MATHEMATICS AND THE DEMARCATION PROBLEM

WIM CHRISTIAENS

Contents

1. The distinction between scientific and non-scientific activities

One of the questions raised in the abstract of the essay contest is:

> Are there, for example, real consequences for physics – including quantum mechanics – of undecidability and non-computability? Are there implications for our understanding of the relations between agency, intelligence, mind, and the physical world?

Undecidability and uncomputability refer mainly to the foundations of mathematics, unpredictability is an issue that arises in the context of empirical science, specifically physics. I will focus on the impact of undecidability and uncomputability (more generally: the foundations of mathematics) on science. Specifically, I will react to another essay that was submitted for this contest: 'Math matters', by Sabine Hossenfelder.[1] In her essay Hossenfelder argues that results in the foundations of mathematics, like those obtained by Gödel or Turing, are not relevant for physics. The reason: physics is science, physics is not mathematics. Hossenfelder's argument hinges on the possibility that one make a sufficiently unambiguous distinction between something that is a science and something that is not a science. A little reflection on the nature of that distinction, leads me to the conclusion that, with respect to that distinction, the foundations of mathematics do matter for physics.

2. Different cases of the distinction

There are many sciences and also many non-scientific activities. There are situations in which making the distinction between a scientific domain of activity

---

[1]https://fqxi.org/data/essay-contest-files/Hossenfelder$_u$n01.pdf

and a non-scientific one is not relevant. We are interested in those in which it is relevant.

In general, why would it be important to make a distinction between a scientific and a non-scientific activity or domain?

Science is part of society. It is one of many practices, and it plays a role in other practices. It can play this role because it has a degree of legitimacy. We can put our trust in it. Hossenfelder gives the example of how quantum mechanics allows us to run the computers we work on. I will add two other examples. First, the way people and governments put their trust in virologists with respect to the corona virus. Second, the role climate science plays in the discussion surrounding climate change.

In each of these examples the legitimacy of the relevant scientific domain plays a role in a larger context (sometimes: a larger debate) that is itself not scientific.

Quantum mechanics plays a key role in our lives because it makes computers and other devices possible. On the one hand, the principles that characterize a scientific domain like quantum mechanics are generally not the ones at play when working with a computer. Computers are used for all kinds of non-scientific activities. On the other hand, few people dispute the claims of physicists when it comes to the basic hardware of computers. The fact that these subjects are very technical, might have something to do with it. Also, the contribution of a specialized domain of physics is not readily apparent. Things are different for the second example. Most people believe what the scientists tell us about the corona virus. Nevertheless there are those that read some Wikipedia pages and dispute the biology underlying the claims of virologists and specialists. There is a larger debate in which scientific results play a role, but that is itself not scientific. A small part of the people participating in the debate, either dispute whether virologists should be listened to, and an even smaller (but vocal) part go so far as to dispute the findings of the scientists altogether. Then there is our third example. With respect to climate change the number of people that reject the findings of scientist is worrying. In the larger debate, the legitimacy of the relevant science, and even the distinction between scientific and non-scientific with respect to climate science, is emphasized by 'believers', rejected by 'deniers'.

One final example. In her book *Lost in Math*,[2] Hossenfelder worries about the reasons physicists give for pursuing a specific research program: they are mainly aesthetic. Her reaction is reasonable: physics is science, physics is not art. Although aspects of non-scientific practices can play a role in science, they should not be decisive when it comes to what is considered scientific. Both physics, mathematics, and art are human practices. They exist alongside each other and many other practices: politics, trade, procreation, religion etc. In many contexts for many reasons it is important to distinguish the scientific from the non-scientific. Science can play in a role in other domains (for example virology can play a role

[2] *Lost in Math*, Basic Books, 2018.

in political decision making), and other domains can play a role in science (for example aesthetics can help help choose between different descriptions of physical laws). But both are only reasonable when one also maintains the distinction: mathematics, art, politics are not sciences, while physics, biology and climate science are sciences. There is the legitimacy of a science for itself (in its own domain, on its own terms), and there is the legitimacy of that science in a larger context: in its own domain other practices might also contribute, but the end result is decided by criteria intrinsic to that scientific domain, not to other practices (in the last analysis aesthetic criteria do not decide what counts as physics); sciences can contribute to non-scientific practices, but it are the principles intrinsic to the latter that take precedence.

