

Is Feasibility in Physics Limited by Fantasy Alone?*

Limits, like fears, are often just an illusion.

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Although various limits on the predicability of physical phenomena as well as on physical knowables are commonly established and accepted, we challenge their ultimate validity. More precisely, we claim that fundamental limits arise only from our limited imagination and fantasy. To illustrate this thesis we give evidence that the well-known Turing incomputability barrier can be trespassed via quantum indeterminacy. From this algorithmic viewpoint, the “fine tuning” of physical phenomena amounts to a “(re)programming” of the universe.

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Take a few moments for some anecdotal recollections. Nuclear science has made true the ancient al-chemic dream of producing gold from other elements such as mercury through nuclear reactions. A century ago, similar claims would have disqualified anybody presenting them as quack. Medical chemistry discovered antibiotics which cure Bubonic plague, tuberculosis, syphilis, bacterial pneumonia, as well as a wide range of bacterial infectious diseases which were considered untreatable only one hundred years ago. For contemporaries it is hard to imagine the kind of isolation, scarcity in international communication, entertainment and transportation most of our ancestors had to cope with.

This historic anecdotal evidence suggests that what is considered tractable, operational and feasible depends on time. One could even extend speculations to the point where everything that is imaginable is also feasible. In what follows, we shall concentrate on some physical issues which might turn out to become relevant in the no-so-distant future, and which might affect the life of the generations succeeding

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ours to a considerable degree. In particular, we shall consider the connections between time, space and the limit velocity of light in vacuum; we shall ponder upon measurement; and we shall discuss physical indeterminism and randomness, and its relations to the possibility of trespassing the Turing incomputability barrier.

I. SPACE-TIME

One of the findings of special relativity theory is the impossibility to trespass the speed of light barrier “from below;” i.e., by starting out with subluminal speed. This fundamental limit applies also to communication and information transfer. Amazingly, this holds true even when quantum mechanics and “nonlocal quantum correlations” are taken into account, stimulating a notion of “peaceful coexistence” between quantum mechanics and special relativity theory. Thereby, superluminal particles, as well as the inclusion of field theoretic effects such as an index of refraction smaller than unity, supercavitation in the quantum ether, or general relativistic effects by locally rotating masses, wormholes or local contraction and expansion of space-time, possibly also related to time travel, to name but a few, cannot be excluded *a priori*.

Recent operational definitions of space-time and velocity, in order to physically represent the former, conventionalised the latter: Initially, the constancy of the velocity of light in vacuum in all reference frames was treated as an empirical fact. Since 1983 it has been frame-invariantly standardised by *Resolution Number 1* of the *17th Conférence Générale des Poids et Mesures (CGPM)* in which the following SI (*International System of Units*) operational definition of the meter has been adopted: “*The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.*” As a result, the empirical fact associated with this convention, as predicted by relativity theory, is the proposition that the length of a solid body depends neither on its spatial orientation, nor on the inertial frame where that body is at rest [1].

Indeed, by a theorem of incidence geometry [2], linear Lorentz-type transformations follow from the frame-invariant standardisation of the velocity of light alone and appear to be a formal consequence of the conventions adopted by the SI. In such an approach, the physics resides in the invariance of Maxwell’s equations and the equations of motion in general, as well as in the invariance of all physical measures based on matter stabilised by them, such as the length or duration of a space or time scale. This, after all, suits the spirit of Einstein’s original 1905 paper, which starts out with conventions defining simultaneity and then proceeds with kinematics and by unifying electric and magnetic phenomena. Of course, for the sake of principle, everybody is free to choose other “limiting speeds,” thereby implicitly sacrificing the form invariant representation of the equations of motion in inertial frames dominated and stabilised by

electromagnetic interactions. In this way it would also not be difficult to adopt special relativity to findings of higher signalling and travel speeds than the velocity of light in vacuum.

Since antiquity, natural philosophers and scientists have pondered about the (in)finite divisibility of space-time, about its (dis)continuity, and about the (im)possibility of motion. In more recent times, the ancient Eleatic arguments ascribed to Parmenides and Zeno of Elea have been revived to “construct” accelerated computations [3] which serve as one of the main paradigms of the fast growing field of hypercomputation.

