The Emergence of Fundamental Physical Concepts By Summing Discrete Vectors

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ABSTRACT

In a manner somewhat reminiscent of Feynman's method of describing QED by summing vectors, I describe how mass, velocity, the ticking of clocks and gravitation can be described using geometry and summing discrete vectors and vector magnitudes. The resulting model agrees completely with Einstein's special relativity and provides an attractive Machian alternative to general relativity which agrees with tests of general relativity.

Introduction

I will be describing a series of thought experiments performed on test particles. The goal is to describe a model in which these test particles behave in a way which agrees with our real world observations. But before I begin I ask the reader to abandon his current ideas about mass, speed and time at the fundamental level with the assurance that these concepts will emerge faithfully from the model. Our thought experiments occur in a universe which only contains test particles. The test particles are matter but are all massless, thus a distinction is made between matter and mass. Mass is an emergent phenomenon which is described as we continue. The particles have no inertia and any change in direction of motion occurs instantly. The particles may travel in any direction but they all travel at the same speed, which also means that there are no motionless particles. The frame of reference is not important as it will be shown later that all frames of reference are equally valid, but for now I ask that you just consider a single frame of reference as a stationary observer equipped with a measuring rod. The properties I have given so far for our test particles bear some resemblance to the properties of light, but I am describing particles of matter.

Motion Without Time

If we have a system which is completely frozen, where everything is motionless we can easily consider doing away with time. However, can we conceive of a system that has motion without time while avoiding any contradiction? It is possible as long as all the particles in the system move at the same speed. If there are no stationary particles and no varying speeds among different particles, we can also do away with time. We can explore this idea by considering what is really meant if all particles move at the same speed. You may be asking what is this single speed of all particles in terms of change in distance divided by change in time? When all particles move at the same speed the idea of change in time becomes irrelevant and the notion should be completely discarded. Before continuing I want to address the potential objection that this must violate Lorentz symmetry. It does not, and the details will be developed as we continue. For now consider yourself a stationary observer, and once I have covered all the details in the following sections you will see that all reference frames are equally valid.

Example 1

> Let's envision a handful of our test particles and give them random but definite starting positions and random but definite directions of motion and let's say they all have a speed of 10 meters a second. You may now imagine the unfolding of events. Now taking the same defined starting positions and directions let's substitute a different speed, any speed. The unfolding of events occurs in exactly the same way. If we represent the unfolding of events as a continuum of instantaneous snapshots we would have an equivalent set of snapshots no matter what speed the particles are given. The change in distance is meaningful, the change in time is not. The concept of change in time is further diminished if we consider that anything we use as a clock should actually exist within our universe of test particles and thus its rate of ticking would be tied to the motion of the particles.

Introducing Discrete Vectors

I will begin referring to the speed of the test particles as c. At the macroscopic level we can have the test particles moving at speeds less than c by defining their motion as a repeating sequence of discrete vectors.

Example 2

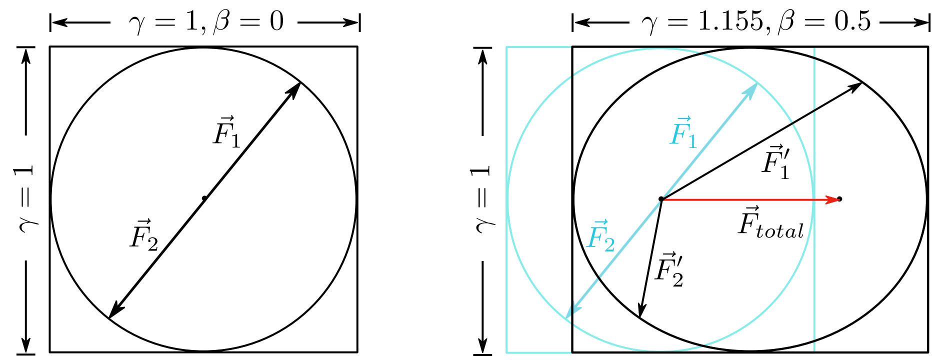
> If we look at a particle and see that its motion consists of moving right one meter at c then moving right another meter then moving left one meter, after which this sequence continually repeats itself, we would see that it's effectively moving to the right at one third of c. What if we use this same sequence but change the distance of each movement to one millimeter. The rate it moves to the right is the same, one third of c, but the motion appears much smoother. Remember that the particles have no inertia and can change direction instantly. If we further reduce the distance of each movement to a length smaller than our ability to measure we might conclude that the particle is indeed moving at one third of c because we cannot detect on the fundamental level that it is in fact moving at c in a sequence of discrete vectors. Using this technique we can see that on a macroscopic level we may observe our test particles moving at various different speeds from zero to c, when in fact on the fundamental level the particles are always moving at c while repeating a sequence of discrete vectors. A particle observed macroscopically to have a speed of zero is at the fundamental level moving at c but its sequence of vectors is comprised of opposing vectors which sum to zero or null vector.