3. The 'science' behind the distinction

As a citizen and a person, I use a simple criterion to make a decision: I believe the expert. Degrees and titles from the appropriate institutions (universities and university schools) indicate who has authority. This means that I make a distinction between the opinion of the non-scientist and the opinion of the scientist when it comes to the field of the scientist, and believe the scientist when he makes claims about his domain. That in turn determines how I will act. Given such an approach, it does not seem hard to make a distinction between scientific pronouncements and non-scientific propositions.

The criterion that I use to make such distinction is pragmatic. But given the fact that we are talking about science, one would expect there to be a more scientific way to distinguish science from other practices. Suppose that the climate scientists are right. How do you convince the deniers? In each of the examples I gave we do not apply a general rule. The distinction between science and non-scientific domains and activities, is a case by case process. At best, we can learn something from another example where the distinction was needed. But at no point do we have a set of general principles which we can then apply to a specific case.

To clarify my point I would like to draw an analogy. We have mature theories, theories about which we feel confident to say that they are scientific. Anybody who has taken the trouble of working his way through less then half the chapters of the first volume of *The Feynman Lectures*, should be convinced that there is a scientific theory about the relative displacement of physical objects, summarized in three principles, called Newtons Laws. People were already able to deal rationally with a number of mechanical phenomena before the discovery of the laws of Newton. But with every new case they had to figure out how to construct a model. They could rely on earlier models, they could compare them; they could contrast mechanical phenomena with non-mechanical phenomena but they did not have an abstract set of principles that covered all the possible models one could construct for mechanical phenomena. It was also much harder to distinguish mechanical

from non-mechanical phenomena. Now, confronted with a mechanical phenome-
non, I can us Newtons general principles to develop a model, even with the bare
minimum of information on the phenomenon. Whatever other aspects there are
to a phenomenon, if it has properties that fall under the laws of Newton, I already
know a lot about that phenomenon. In other words, no matter how specific a
context, no matter how complex or muddled a context (not every phenomenon
allows one to look at specific properties in isolation), I do not have to start from
scratch with respect to the mechanical aspect of the phenomenon, nor do I have
to use comparison with other mechanical phenomena.

We have similar theories for other physical properties. We also have them in
chemistry and other scientific domains. Things are trickier in the human sciences.
(I will say something about that in a moment.) But do we have a general set of
principles that allows us distinguish a scientific theory from a non-scientific theory?
Do we have a general set of principles that allows one to classify a bunch of human
practices together under the banner: science?

When I was younger I did believe these procedures existed. There were a few
promising candidates: Kants criticism, logical positivism, Husserls phenomenology,
Popper and falsification etc. But even though most of them aspired to be more
than that, in the end they all remained 'only philosophical theories. There is no
theory that has anything close to the level of maturity of the natural sciences,
which teaches me the distinction between scientific and non-scientific. One either
uses vague and trivial/wrong ideas or one relies on philosophical theories. One can
of course simply rely on my pragmatic criterion which I mentioned earlier. For
an individual finding his way in the world, living his life, such a criterion might
seem acceptable. For macro-social decision making – the political domain – it is
inadequate.

If the practice that makes the distinction between the class of practices consisting
of classical mechanics, electro-magnetism, thermo-dynamcs, quantum mechanics
etc. on the one hand, and the class of theories comprised of arithmetic, geometry,
calculus, topology, algebra etc. on the other (similarly the distinction between
that first class and other classes like trade, politics, art, friendship, sports, procre-
ation etc.) is not of the same nature as the first domain (if it is not sufficiently
theoretical),then anybody can reject it for some everyday pragmatic reason. The
way a science plays a role in a macro-social decision on an issue in which that
science has legitimate authority, is pragmatic not systematic. Because the only
way to convince oneself of that legitimate authority is to work one's way through
all three volumes of *The Feynman Lectures* and then some. One can only estab-
lish the legitimacy from within; the legitimacy also requires one to stay inside the
theory, continue to practice it. There is no principle of physics that tells me the
what physics can and cannot do.

More generally, we do not have a theory about science. There is no set of principles that gather together the theories of physics, chemistry, biology in terms of a property or structural characteristic 'scientific.

