## Physics is a Branch of Mathematics

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A hundred years ago a theory of gravitation was developed by Albert Einstein between 1907 and 1915 called general relativity, positing that the observed gravitational effect between masses results from their warping of spacetime - the groundwork for which had been well established by the successful application of special relativity. Thus, that a four-dimensional geometry was intrinsic to the description of a physical force of nature.

Several effects that are unexplained by Newton's law of gravity, such as minute anomalies in the orbits of Mercury and the bending of light rays by masses accounted for by general relativity have been predicted and confirmed by observation and experiment.

Similarly, early in the last century, quantum mechanics was initially developed to provide a better explanation and description of the atom, especially in the spectra of light emissions beyond that provided by classical mechanics and electromagnetism.

At that time, in both instances, the physics community was sufficiently flexible enough to accept new concepts that gave computational predictions consistent to a high degree of accuracy with observation and experiment.

The current environment seems to be one of holding on to current theories because the holders of high degrees and department offices have that knowledge and new theory would tend to move them out of those positions of authority (and the accompanying high salaries).

Alfred Wegener's continental drift proposal suffered similar intransigence in 1912.
Thus, for the last thirty years, physics has essentially been held hostage to string/membrane theory and supersymmetry/gravity. That symmetry and gauge invariance are somehow fundamentals rather than results of something more fundamental should be a troubling idea.

Further, that certain physical objects (particles) obey one particular set of mathematical laws (Lagrangians) and other such particles obey other differing laws (Lagrangians) leads one to believe that some deranged creator must have formed it as some patchwork to confuse us all - quite a different idea than what Stephen Hawking proposes out of the Standard Model. The Higgs mechanism and Lagrangian sector just another example of some gods or demons dancing on the denizens of the universe.

It should also be troubling how great the rush was to declare the Huggs a fact from a single experiment, especially considering past errors and the general human propensity to act in their own interests.

Humanity realizing that there is some order in reality goes back as far as mankind itself.
That it's dark and cold at night and brighter and warmer in the daytime; warmer in summer than winter.

That rocks are harder than feathers - enemy skulls can be bashed in by rocks, and throats slashed by sharp rocks.

The better your tribe/clan/nation's control and manipulation of physical phenomena (and thus analysis of it ), the better your people are at defending themselves and making war imposing your will on others - stone age - bronze age - iron age - ... - space age.

A natural progression bcoming an analysis of physical phenomena; and as mathematics (including logic) is a more precise form of analysis, turning to mathematics in making analysis in physics is another natural outgrowth (just as it is commonly used in chemistry, biology,
management, logistics, and all forms of warfare).
Engineering is the application of the laws of physics (and other disciplines), and a high level of mathematics is indispensible in architecture, construction, and all other engineering projects. Faults in engineering can lead to disasters - like the Titanic.

Battleships don't appear out of nowhere, nor F-16's, nor aircraft carriers, nor drones, nor AK-47's, nor smart bombs, nor TV's, nor smart phones. No, that reality has mathematical description is surely beyond doubt.

The billions of cathode ray tubes in televisions around the world for the last 60 years prior to the replacement by flat-screen TV's attest to the accuracy and power of Maxwell's equations - and as trillions upon trillions of independent tests of the validity of the laws of elecromagnetism. Of course, so does most of the electronic devices produced and functioning, especially since Oersted's and Marconi's achievements. Television and radio antenna transmitters still function according to these same electromagnetic laws.

So, having sufficient evidence of mathematical basis for explanation of physical phenomena, a natural question to ask is it's extent. Is there a sufficiently fundamental basis for it's foundation?