This idea that particles are always moving at c through spacetime, three spatial dimensions and one time dimension, is already the accepted view. By having motion comprised of a sequence of discrete vectors as my model suggests, particles can always move at c in three spatial dimensions without the need for a dimension of time while also agreeing with observations. Having a time dimension is only helpful if particles can move at varying speeds in the three spatial dimensions. There is also a relation to Dirac particles which always have an instantaneous speed of c yet their empirical speed is an average determined by measurable changes in position and time [A].

Discrete Vectors Undergoing Lorentz Boost

Let's consider a particle that macroscopically has a speed of zero and moves according to a repeating sequence of vectors $\vec{F}_{1..n}$. For simplicity let's comprise $\vec{F}_{1..n}$ with pairs of opposing vectors of equal magnitude. There is no requirement that the vectors need to be organized in this manner, this is just for ease of explanation. Later we will describe a Machian origin for these vectors in greater detail, but for now we will simply say that each vector is associated with a distant particle which the vector always remains pointed towards.

I will start with the simplest example of a particle with $\vec{F}_{1..n}$ with n equal to 2, after which we will understand the behavior of the vectors when n is large. The vectors are labeled and $\vec{F}_1$ and $\vec{F}_2$ in the below left diagram and both have a magnitude of 0.5. Other than being opposed, the direction of the vectors is arbitrary and the example will work correctly with any given direction. The sum of the vector magnitudes is one and correspondingly the γ factor is one. The sum of the vectors is zero or null vector and correspondingly β or its speed as a fraction of c is zero.



Now a photon represented as a vector $\vec{J}$ with a magnitude of 0.155 pointing right is going to strike the particle from the left side. While ignoring heat, the particle will fully absorb $\vec{J}$ and be propelled to the right. The γ factor for the particle becomes 1.155 and its speed as a fraction of c is determined to be 0.5 by using the Lorentz γ factor formula below.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad or \qquad \beta = \sqrt{1-\frac{1}{\gamma^2}}$$

Now we need to calculate how the angles and magnitudes of $\vec{F}_1$ and $\vec{F}_2$ have changed. Remember that the vectors have a Machian connection to other distant particles and the vectors always remain pointed at these distant particles. Because of aberration $\vec{F}_1$ and $\vec{F}_2$ transform to $\vec{F}'_1$ and $\vec{F}'_2$ respectively, which is shown in the figure above and to the right. From the particles frame of reference the vectors are always pointed at their distant targets. The equations Einstein used to calculate the aberration of light can be repurposed to calculate the new angles and magnitudes of the vectors and are shown below respectively. Originally v represented frequency, but we will use v to represent the vector magnitude.

$$\cos\theta' = \frac{\cos\theta-\beta}{1-\cos\theta\cdot\beta} \qquad\qquad v' = v\frac{1-\cos\theta\cdot\beta}{\sqrt{1-\beta^2}}$$