The most famous argument ascribed to Zeno is the impossibility for “Achilles” to overtake a turtle if the turtle is granted to start some finite distance ahead of Achilles, even though the turtle moves, say, one hundred times slower than Achilles: for in the finite time it takes Achilles to reach the turtle’s start position, it has already moved away from it and is still a (tenth of the original) finite distance apart from Achilles. Now, if Achilles tries to reach that new point in space, the turtle has made its way to another point and is still apart from Achilles. Achilles’ vain attempts to reach and overtake the turtle could be considered *ad infinitum*; with him coming ever closer to the turtle but never reaching it. By a similar argument, there could not be any motion, because in order to move from one spatial point to another, one would have first to cross half-distance; and in order to be able to do this, the half-distance of the half-distance, . . . again *ad infinitum*. It might seem that because of the infinite divisibility of space, unrestricted motion within it is illusory because of this impossibility.

The modern-day “solution” of this seemingly impossible endeavour to move ahead of a slower object resides in the fact that it takes Achilles an ever decreasing outer (extrinsic, exterior) time to reach the turtle’s previous position; so that if one takes “the limit” by summing up all infinitely many outer space and time intervals, Achilles meets (and overtakes) the turtle in finite outer time, thereby approaching an infinity of space-time points. Of course, Achilles’ approach is even then modelled by an infinite number of steps or trip segments, which can be used to create an inner (intrinsic) discrete temporal counter. If this inner counter could in some form be associated with the cycle of an otherwise conventional universal computer such as a universal Turing machine or a universal cellular automaton [4, 5], then these “machines” might provide “oracles” for “infinite computations.” In this respect, the physics of space and time, and computer science intertwine.

The accelerated Turing machine (sometimes called Zeno machine) is a Turing machine working in a computational space analogue to Zeno’s scenario. More precisely, an accelerated Turing machine is a Turing machine that operates in a universe with two clocks: for the exterior clock each step is executed in a unit of time (we assume that steps are in some sense identical except for the time taken for their execution) while for the inner clock it takes 2^{-n} units of time (say seconds) to perform its n th step. Accelerated Turing

machines have been implicitly described by Weyl in 1927 and studied in many papers and books.

Because an accelerated Turing machine can run an infinite number of steps (as measured by the exterior clock) in one unit of time (according to the inner clock), such a mechanism may compute incomputable functions, for example, the characteristic function of the halting problem.

How feasible are these types of computation? This is not an easy question, so not surprisingly there is no definitive answer. One way to look at this question is to study the relation between computational time and space. As expected, there is a similarity between computational time and space; however, this parallel is not perfect. For example, it is not true that an accelerated Turing machine which uses unbounded space has to use an infinite space for some input. An accelerated Turing machine that uses a finite space (not necessarily bounded) for all inputs computes a *computable function* (the function is not necessarily computed by the same machine) [6]. Hence, if an accelerated Turing machine computes an incomputable function, then the machine has to use an infinite set of configurations for infinitely many inputs. Re-phrasing, going beyond Turing barrier with an accelerated Turing machine requires an infinite computational space (even if the computational time is finite); the computational space can be bounded (embedded in the unit interval), but cannot be made finite. Do we have such a space? It seems that relativistic computation offers a physical model.

II. MEASUREMENT

Another challenging question has emerged in the quantum mechanical context but it equally applies for all reversible systems: what is an irreversible measurement? Because if the quantum evolution is uniformly unitary and thus strictly reversible, what is to be considered the separated “measurement object” and the “measurement apparatus” can be “wrapped together” in a bigger system containing both, together with the “Cartesian cut;” i.e., the environment supporting communication between these two entities. Any such bigger system is then uniformly describable by quantum mechanics, resulting in total reversibility of whatever might be considered intrinsically and subjectively as a “measurement.” This in turn results in the principal impossibility of any irreversible measurement (not ruling out decoherence “fapp;” i.e., for all practical purposes); associated with the possibility to “reconstruct” a physical state prior to measurement; and to “undo” the measurement [7]. The quantum state behaves just as in Schrödinger’s interpretation of the Ψ function as a *catalogue of expectation values*: this catalogue can only be “opened and read” at a single page; yet it may be “closed” again by “using up” all knowledge obtained so far, and then reopened at another page.

Two related types of unknowables which have emerged in the quantum context are complementarity and value indefiniteness. *Complementarity* is the impossibility to measure two or more observables in-

stantaneously with arbitrary accuracy: in the extreme case, measurement of one observable annihilates the possibility to measure another observable, and *vice versa*. Despite attempts to reduce this feature to a “completable” incompleteness of the quantum formalism, and thus to temporary epistemological deficiencies, the hypothetical “quantum veil,” possibly hiding the “physical existence” of the multitude of all conceivable (complementary) observables, has maintained its impermeability until today.