A set of axioms which is a consequence of a smaller set of axioms is clearly less fundamental than the smaller. Indeed, if a larger set of axioms is preferable to a smaller, than the set of axioms including the results of all experimental results - ever - is the most preferred. However, this does little to predict future phenomena, which a smaller succesful set may do. In set theory, certain axiom sets follow from others, indicating an equivalence. But, with essentially the same axiom count, no preference seems indicated. However, before Bertrand Russell uncovered an essential contradiction in an axiom set, it appeared a valid fundamental set theory foundation. Until similar circumstances may arise, the Zermelo-Fraenkel set theory with the axiom of choice (ZFC) (or Zorn's Lemma or the well-ordering theorem); or equivalent axiom set collection is as noted in Wikipedia "the standard form of axiomatic set theory and as such is the most common foundation of mathematics"; although others, such as Von Neumann-Bernays-Gödel set theory (NBG), are alternatives.

The foundations of mathematics is a difficult rigorous subject, and deeper research in that area continues.

Questions such as the independence of the continuum hypothesis from ZFC, and of the axiom of choice from the remaining ZFC axioms, and that the consistency of a theory such as ZFC cannot be proved within the theory itself have been established.

Gödel's second incompleteness theorem may play a role in physical theory, but as the foundations of mathematics develope, such considerations are mere speculation. Indeed, attempts to question foundations of mathematics from a physical theory must be aware at the very least of classic paradoxes of naive set theory: Russell's paradox, the Burali-Forti paradox, and Cantor's paradox.

It is worth noting here that although an observation/experiment may eliminate a mathematical model from consideration as modeling a physical phenomena; it can neither prove nor disprove mathematical consistency.

When derivatives and integrals are evaluated infinity is confronted. That the volume of a sphere is four-thirds pi-r-cubed is derived using triple integrals. Maxwell's equations have both differential and integral form - both of which have common applications well verified. The
sine and cosine functions have infinite series form, and are instrumental in acoustic and wave analysis in general, as well as Fourier analysis and solution to differential equations.
Mathematical derivations for the hanging cable, vibrating string, heat conduction utilize an infinity process of limits to yield well verified differential equations. There may be some question of infinities regarding source/sink points of fields, but complex function residue theory reassures us that all is well in the space, and even provides much extra to be known of the function. Indeed, harmonic functions in the complex plane have a particularly simple expression using the Poisson kernel, strikingly similar to the potentials of the fundamental forces of nature. Methods of modern calculus use convergent series to explain paradoxes of Zeno and others. Thus, it is clear that infinities are an intrinsic part of reality, and physicists must be well acquainted with the mathematical tools dealing with it - and not be afraid of it.

Noting above similarities of the harmonic functions and the complex plane to three-space and time questioning whether such a four-space might be constructed similar to the complex plane and follow much of the work already done with the complex plane could be in order. Indeed, it turns out that such is the case.

First, note that the complex plane is a vector space with basis $\{1, i\}$.
Second, note that the basis $\{1, i\}$ is homomorphic to the basis

$$
\left\{\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right),\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right\} \text { under ordinary matrix multiplication. }
$$

Since the first matrix is the $2 \times 2$ identity matrix the first three products are like multiplying by one.

And the last product is easily seen to be the negative of the first matrix:

$$
\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)=\left(\begin{array}{cc}
0 & -1 \\
-1 & 0
\end{array}\right)=-\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right)
$$

Third, the Cauchy-Rieman equations of a complex analytic function of a complex variable may be generalized in an associtive algebra for a vector function of any integral dimension, as follows:

For an associative space: $(\mathbf{A} \circ \mathbf{B}) \circ \mathbf{C}=\mathbf{A} \circ(\mathbf{B} \circ \mathbf{C})$

$$
\mathbf{Z}=\mathbf{Z}\left(x^{1}, x^{2}, \ldots, x^{n}\right)=\sum_{i=1}^{n} Z^{i}\left(x^{1}, x^{2}, \ldots, x^{n}\right) \mathbf{u}_{i}
$$

where $\mathbf{u}_{i}$ are basis vectors of the vector space and $Z^{i}$ functions of the variables of the vector space $V$, just as in ordinary vector calculus/analysis classes/texts.

If a differential of such function exists and this is a closed operation, then it is also a vector in the space.