We can achieve the same results as the equations in a more visual way by using the geometry of an ellipsoid or in this case an ellipse which is easier to diagram. A stationary particle is represented by a circle. A circle is an ellipse with both foci at the same location. In this case we have two opposing vectors of equal magnitude, $\vec{F}_1$ and $\vec{F}_2$, which again represents a stationary particle since the sum of the vectors is zero. The sum of the vectors being zero relates to the distance between the foci which is also zero. When using two opposing vectors the sum of the vectors is always a vector originating on one focus point and terminating on the other focus point and this holds true even after a Lorentz boost. The magnitude of the sum of the vectors divided by the sum of the vector magnitudes equals the eccentricity of the ellipse which is β or the speed of the particle as a fraction of c. The sum of the vector magnitudes is equal to the length of the major axis which is γ [B]. When the circle receives a Lorentz boost to the right the circle is stretched into an ellipse uniformly along the x axis by the γ factor. The ends of the vectors remain inscribed to the points on the curve while the curve is uniformly stretched. While the foci move away from each other, the origin of the vectors remain at the first focus point which give us $\vec{F}'_1$ and $\vec{F}'_2$. This geometric technique works no matter how the two opposing vectors are oriented which is important when considering a stationary particle which has an innumerable number of vectors. We can now find that a stationary particle with innumerable vectors is related to a set which contains many pairs of opposing vectors of equal magnitude [C]. This means we don't need to specifically know all the directions and magnitudes of all the vectors but instead we can encapsulate all the pertinent information these discrete vectors represent into something easier to work with, namely the sum of the vectors and the sum of the vector magnitudes from which we will know the velocity and relativistic mass of the particle. From this more manageable information we can readily perform Lorentz transformations without needing to know the specific details of each discrete vector.

These operations in this example can also be performed in reverse where a moving particle is slowed down on the macroscopic level when giving off energy in the direction of its motion.

By now it is clear that our particles which are massless on the fundamental level would be observed to have mass or inertia at the macroscopic level in a way which agrees with special relativity. I have not addressed what determines the dimensions of our test particles in this paper and will only say that whatever dimensions a particle has will undergo Lorentz contraction in the direction of motion.

Reducing Physics Using Ratios of Similar Terms

The velocity of a particle can be described by the following formula which sums the vectors and divides by the sum of the vector magnitudes. The velocity is a fraction of *c*.

$$\vec{V} = \frac{\sum \vec{F}_n}{\sum ||\vec{F}_n||}$$

Since we are creating a ratio of similar terms, the unit of length used to measure the vectors is not important and the resulting velocity remains invariant no matter what unit is used as long as that unit is used consistently for all vectors. The mass of a particle is described by the following formula which sums the vector magnitudes. This is relativistic mass so any kinetic energy is included.

$$M = \sum ||\vec{F}_n||$$

Again let's address the use of ambiguous units. The concept of mass is only meaningful when comparing the masses of two or more things so we are yet again creating a ratio of similar terms. Defining a kilogram to equal a specific number of particles in a particular location in the universe is arbitrary and not important to our fundamental understanding of mass. We can also compare a particle's mass to a quantity of energy without problem, for example we can represent a photon of a given energy as a vector which is measured in the same units as the vectors of the particle it will strike since mass and energy are equivalent. The momentum of a particle is described by the following formula which sums the vectors.

$$\vec{P} = M\vec{V} = \sum ||\vec{F}_n|| \frac{\sum \vec{F}_n}{\sum ||\vec{F}_n||} = \sum \vec{F}_n$$

I find the ability to represent traditionally very dissimilar concepts using similar terms very satisfying.

Gravitational Potential

In preparation for the next section which describes the Machian aspect of the model, it is important to first take a look at gravitational potential from a different perspective. Gravitational potential is represented as U. U is traditionally negative allowing it to increase to zero as distance r approaches infinity. The equation for U can be found by integrating the equation to calculate gravitational force, F_g, so we can alternatively visualize U as the area under the curve for F_g from r to infinity.

$$F_g = \frac{GMm}{r^2} \qquad\qquad U = -\frac{GMm}{r}$$

Instead of looking at gravitational potential which is a negative value or a kind of debt, I prefer to consider a positive value or $-U$ which we can think of as a mass contribution. This positive value is the area under the curve for F_g from r to infinity and is a positive quantity of energy which is a acquired or becomes a property of a particle at different distances of r. As we change the distance r we can visually make the connection that the energy or mass of the particle will change by an amount proportional to the change in area under the curve for F_g from r to infinity. Now let's consider an example which will allow us to more easily introduce the next section on the Machian connection. This example will show how a positively valued mass contribution relates to special and general relativity.