As new evidence emerged, the lack of classical comprehensibility has gotten even worse: whereas quasi-classical systems—such as generalised urn or finite automaton models [8]—feature complementarity, some quantised systems with more than two measurement outcomes cannot be thought of as possessing any global “truth function.” As the Kochen-Specker theorem [9, 10] shows, they are *value indefinite* in the sense that there exist (even finite) sets of observables which, under the hypothesis of non-contextuality, cannot have definite values independent of the type of measurement actually being performed. The contradiction can be pinned down to the fact that quantum mechanical observables need not be commutative, making it impossible to embed, under the given assumption, the algebra of these observables in a commutative algebra. Some prefer a resolution in terms of *contextual* realism: measurement values “exist” irrespective of their “actual measurement,” but they depend on what other observables are measured alongside of them. Another possibility is to abandon classical omniscience and assume that an “elementary” quantum system is only capable of expressing a *single* bit (or dit for d potential measurement outcomes) [11] or context; all other conceivable measurements are mediated by a measurement apparatus capable of context translation.

III. INDETERMINACY AND HYPERCOMPUTABILITY

In 1926, Max Born stated that (cf. [12, p. 866], English translation in [13, p. 54])

“From the standpoint of our quantum mechanics, there is no quantity which in any individual case causally fixes the consequence of the collision; but also experimentally we have so far no reason to believe that there are some inner properties of the atom which condition a definite outcome for the collision. Ought we to hope later to discover such properties [...] and determine them in individual cases? Or ought we to believe that the agreement of theory and experiment — as to the impossibility of prescribing conditions? I myself am inclined to give up determinism in the world of atoms.”

Born’s departure from the *principle of sufficient reason* — stating that every phenomenon has its explanation and cause — by postulating irreducible randomness [14] in the physical sciences did not specify formally the type of “indeterminism” involved. More recent findings related to the Boole-Bell, Greenberger-

Horne-Zeilinger as well as Kochen-Specker theorems for Hilbert spaces of dimension three onwards derive physical indeterminism from value indefiniteness.

Since indeterminism and randomness are defined by algorithmic “lawlessness” and “incompressibility” [15] any physical system featuring indeterminism and randomness cannot be simulated by a universal computer; it “outperforms” any known computing machinery in terms of unpredictability. Physical value indefiniteness and randomness can thus be seen as valuable resources capable of serving as “oracles” for example, for Monte Carlo methods and primality testing requiring them. Indeterminism becomes an *asset* rather than a deficiency.

Contemporary realisations of quantum random number generators involve beam splitters. Thereby it should be noted that lossless beam splitters are reversible devices formalised by unitary transformations, and that the single photons used constitute a two-dimensional Hilbert space which may be “protected” from cryptanalytic attacks “lifting the hypothetical quantum veil” by quantum complementarity only.

Imperfections in measurements are typically corrected with von Neumann’s procedure of normalisation — “compressing” a bit sequence *via* the map $00,11 \mapsto \{\}$ (00 and 11 are discarded), $01 \mapsto 0$, and $10 \mapsto 1$. The algorithm works under the hypotheses of independence and stationarity of the original sequence, conditions which may not be satisfied in beam splitting experiments—for instance due to multiparticle statistics like the Hanbury Brown and Twiss effect.

Some quantum systems are protected by value indefiniteness grounded in the Kochen-Specker theorem from Hilbert space dimensions three onwards. As the Kochen-Specker theorem requires complementarity, but the converse implication is not true, it follows that a system of two entangled photons in a singlet state or systems with three or more measurement outcomes may be more suitable for generating quantum random bits. Getting more than two outcomes is not problematic as, if in a sequence of random elements drawn from an alphabet with $n > 2$ symbols a fixed symbol is systematically removed, the resulting sequence is still random (over an alphabet of $n - 1$ symbols).

IV. REPROGRAMMING THE UNIVERSE

In the Pythagorean tradition, the universe computes. Thus any method and measure to change its behaviour amounts to (re)programming. If one remains within this metaphor, the character and “plasticity” of the “substratum” software and hardware needs to be exploited. Presently, the Church-Turing thesis confines the universe to universal computability formalised by recursion theory, but is it conceivable that some physical processes transcend this realm?