Variable bases may be developed using covariant derivatives, but for constant bases:

$$
d \mathbf{Z}=\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial Z^{i}}{\partial x^{j}} \mathbf{u}_{i} d x^{j}
$$

Now, for an algebra of the vector space there is an operation $\circ$, such that $\forall a, b \in V: a \circ b \in V$

So, given the basis element products: $\mathbf{u}_{i} \circ \mathbf{u}_{j}=\sum_{m=1}^{n} \beta_{i j}^{m} \mathbf{u}_{m}$

$$
\text { whenever: } d \mathbf{Z}=\mathbf{Z}_{L} \circ d \mathbf{r} \Rightarrow \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial Z^{i}}{\partial x^{j}} \mathbf{u}_{i} d x^{j}=\mathbf{Z}_{L} \circ \sum_{j=1}^{n} \mathbf{u}_{j} d x^{j}=\sum_{j=1}^{n} \mathbf{Z}_{L} \circ \mathbf{u}_{j} d x^{j}
$$

$$
\Rightarrow \sum_{i=1}^{n} \frac{\partial \mathbf{Z}^{i}}{\partial x^{j}} \mathbf{u}_{i}=\mathbf{Z}_{L} \circ \mathbf{u}_{j}
$$

Thus:

$$
\begin{aligned}
& \left(\mathbf{Z}_{L} \circ \mathbf{u}_{j}\right) \circ \mathbf{u}_{k}=\mathbf{Z}_{L} \circ\left(\mathbf{u}_{j} \circ \mathbf{u}_{k}\right)=\mathbf{Z}_{L} \circ\left(\sum_{m=1}^{n}\left(\beta_{j k}^{m} \mathbf{u}_{m}\right)\right)=\sum_{m=1}^{n}\left(\beta_{j k}^{m} \mathbf{Z}_{L} \circ \mathbf{u}_{m}\right) \\
\Rightarrow & \sum_{i=1}^{n} \frac{\partial Z^{i}}{\partial x^{j}} \mathbf{u}_{i} \circ \mathbf{u}_{k}=\sum_{m=1}^{n}\left(\beta_{j k}^{m}\left(\sum_{i=1}^{n} \frac{\partial Z^{i}}{\partial x^{m}} \mathbf{u}_{i}\right)\right)^{n}=\sum_{m=1}^{n} \sum_{i=1}^{n} \beta_{j k}^{m} \frac{\partial Z^{i}}{\partial x^{m}} \mathbf{u}_{i} \\
\Rightarrow & \sum_{i=1}^{n} \frac{\partial Z^{i}}{\partial x^{j}} \sum_{m=1}^{n}\left(\beta_{i k}^{m} \mathbf{u}_{m}\right)=\sum_{m==}^{n} \sum_{i=1}^{n} \frac{\partial Z^{i}}{\partial x^{j}} \beta_{i k}^{m} \mathbf{u}_{m}=\sum_{m=1}^{n} \sum_{i=1}^{n} \beta_{j k}^{i} \frac{\partial Z^{m}}{\partial x^{i}} \mathbf{u}_{m} \\
\Rightarrow & \sum_{i=1}^{n}\left(\beta_{j k}^{i} \frac{\partial Z^{m}}{\partial x^{i}}-\beta_{i k}^{m} \frac{\partial Z^{i}}{\partial x^{j}}\right)=0
\end{aligned}
$$

Similarly, whenever: $d \mathbf{Z}=d \mathbf{r} \circ \mathbf{Z}_{R} \Rightarrow \sum_{i=1}^{n}\left(\beta_{j k}^{i} \frac{\partial Z^{m}}{\partial x^{i}}-\beta_{k i}^{m} \frac{\partial Z^{i}}{\partial x^{j}}\right)=0$
And when both are true: $\sum_{i=1}^{n}\left(\beta_{i k}^{m}-\beta_{k i}^{m}\right) \frac{\partial Z^{i}}{\partial x^{j}}=0$
Using this fact, it may be shown in any such commutative algebra the Cauchy Integral Theorem may be proven.