Example 3

> We will consider $body_1$ and $body_2$. $Body_1$ is very massive and $body_2$ has very little mass so for any gravitational attraction between them we can treat any movement of $body_1$ as insignificant and just ignore it for the sake of the example. If $body_1$ and $body_2$ are separated by a very great distance such that the area under the curve for F_g from r to infinity is close to zero, then $body_2$ at rest from that distance r falls toward $body_1$. As $body_2$ continues to fall, r becomes smaller and the area under the curve for F_g from r to infinity grows. Because $body_2$ is picking up greater and greater speed as it falls its kinetic energy is increasing and thus equivalently its relativistic mass is increasing as described by special relativity. This increase in relativistic mass due to special relativity is directly related to the increase in the area under the curve for F_g from r to infinity.
>
> There is also a separate increase in mass described by general relativity which for this example is equal to the increase in mass from special relativity [D]. This increase in mass from general relativity is not dependant on the $body_2$'s kinetic energy but only its proximity to $body_1$. The increase in mass due to general relativity is also directly related to the increase in the area under the curve for F_g from r to infinity.

When determining the properties of the discrete vectors of a test particle there are two separate considerations that need to be made. The first is the Machian connection which will be covered in the next section. This Machian connection deals with the position of the test particle in relation to all the other particles in the universe. The second consideration is the test particle's history of accelerations which come from a series of Lorentz boosts which modify the direction and magnitude of the discrete vectors.

The Machian Connection

The origin of a particle's vectors is a Machian connection to every other particle in our universe. Each individual vector is associated with another unique individual particle and the direction and magnitude of the vector is based on the position of the particles with respect to each other. From example 3 above we find that when looking at $body_2$'s position only (ignoring kinetic energy) with respect to $body_1$ we can conclude that there is a positive contribution to $body_2$'s total mass which is inversely proportional to r.

Let's now see how this idea relates to the properties of the discrete vectors of our test particle. At this point I will also make a distinction between vector concepts. A vector using the notation $\vec{F}_1$ is determined by its Machian connection which we are about to cover as well as any changes detailed from the section on Lorentz boosts. A different notation $\bar{F}_1$ which uses a bar accent will be used when we are considering the vector solely based on its Machian connection and ignoring any changes in direction or magnitude attributed to Lorentz boost.

Example 4

Let's define the universe in our thought experiment to have a multitude of test particles distributed in an even homogenous manner. Let's focus on a single test particle which we will label P_0 and determine its sequence of discrete vectors. All other particles are labeled P_1 to P_n. We will start with P_0 and P_1. The distance between them is r. The first vector for P_0 has its origin at the location of P_0 and its direction points to P_1. This vector will be labeled $\bar{F}_1$ and is considered to be associated with P_1. The magnitude of $\bar{F}_1$ is equal to $1/r$. If P_0 and P_1 move in relation to each other, the direction of $\bar{F}_1$ will also adjust accordingly always originating at P_0 and pointing to P_1. The magnitude will also adjust remaining equal to $1/r$ as the distance between the particles change due to changes in position. These adjustments occur once per cycle $\bar{F}_{1..n}$. After P_0 moves upon $\bar{F}_1$ the vector is adjusted to $\bar{F}'_1$ based on its new position. $\bar{F}'_1$ becomes the vector moved upon during the following cycle $\bar{F}_{1..n}$. The remaining vectors $\bar{F}_{2..n}$ are determined in the same fashion and subsequent adjustments also occurs for the vectors $\bar{F}_{2..n}$ as they are moved upon. P_0 has a vector which is associated with all other particles from $P_{1..n}$ thus P_0 will have vectors from $\bar{F}_{1..n}$.

As to the question of what unit measure of length should be used to determine the distance between particles and thus the magnitude of the vector, the answer is it doesn't matter just as long as you consistently use the same units. The reason it doesn't matter is because the results of our calculations on these vectors do not change as long as we scale all the vectors up or down proportionally. The results come from ratios of similar terms and can be understood without reference to specific units as was demonstrated in example 2. For most people working without units may seem unacceptable and in that case you may go ahead and use meters as the unit of length, which will work fine. However, you will see that the results will be the same no matter what unit of measure is used, and thus the units are largely irrelevant. Again this is because the results are determined by ratios. The only caveat is that an excessively large unit of measure will result in macroscopic observations which appear more granular rather than smooth and analog.