Apart from the distinction Hossenfelder makes between scientific and non-scientific, she also expresses views on determinism and reductionism. I neither agree nor disagree with any pronouncement Hossenfelder makes about determinism or reductionism. The situation here is very similar to the demarcation problem. We do not have a theory about reductionism, we only have examples of reduction. We do not have a non-trivial model of determinism that one can then apply in different domains. I would like to have other ways than the ones I use to solve my everyday problems when it comes to deciding issues of determinism, reductionism etc.

In the film *Good Will Hunting* Matt Damon plays an uneducated working class kid that is an unacknowledged genius. He crushes an arrogant history student by quoting the original sources the latter's classes are based on, dominates the most brilliant therapists, and outclasses a Fields medalist. The consequence of any distinction in principle of science from other domains, is that anybody with an internet connection can simply check Wikipedia and such, and believe himself an expert in a matter of minutes. On the one hand, fringe theories and conspiracy theories abound, for example with respect to climate change. On the other hand, reacting against this trend, the word 'fact has gained in popularity as a last line of defense among those non-scientists defending science (in this example: climate science).

4. A paradigm case for a systematic distiction

A contextual and pragmatic approach is heavily dependent on heuristics and even more so on interpretation. Even if it would be true that one cannot hope for more (given Hossenfelder's approach in her essay, in her book and in her Youtube-videos, I expect she would choose this option), then I would like it to be expressed in a general principle an impossibility theorem. In other words, I would like the impossibility of drawing a distinction between scientific domains and other other domains except case by case, to be itself a principle, a theoretical result, and not just an opinion or attitude.

Hossenfelder is unambiguous: 'impossibility theories are mathematical curiosities without scientific relevance.'

I disagree. There is something to be learned by physics from successful impossibility results. For example, in the human sciences it is *in principle* impossible to achieve the same level of objectivity as in physics, because the scientist cannot subtract himself from the phenomenon he studies, the way a physicist has been able to for so many physical phenomena. However, not the human sciences are the subject of this essay contest, but rather impossibility theorems in the foundations of mathematics.

Hossenfelder describes a typical example of this, Turings Halting problem, as follows:

> An algorithm will either finish running at finite time or continue calculating forever. The problem is, Turing showed, that there is no meta-algorithm that can decide for any given algorithm whether the algorithm will or won't halt. However, the major difficulty in solving the Halting Problem is that the meta-algorithm must work for all possible input, which is an infinitely large class of algorithms. Nothing real is infinite, therefore the whole formulation of the problem is scientifically meaningless. In practice, we never need an algorithm that can correctly answer infinitely many questions.

Notice the nature of this and other impossibility theorems in mathematics.

There exist algorithms. To look at an algorithm in natural reality or in a real computer, is very different from looking at the concept of algorithm, which is a purely mathematical model. When confronted with an algorithm in non-mathematical context involving many other non-mathematical properties and activities, one can abstract from the latter, and concentrate solely on what is specifically and intrinsically algorithmic. This is not a case by case process. It is not that each and every time, one encounters something that also has algorithmic properties, one has to proceed by comparison and construction. Turing gave us a general principle that is true for anything algorithmic. So there is good deal of abstraction involved, but it pays of. Similarly to what we said about Newton's laws, just by considering the algorithmic aspect of a phenomenon, the general mathematical theorems tell me a good deal about the phenomenon, without knowing anything else about it.

It might well be that mathematics is not science, as Hossenfelder claims, but it certainly managed to demarcate itself, to the benefit of everybody. The theorems in the foundation of mathematics (more so than any axiomatic foundation of the whole of mathematics like ZF), delimit mathematics unambiguously from other activities, by as clearly as possible expressing the limits mathematics has with respect to itself. It is clear within mathematics what mathematics can and cannot do, on a high level of generality; and this tells us, also on a high level of generality, how mathematics can play a role in other domains. When there is a phenomenon, those aspects of it that are mathematical, are constrained with respect to what one can mathematically do with them *in principle*. No matter in what kind of a larger context, mathematics is involved, what it can and cannot do and is clear, and this also allows for a clear (albeit very abstract, i.e., high-level) demarcation between itself and other human practices it interacts with. As an example that this can be done, the foundations of mathematics are relevant for a domain that can only do this case by case, but not in principle, like physics.