Arguably, the most (in)famous result in theoretical computer science is Turing’s theorem saying that it

is undecidable to determine whether a general computer program will halt or not. This is formally known as the *halting problem*. More precisely, there is no computer program `halt` which given as input an arbitrary program `p` runs a finite-time computation and returns 1 if `p` eventually stops and 0 if `p` never stops (here we use a fixed universal Turing machine to run programs).

There are two essential conditions imposed on `halt`: a) `halt` has to stop on every input, b) `halt` returns the correct answer. It is easy to construct a program `halt` that satisfies the above two conditions for many very, very large sets of programs, even for infinite sets of programs, but *not*, as Turing proved, for *all* programs.

So, one way to trespass the Turing barrier is to provide a physical mechanism which computes the function `halt` discussed above. There are many proposals for such devices.. Let's first present a negative result: using an information-theoretic argument, the possibility of having access to a time-travel machine would not solve the halting problem, unless one could travel back and forth in time at a pace exceeding the growth of any computable function.

Would some quantum processes transcend the Turing barrier? Surprisingly, the answer is yes [16], and the main reason is the incomputability of quantum randomness.

Quantum randomness appears to occur in two different scenarios: (i) the complete impossibility to predict or explain the occurrence of certain *single* events and measurement outcomes from any kind of operational causal connection; the hidden “parameter models” for the quantum phenomena which have been proposed so far do not provide more insight for the predictions of intrinsic observers embedded in the system; and (ii) the concatenation of such single quantum random events forms sequences of random bits which can be expected to be equivalent stochastically to white noise. White noise carries the least correlations, as the occurrence of a particular bit value in a binary expansion does not depend on previous or future bits of that expansion. We concentrate on the second scenario and assume the following hypotheses:

- The single outcome, from which quantum random sequences are formed, occurs unbiased; i.e., for the i th outcome x_i , there is a 50:50 probability for either 0 or 1:

$$\text{Prob}(x_i = 0) = \text{Prob}(x_i = 1) = \frac{1}{2}. \quad (1)$$

- There is a total independence of previous history, such that no correlation exists between x_i and previous or future outcomes. This means that the system carries no memories of previous or expectations of future events. All outcomes are temporally “isolated” and free from control, influence and determination. They are both unbiased and self-contained.

Assume that we have a quantum experiment which at each stage produces a quantum random bit, and we assume that this experiment is run for ever generating an infinite binary sequence:

$$X = x_1x_2x_3 \cdots x_i \cdots \quad (2)$$

In this scenario, the first condition shows that the limiting frequency of 0 and 1 in the sequence X is $1/2$. Locally, we might record significant deviations, i.e., X may well start with a million of 1's, but in the limit these discrepancies disappear.

The “lack of correlations” postulated above is more difficult to understand. First, *finite correlations* will always exist, because of the asymptotic nature of “randomness”. Secondly, even *infinite correlations* cannot be eliminated because they have been proven to exist in *every infinite sequence*; for example Ramsey-type correlations. So, what type of correlations should be prohibited? There are many possible choices, but the ones which come naturally in the “lab”-context are “computable defined correlations.” In other terms, correlations—finite or infinite—which can be detected in an effective/algorithmic way, should be excluded.

A quantum process protected by Kochen-Specker theorem and satisfying the above two conditions generates an incomputable sequence (2). Even more, the sequence (2) is not only incomputable, but also bi-immune, i.e. there is no algorithm capable of computing infinitely many bits of it. In other words, any algorithmic prediction is capable of correctly predicting only finitely many scattered bits of (2). This property is a weak form of algorithmic randomness, so the main open question is *whether (2) is indeed algorithmic random?*

Does the process producing the sequence (2) trespass the Turing barrier? The answer is yes. Can a Turing machine, using as an oracle unboundedly, but finitely many bits of (2), solve the halting problem? The problem is open.

Final remarks

There are exciting times ahead of us. The limits which seem to be imposed upon us by various constraints might decay into “thin air” as the conditions upon which these constraints are founded will lose their applicability and necessity, or even lose their operational validity. Thus, we perceive physical tractability and feasibility wide open, positive, and full of unexpected opportunities. Indeed, we just quiver at the extension of our imaginable ignorance; let alone the possibilities which we even lack to fantasise. Any further scientific exploration of this realm has to be strongly encouraged.

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