Now let's see how P_0 behaves as it moves sequentially from $\bar{F}_{1..n}$ and then continually repeats the sequence from $\bar{F}_{1..n}$. Since our universe is homogenous we would expect the sum of $\bar{F}_{1..n}$ to be zero or null vector. This is because the directions and magnitudes of the $\bar{F}_{1..n}$ should be distributed evenly with opposing vectors cancelling each other out to arrive at a zero sum. We would then observe macroscopically that P_0 is not moving. Perhaps we should also consider what the result would be if we used a different unit of length to confirm that results remains invariant. Using different units would scale each discrete vector in $\bar{F}_{1..n}$ up or down proportionally and will not change the speed of P_0 as a fraction of c, in this case the speed remains zero.

What if our test universe is not homogenous? Let's move P_1 so that it is now very close to P_0. The remaining $P_{2..n}$ are still distributed homogenously. The sum of $\bar{F}_{2..n}$ is zero, but now $\bar{F}_1$ has a greater magnitude due to P_1's greater proximity to P_0. We would see that the sum of $\bar{F}_{1..n}$ is no longer zero and that P_0 now has a some velocity macroscopically. To determine the velocity due to the new prominence of $\bar{F}_1$ we apply energy $\vec{J}_1$ to $\vec{F}_{1..n}$ which gives us the updated boosted vectors. We calculate $\vec{J}_1$ with the following equation.

$$\vec{J}_1 = \bar{F}_1' \left(\frac{||\bar{F}_1'|| - ||\bar{F}_1||}{||\bar{F}_1'||} \right)$$

If there is no history of prior boosts or all prior boosts have been cancelled out then $\vec{F}_{1..n}$ simply equals $\bar{F}_{1..n}$. After applying $\vec{J}_1$ we continue with $\vec{J}_{2..n}$ as we sequentially move on the associated vectors $\vec{F}_{2..n}$. In this case we can ignore $\vec{J}_{2..n}$ because in this example $P_{2..n}$ are distributed homogenously so those boosts will cancel each other out and do not contribute to macroscopic acceleration. The motion we are describing for P_0 behaves as gravitation [E].

We have now replaced the usual vagueness of Mach's principle with a precise description which determines the discrete vectors of a particle by its relationship with every other particle in the universe.

The Ticking of Clocks

If the movement of particles can be described by $\vec{F}_{1..n}$ which is a repeating sequence of discrete vectors. One iteration of this repeating sequence is also the smallest meaningful unit of time. Time being what is measured by the ticking of clocks. If a clock functions by utilizing the regularity of some physical process, on a fundamental level the ticks of a clock depend on the ability to detect this regular physical process. If we were to construct a clock that ticked when it detected that a particle performs a move on a single vector in a sequence, for example $\vec{F}_1$, that would be the most precise clock that could be constructed. A clock that ticked multiple times during one iteration of $\vec{F}_{1..n}$ could not be known to have ticks that are consistent in the duration from one tick to another without having detailed knowledge of all the discrete vectors.

From our real observations we know that relativistic mass and the duration between the ticks of a clock are related and are both affected equally by the γ factor. This relationship is reproduced faithfully by my model allowing it to agree with special relativity and tests of general relativity.

Equivalence Principle

We have described acceleration due to gravity using the same method as any other form acceleration. If during each cycle $\vec{F}_{1..n}$ a person receives boosts accelerating him towards the ground then in order to remain motionless equal and opposite boosts to the bottoms of his feet are needed to prevent him from sinking though the earth. Perhaps equivalence is too weak a term and we should say that the feeling of acceleration beneath our feet on earth is exactly equal in nature to the acceleration felt in a rocket far from earth.

Why *G* Appears Constant Locally

The constancy of G is dependent on the constancy of rest mass. In my model the rest mass of a particle is the sum of its vector magnitudes of $\bar{F}_{1..n}$. The quantity of the vectors is determined by the quantity of other particles in the universe and the magnitude of each vector is the inverse of distance to its associated particle. The interplay of the quantity of other particles and their distance can make detecting the variability of rest mass difficult if the vast majority of a particle's mass is derived from other particles which are exceedingly distant and numerous. Because rest mass appears constant locally, G also appears constant locally. If we view the mass contribution of one particle to another as a curve that falls with the inverse of distance we see that the greater the distance the flatter the curve becomes. At great distances the curve may be so flat that changes in distance even as great as the size of our solar system may have a negligible effect on mass contribution. Of course at such great distances the mass contribution will also be very small but this is made up for by the enormous number of particles found at great distances. Also if we focus on distant particles outside our galaxy we find that the curves offset each other due to the homogenous distribution of matter in the universe. These offsetting curves which are virtually linear at great distances make detecting variations in rest mass very difficult. Detecting variations in mass at different distances from our galactic core should be possible but our ability to conduct these experiments so far from earth is not possible. We are able to make observations of stars in spiral galaxies orbiting their galactic core and find that orbital speed often remains constant for all orbital distances from the core. Possible explanations include a modification of gravity while retaining invariant rest mass, or equivalently modifying rest mass while retaining currently accepted gravitation (gravitational attraction determined by matter not mass). If the constituent matter of a spiral galaxy had a variable rest mass which varied based on its distances r from the core by a factor proportional to $1/r$ then we would expect flat spiral galaxy rotation curves.