These may be regarded as field equations for a 'smooth' function; however, this specification is too strict to allow field equations as robust as Maxwell's equations.

In fact, since dot products are defined in any vector space, norms and metrics or pseudometrics may be defined - leading one to look to algebras as a more fundamental approach to geometry (and via that to a physics, as noted at the outset).

But, assuming an algebra exists and building an empire of geometry and physics upon it is subject to faith in it's existence and vulnerable to question and proof of non-existence. Therefore, developing and utilizing a more general matrix product called weighted matrix multiplication enterd the picture in order to construct an algebra of desired characteristics.

$$
\begin{array}{ll}
\mathbf{A} \equiv\left(A_{i j}\right), \quad \mathbf{B} \equiv\left(B_{j h}\right), & \boldsymbol{\Phi}_{\alpha} \equiv\left(\Phi_{\alpha i j}\right): \text { a set of Weight Matrices } \\
\mathbf{A} \circ \mathbf{B} \equiv\left(\sum_{\alpha=1}^{c( } \Phi_{\alpha i j} A_{i \alpha} B_{\alpha j}\right), & \text { where: } c(A): \text { the number of columns of } \mathbf{A}
\end{array}
$$

Indeed, the permutation matrices may be use as bases for any vector space with dimension power of two, and a simple framework for dot an cross product definitions - even the quaternions may be so constructed: .

$$
\begin{aligned}
& \forall u, v \in \mathbf{S}, u \circ v=u^{*} \odot v+u^{*} \times v, \\
& u \odot v=\frac{1}{2}\left(u^{*} \circ v+v^{*} \circ u\right), \quad u \times v=\frac{1}{2}\left(u^{*} \circ v-v^{*} \circ u\right)
\end{aligned}
$$

or:

$$
\begin{aligned}
& u \circ v=u \odot v+u \times v, \\
& u \odot v=\frac{1}{2}(u \circ v+v \circ u) \quad, \quad u \times v=\frac{1}{2}(u \circ v-v \circ u)
\end{aligned}
$$

(depending on the space-type)

In a particular algebra, a less strict definition for 'smoothness' criteria

$$
\vee \mathbf{f}(\mathbf{x}) \equiv \sum_{m=1}^{n}\left[\lim _{\substack{\delta x^{m} \rightarrow 0 \\ \delta x^{h}=0,(h \neq m)}}\left\{(\delta \mathbf{x})_{R}^{-1} \circ \delta \mathbf{f}(\mathbf{x})\right\}\right]
$$

yields an analogue to the divergence and curl with the same form as the dot and cross product, yielding a divergence/curl symmetry in Maxwell's equations - the symmetry is a
result, not a cause.

$$
\begin{aligned}
\boldsymbol{\nabla} \odot \mathbf{f} \equiv 1 / 2\{( & \left.\left.\sum_{m=1}^{4}\left[\lim _{\substack{\delta x^{m} \rightarrow 0 \\
\delta x^{n}=0,(h m)}}\left\{(\delta \mathbf{x})_{R}^{-1} \circ \delta \mathbf{f}(\mathbf{x})\right\}\right]\right)-\left(\sum_{m=1}^{4}\left[\lim _{\substack{\delta x^{m} \rightarrow 0 \\
\delta x^{\prime}=0,(h m)}}\left\{(\delta \mathbf{x})_{R}^{-1} \circ \delta \mathbf{f}(\mathbf{x})\right\}\right]\right)^{*}\right\} \\
& =1 / 2\left[\odot \mathbf{f}(\mathbf{x})-(\gtrdot \mathbf{f}(\mathbf{x}))^{*}\right]=1 / 2\left[\nabla \circ \mathbf{\nabla}-(\nabla \circ \mathbf{\nabla})^{*}\right],
\end{aligned}
$$

