using software designed for that purpose (Cook et al., 1999). The mathematically unambiguous isomorphism between quantum space and lattice space has been elaborated on in many publications over the past three decades, and recently summarized in a monograph (Cook, 2010). The implications for the establishment of QND are, however, new and are outlined below.

B. Relation to nuclear states

The essential difference between the conventional IPM of the nucleus and its lattice version lies in the assumptions concerning the nuclear force. They both produce – identically – the same set of quantal states for any given number of protons and neutrons and are equivalent descriptions of the known independent-particle character of nuclei, but there is nonetheless a huge difference between the two. That is, in the conventional IPM, there is no realistic possibility of calculating the local forces acting on nucleon ***a*** because nucleon ***a*** is assumed to be imbedded in the “mean field” of all other nucleons orbiting within the nucleus and interacting with other nucleons, ***b, c, d, …, z*** to varying and completely unknown degrees. In contrast, the same nucleon state in the lattice has an explicit set of local nucleon-nucleon interactions for 1st, 2nd and 3rd (etc.) nearest-neighbors, as is implied by the lattice geometry. Computationally, that difference is significant because the lattice geometry is a (fairly complex, but) tractable problem. What that implies is that, for a given number of protons and neutrons, either approach can account post hoc for the experimentally-known set of excited states with specific energies, ***J***-values, parities and magnetic moments, but only the lattice version can state unambiguously that nucleon ***a*** with known quantal characteristics and known position within the lattice is a specific distance and orientation in relation to nucleon ***b*** with its own quantal characteristics – and similarly for nucleons ***c*** through ***z***, and beyond.

In this regard, it should be said that the conventional IPM is essentially correct in its quantal *description* of nuclear states. However, in utilizing a mean-field nuclear force, the conventional model is inherently incapable of specifying the nature of the local nucleon-nucleon interactions for any particular nucleon. On the other hand, because of the lattice geometry, the lattice version of the IPM necessarily includes a complete specification of all of the local nucleon-nucleon interactions that any particular nucleon imbedded in the lattice experiences. The nuclear lattice does not of course address issues of nucleon substructure or the interpretation of quantum theory itself, and many aspects of quantum "weirdness" remain enigmas in the lattice. Nevertheless, the nucleon lattice has a comprehensible substructure that is entirely absent in a nuclear "gas."

In effect, both the conventional and the lattice versions of the IPM can be used to *describe* any nuclear state and the transitions among the stable and excited states that are allowed, forbidden or super-allowed. But drawing parallels between the experimental data and theory is not proof of either structure. Both versions exhibit the same quantal descriptive powers, but the lattice also makes possible the calculation of local two-body nucleon-nucleon interactions. It is for this reason that the lattice version has the potential for being the basis for a rigorous QND, whereas the gaseous version remains inherently "too difficult" – even with supercomputer assistance – and ends up with, at best, a vast array of adjustable parameters that must then be "fitted" without theoretical foundation to the empirical data.

V. Conclusion

Subsequent to the (re)discovery of the fact that the internal symmetries of an fcc lattice reproduce the well-established symmetries of the IPM, the lattice model has been developed in a variety of ways. Arguments concerning the "unification" of the nuclear models and visualization of nuclear structure remain of peripheral interest, but a far more valuable step would be the accurate prediction of nuclear properties on the basis of lattice symmetries *without going through the theoretical contortions* of the fictitious long-range nuclear force of the gaseous IPM. In that respect, the establishment of a computational, lattice-based QND should be welcomed by theorists of all backgrounds and would essentially eliminate the need to choose a nuclear model before engaging in quantitative work: One "chooses" quantum mechanics and then calculates the full set of two-body interactions implied by the lattice representation of IPM. Three-body and higher-order interactions within the lattice might provide greater precision, but the known lattice dimensions and symmetries already provide a first-order description of nuclear states that is deducible solely from two-body interactions.