The variability of rest mass is helpful in explaining the flat rotation curves but is not likely to fully account for the observed orbits. Because matter is evenly distributed in the universe we find that abundance of matter in the universe at different distances r is proportional to r^2 but the mass contribution only falls with $1/r$. While ignoring the clumpiness of matter in the universe, a particle's mass contribution from distant matter increases proportionally with distance. Which would mean that variations in rest mass would be virtually undetectable. But since matter does have a clumpy distribution, there is a greater significance of mass contributions from matter within a local clump. However this clumpiness is unlikely to be sufficient to vary rest mass to the extent needed to explain flat rotation curves.

Other Considerations

There are other considerations which need more attention than can be given in this paper and will only be touched upon here. Throughout this paper I have described the motion of our test particle by focusing on $\vec{F}_{1..n}$. The particle has a position then after one iteration of $\vec{F}_{1..n}$ it has a new position. $\vec{F}_{1..n}$ takes into account the history of accelerations the particle has experienced. The way I have described the behavior of $\vec{F}_{1..n}$ has ignored the fact that the unboosted $\bar{F}_{1..n}$ most likely do not sum to zero so there should be some underlying particle motion even before considering accelerations from gravitational potential or from other energy. Is this underlying motion significant? Yes and no. If we assume our universe operates as my proposed model, then this underlying motion would not be easy to observe on scales the size of our solar system or smaller, but the effects may be significant on larger scales.

Conclusion

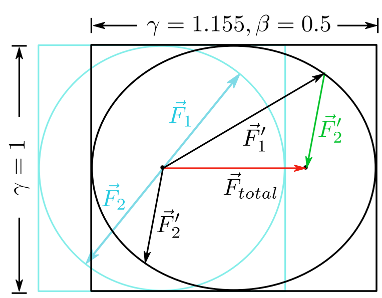
During the conception of these ideas I often found myself aided by a vision of Feynman describing a probability amplitude by waving his finger in circles with gusto. By drawing a bunch of arrows head to tail we can again better our understanding of how nature works. By summing discrete vectors we can produce a fundamental model which describes the emergence of mass, ticking of clocks, why observed speeds are restricted to c or below and a proper Machian theory of gravity. Reductionism is further served by the ability to describe the formerly disparate concepts of velocity, mass, momentum and the ticking of clocks by using similar terms or ratios of similar terms.

Appendix

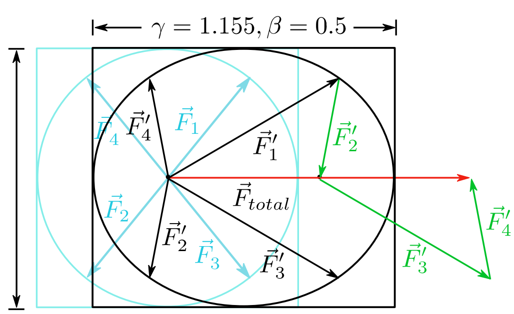
A. The trajectories the particles in my model can be likened to Zitterbewegung. I don't address any preferred order which describes how $\vec{F}_{1..n}$ are organized such as why one vector should precede of follow another. Assuming there is a preferred ordering of vectors, we can identify an ordering which for a macroscopically stationary particle will produce at the fundamental level circular trajectories around an axis while oscillating back and forth along that axis. If the particle were to move along this given axis with a macroscopic speed then on the fundamental level we would continue to see a circular trajectory with oscillations back and forth but the oscillations will favor one direction more than the other to agree with the observed macroscopic speed.