and

$$
\begin{aligned}
& \boldsymbol{\nabla} \times \mathbf{f} \equiv 1 / 2\left\{\left(\sum_{m=1}^{4}\left[\lim _{\substack{\delta \mathrm{m}^{m} \rightarrow 0 \\
\delta \chi^{n}=0,(h \neq m)}}\left\{(\delta \mathbf{x})_{R}^{-1} \circ \delta \mathbf{f}(\mathbf{x})\right\}\right]\right)+\left(\sum_{m=1}^{4}\left[\lim _{\substack{\delta x^{m} \rightarrow 0 \\
\delta x^{h}=0,(h \neq m)}}\left\{(\delta \mathbf{x})_{R}^{-1} \circ \delta \mathbf{f}(\mathbf{x})\right\}\right]\right)^{*}\right\} \\
& =1 / 2\left[\bigcirc \mathbf{f}(\mathbf{x})+(\bigcirc \mathbf{f}(\mathbf{x}))^{*}\right]=1 / 2\left[\nabla \circ \mathbf{f}+(\nabla \circ \mathbf{f})^{*}\right] \text {, }
\end{aligned}
$$

Independent of such algebra (yet a consequence of it), however, a Helmholtzian operator factorization generalization of the d'Alembertian operator factorization of the wave equation yielding Maxwell's equations yields an anlogous factorization of the four-vector Klein-Gordon equation (which all particles satisfy). The Dirac equation resulting from a factorization of the Klein-Gordon equation may be shown to be a special case this Helmholtzian operator factorization.

Thus, instead of requiring a Higgs mechanism, Higgs boson, and Higgs sector to add mass to wave equation Laplacians and seperate Laplacians for strong and weak forces - mass is already there, a fairly simple relationship between all the fermion masses in the Helmholtzian factorization,

| $f(0)=1$ | $m_{v_{e}}=m(0,1)=m(0,1) f(0)=m(0,1)$ |
| :--- | :--- |
| $f(1)=2$ | $m_{d}=m(1,1)=m(2,1) f(1)=2 m(2,1)$ |
| $f(2)=5$ | $m_{u}=m(2,1)=m(3,1) f(2)=5 m(3,1)$ |
| $f(3)=10^{5} \cdot(2 k)$ | $m_{e}=m(3,1)=m(0,1) f(3)=10^{5} \cdot(2 k) m(0,1)$ |
| $\frac{m(0,2)}{m(0,1)}=1$ | $\frac{m(0,3)}{m(0,1)}=1$ |
| $\frac{m(1,2)}{m(2,1)}=\left(\frac{23}{25}\right) \cdot(k)$ | $\frac{m(1,3)}{m(2,1)}=\left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot(k)^{2}$ |
| $\frac{m(2,2)}{m(1,1)}=1 \cdot(6 k)$ | $\frac{m(2,3)}{m(1,1)}=1 \cdot\left[\left(\frac{3}{1004}\right)(6 k)^{2}\right]^{2}$ |
| $\frac{m(3,2)}{m(3,1)}=1 \cdot(5 k)$ | $\frac{m(3,3)}{m(3,1)}=1 \cdot\left[\left(\frac{2}{1450}\right)(5 k)^{2}\right]^{2}$ |

and even an affine transformation serves as a foundation to determine these masses from merely two numbers dependent on $\pi, e$, and a few rational fractions.

$$
\begin{aligned}
\frac{m(3,1)}{10 e} / 1 \mathrm{Mev} / \mathrm{c}^{2} & =\frac{15}{8}+\left(\frac{486}{25}\right) \frac{1}{4000} \\
k & =\frac{15}{8}+\left(\frac{20}{21}\right) \frac{1}{4000}+4 \pi^{2}
\end{aligned}
$$

Thus, the geometry inhabited and physics encountered have constructable algebraic
mathematical description, and since this may be extended to higher dimension power of two, it may serve to quantify higher value-based subjective-type concepts such as art, beauty, love, other emotional states, and so-called supernatural phenomena.

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