Given the identity between a gaseous-phase IPM and a lattice IPM, the theoretical situation in nuclear structure physics in the early 21st century is curiously similar to that in chemistry in the middle of the 19th century. In both fields a foundation of empirical findings was first established from painstaking laboratory work, where the primary data were masses and dissociation energies. Initially, 3D configurations of particles were *not* thought to be either realistic as depictions of the physical reality or useful as heuristics for theoretical study. The most notorious example of the *disregard* for geometrical considerations in chemistry concerns the benzene molecule. On the basis of experimental work, benzene had been determined to consist of 6 carbon atoms and 6 hydrogen atoms, C_6H_6. Kekulé proposed a hexagonal ring of carbons, but for *decades* the academic authorities in chemistry rejected all notions of molecular structure – both Kekulé's structure for benzene and van't Hoff's notion of a geometrically asymmetrical carbon atom. Journal editors, such as A.W.H. Kolbe, famously argued that stereochemistry was “loose speculation parading as theory” indulged in by those with “no liking for exact chemical investigation.” Eventually, of course, Kekulé became known as “the father of modern stereochemistry,” and three of his students, including van’t Hoff, won Nobel Prizes in chemistry in the early 20th century.

It is relevant to note that the rejection of notions of 3D structure in 19th century chemistry had nothing to do with the philosophical quandaries of the interpretation of the uncertainty principle, the wave/particle dilemma or the collapse of the wave equation, etc. Indeed, quantum mechanics did not emerge until several decades later, but there was nonetheless, already in the mid-19th century, a strong reluctance among practicing chemists to “speculate” about spatial structure. Understandably, perhaps, most chemists were wary of the daunting complexity of the structural permutations implied by stereochemical considerations, but eventually inclusion of the constraints of molecular geometry proved necessary. Ultimately, the blanket dismissal of the complexities of 3D structure by “old school” laboratory chemists proved to be unfounded, and stereochemistry has of course become a mainstream issue in all aspects of chemistry, biochemistry and molecular biology.

In the early 21st century, nuclear physics has arrived at a similar fork in the road, where “old school” experimentalists would maintain that there is no nuclear substructure inherent to the pattern of data, such as shown in Figure 2. In effect, they argue that the structural deconvolution of the wave-equation into structural subcomponents is impossible. Many theorists are in fact hopeful that longstanding theoretical difficulties might eventually be overcome by developments in computer hardware without addressing issues of 3D structure and some are even convinced that the very idea of nuclear substructure “violates” quantum mechanics. The lattice representation of the nuclear IPM symmetries, however, indicates a possible way forward for those who are willing to “speculate” on the internal structure of the atomic nucleus. Whether or not quantitative QND lies just over the hill remains to be seen.