If we consider a particle with $\vec{F}_{1..n}$ each vector having equal magnitude and pointing evenly in all directions and for ease of description lets locate the particle in the center of a spherical shell. Each vector points to a location on the surface of the shell so we can identify each vector by a location on the shell using a global coordinate. If the particle takes a trajectory comprised of only the vectors near the equator beginning at the Prime Meridian and systematically moving westward, the particle moving on those vectors in order will complete a circle. We can complete two circles with half the circumference by moving on every other vector for the first circle and then the remainder in order. In this manner we can achieve various sizes of circular trajectories. If we perform the same exercise on a higher latitude we achieve a helical trajectory. We can again adjust the tightness of the trajectory by skipping different numbers of vectors and we can also intersperse vectors of different latitudes to alter the tightness of the trajectory and rate the trajectory moves North or South.

B. It may be seen more clearly that the sum of $\vec{F}'_1$ and $\vec{F}'_2$ is equal to the foci separation if we move $\vec{F}'_2$ (shown in green) so that its tail is positioned on the point of $\vec{F}'_1$. We also see that the sum of the magnitudes of $\vec{F}'_1$ and $\vec{F}'_2$ is equal to the major axis which is easier seen if you also image what the diagram would have looked like if $\vec{F}_1$ and $\vec{F}_2$ had originally been oriented to be aligned with the major axis.



C. When using the ellipse technique with two opposing vectors, the sum of the vectors is equal to the foci separation and the sum of the magnitudes is equal to the major axis. If we have two pairs of opposing vectors (or more) the results are proportional and no longer equal to the ellipse attributes. Using the ellipse is just to demonstrate the behavior of the vectors. With the behavior known it is not necessary to think about all the individual vectors but rather just consider the sum of the vectors and the sum of the vector magnitudes.



From this we know the relativistic mass and velocity. I can then alternatively reconsider the ellipse without vectors by just considering it to have an area equal to mass and its eccentricity equal to β which preserves the expected outcomes under Lorentz boost. Thinking in terms of ellipse area and eccentricity avoids messy vectors which quickly get out of hand.

The above diagram starts with four vectors each with a magnitude of 0.5 thus the mass of the particle is twice that shown in the prior diagram [A]. To boost the particle to half of *c* as before we need to apply energy $\vec{J}$ with a magnitude of 0.31 or twice the energy as before. If for example $\vec{J}$ was 0.155 as before we would find β to 0.372 so we find that mass can faithfully emerge from the model.

D. If we have a body$_2$ with mass *m* separated by a distance R_2 from a second body$_1$ with mass *M*, we can determine the increase in kinetic energy of body$_2$ if it is allowed to fall freely toward body$_1$ reaching a distance R_1. This is shown on the right side of the equation below by finding the difference in gravitational potential. This value can be equated to a separate increase in body$_2$'s energy due to gravitational mass dilation (mass converted to energy by multiplying by c^2) which is shown on the left side. The left side of the equation is based on the position of body$_2$, while the right side is based on the speed of body$_2$.

$$\left(\frac{m}{\sqrt{1-\frac{2GM}{R_1c^2}}}-\frac{m}{\sqrt{1-\frac{2GM}{R_2c^2}}}\right)c^2 \approx \frac{GMm}{R_1}-\frac{GMm}{R_2}$$

E. It may seem strange to calculate the trajectory of gravitating bodies using an equivalent of gravitational potential which follows a $1/r$ relationship rather than using a $1/r^2$ gravitational force as is the custom. If we know the position and $\vec{F}_{1..n}$ of our test particle, we can easily know its new position once it has completed one cycle of $\vec{F}_{1..n}$. As the particle moves sequentially on $\vec{F}_{1..n}$ we can determine changes in gravitational potential with respect to the other particles in the universe from which we learn the magnitude and direction of the boosts which are applied. These boosts alter $\vec{F}_{1..n}$ and consequently the trajectory of the particle. We have all the information needed to calculate the particle's position and speed as a fraction of *c* after each iteration of $\vec{F}_{1..n}$. Using gravitational potential may seem to present a problem if we have a particle moving opposite a gravitational attraction at less than escape velocity. It would seem that calculating the trajectory with gravitational potential would result in the particle reaching apogee and then remain stopped rather than falling back down. This isn't a problem since there is always an underlying jiggling motion seen in the unboosted $\bar{F}_{1..n}$ which prevents the trajectory from getting asymptotically stuck.