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Appendix

5	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
6			(Keyboard Input)			Quantum Numbers and Parities Calculated							Lattice Sites Calculated from the			X,Y,Z
7	Nucleon Sequence		FCC Lattice Sites			from the lattice sites (Eqs. 4-10)							Quantum Numbers (Eqs. 1-3)			coords.
8	Proton No.	Neutron No.	X	Y	Z	*n*	*L*	*j*	*m*	*s*	*i*	*parity*	X	Y	Z	Check
9		1	1	-1	-1	0	0	1/2	1/2	1/2	-1	1	1	-1	-1	ok
10	1		1	1	1	0	0	1/2	1/2	1/2	1	1	1	1	1	ok
11		2	-1	1	-1	0	0	1/2	-1/2	-1/2	-1	1	-1	1	-1	ok
12	2		-1	-1	1	0	0	1/2	-1/2	-1/2	1	1	-1	-1	1	ok
13		3	1	3	-1	1	1	3/2	1/2	1/2	-1	-1	1	3	-1	ok
14	3		3	-1	1	1	1	3/2	-3/2	-1/2	1	-1	3	-1	1	ok
15		4	-1	-3	-1	1	1	3/2	-1/2	-1/2	-1	-1	-1	-3	-1	ok
16	4		-1	3	1	1	1	3/2	-1/2	-1/2	1	-1	-1	3	1	ok
17		5	3	1	-1	1	1	3/2	-3/2	-1/2	-1	-1	3	1	-1	ok
18	5		-3	1	1	1	1	3/2	3/2	1/2	1	-1	-3	1	1	ok
19		6	-3	-1	-1	1	1	3/2	3/2	1/2	-1	-1	-3	-1	-1	ok
20	6		1	-3	1	1	1	3/2	1/2	1/2	1	-1	1	-3	1	ok
21		7	-1	1	3	1	0	1/2	-1/2	-1/2	-1	-1	-1	1	3	ok
22	7		1	1	-3	1	0	1/2	1/2	1/2	1	-1	1	1	-3	ok
23		8	1	-1	3	1	0	1/2	1/2	1/2	-1	-1	1	-1	3	ok
24	8		-1	-1	-3	1	0	1/2	-1/2	-1/2	1	-1	-1	-1	-3	ok
25		9	3	-3	-1	2	2	5/2	-3/2	-1/2	-1	1	3	-3	-1	ok
26	9		3	3	1	2	2	5/2	-3/2	-1/2	1	1	3	3	1	ok
27		10	-3	3	-1	2	2	5/2	3/2	1/2	-1	1	-3	3	-1	ok
28	10		-3	-3	1	2	2	5/2	3/2	1/2	1	1	-3	-3	1	ok
29		11	5	-1	-1	2	2	5/2	5/2	1/2	-1	1	5	-1	-1	ok
30	11		5	1	1	2	2	5/2	5/2	1/2	1	1	5	1	1	ok
31		12	-5	1	-1	2	2	5/2	-5/2	-1/2	-1	1	-5	1	-1	ok
32	12		1	5	1	2	2	5/2	1/2	1/2	1	1	1	5	1	ok
33		13	1	-5	-1	2	2	5/2	1/2	1/2	-1	1	1	-5	-1	ok
34	13		-1	-5	1	2	2	5/2	-1/2	-1/2	1	1	-1	-5	1	ok
35		14	-1	5	-1	2	2	5/2	-1/2	-1/2	-1	1	-1	5	-1	ok
36	14		-5	-1	1	2	2	5/2	-5/2	-1/2	1	1	-5	-1	1	ok
37		15	3	1	3	2	1	3/2	-3/2	-1/2	-1	1	3	1	3	ok
38	15		-1	3	-3	2	1	3/2	-1/2	-1/2	1	1	-1	3	-3	ok
39		16	1	3	3	2	1	3/2	1/2	1/2	-1	1	1	3	3	ok
40	16		1	-3	-3	2	1	3/2	1/2	1/2	1	1	1	-3	-3	ok
41		17	-1	-3	3	2	1	3/2	-1/2	-1/2	-1	1	-1	-3	3	ok
42	17		3	-1	-3	2	1	3/2	-3/2	-1/2	1	1	3	-1	-3	ok
43		18	-3	-1	3	2	1	3/2	3/2	1/2	-1	1	-3	-1	3	ok
44	18		-3	1	-3	2	1	3/2	3/2	1/2	1	1	-3	1	-3	ok
45		19	-1	1	-5	2	0	1/2	-1/2	-1/2	-1	1	-1	1	-5	ok
46	19		1	1	5	2	0	1/2	1/2	1/2	1	1	1	1	5	ok
47		20	1	-1	-5	2	0	1/2	1/2	1/2	-1	1	1	-1	-5	ok
48	20		-1	-1	5	2	0	1/2	-1/2	-1/2	1	1	-1	-1	5	ok
49	Proton No.	Neutron No.	X	Y	Z	*n*	*L*	*j*	*m*	*s*	*i*	*parity*	X	Y	Z	Check

Snapshot of a spreadsheet in which nucleon quantum values (Columns F~L) are calculated from the lattice coordinates (Columns C~E). Just as there are no two nucleons with identical Cartesian coordinates, there are no two nucleons with the same set of quantum numbers. The spreadsheet and computer algorithms (in the C-language) for calculating nuclear lattice properties can be downloaded at http://www.res.kutc.kansai-u.ac.jp/~cook/40%20NVSDownload.html.