The Flip

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> *"Take an ordinary sentence, say, 'She went'... [It] might mean that Susan Jones had gone home, that the cat had one day left the house and had never returned, or that the RMS Queen Mary 2 had just left the harbor. [It] might mean that the neighbor carried out her threat to go to the police; or, ironically, that her interlocutor had been a fool to imagine that their neighbor would carry out that threat. It might mean metaphorically that Nancy Smith had, at some point, wholly ceased paying attention. And so forth. None of these meanings is fully encoded by the sentence; some are not even partially encoded."*
>
> – Dan Sperber[1]

In the 1850's construction of a global telegraph network was well underway. The media at the time widely reported that this new technology would bring about world peace:

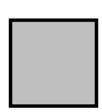
> "This binds together by a vital cord all the nations of the earth. It is impossible that old prejudices and hostilities should longer exist, while such an instrument has been created for the exchange of thought between all the nations of the earth."[2]

Now we have twitter, which binds a vital cord between a significant portion of all the people of the earth, and world peace is nowhere in sight. What is going wrong?

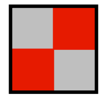
Unpacking this question leads to many others. What is meaning, or information? What is language? What is understanding? How do I know you get what I'm sending? Sometimes these questions are best for taste testing only, swilled around the mouth before being spat out. But these questions also have incarnations within the realm of mathematics, physics and logic.

The influential mathematician David Hilbert said of the axioms of geometry that we could replace the words "point", "line", etc. with any other words such as "table", "chair", and we would still be talking about geometry. If so, where is the meaning? Now we have a question worth imbibing, whose answer turns out to be completely evasive.

Travelling back in time to 1872, we find that Felix Klein had already tackled this problem of what is geometry. Consider a square:



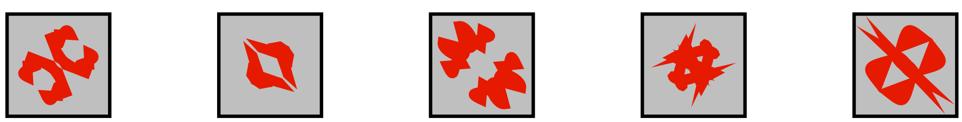
If this was a painting how would you know which way to hang it? There are four equivalent ways, including reflections there are eight. To help us remember which way to hang the painting we may paint some distinguishing features:



Now we can see if someone rotated the painting by 90 degrees, but there is still a fourfold freedom. Clearly we used too much paint. If we decide to paint just one of the corners:

this still leaves two symmetries, the identity and a reflection. Finally, if we are even more frugal with the paint:

all symmetry is gone. What we are doing with the paint is adding *structure* to the square. The original square has the "nothing" structure. The next one we could call a "diagonal" because the red marking highlighted a diagonal. Then we had red in a corner, so we call this a "point" structure, and finally a "frame" structure. Notice there are many different ways to paint these structures. For example, any of the following decorations could serve to convey the idea of a diagonal:

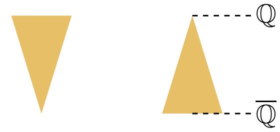


These are Hilbert's "cups" and "chairs". Klein was faced with this same problem: the geometers of the 19th century had many different ways of describing their theories. Evidently, the key idea is that these diagrams all have the same four-fold symmetry. This is in summary what Klein did with the foundations of geometry: structure is described via the symmetries thereof, not the structure itself.[3] This is our first evasive answer.

As physicists, we think when we find some symmetry that means we now know *more* about that thing. But what Kleinien geometry tells us is that symmetry is knowing *less* about that thing. The more symmetry there is, the less structure. And vice-versa. We can draw this contra-variant relationship schematically using two adjacent wedges:

symmetry structure

The study of symmetry has been dubbed *group theory*, and goes back at least to the mathematician Évariste Galois. Galois' brilliant idea was that the symmetry possessed by the roots of a polynomial cannot be broken by any algebraic expression for those roots. For an example of this kind of symmetry consider the imaginary unit: it is defined to be that number which squared gives minus one. But which number? There are two of these numbers, equally deserving to be called the imaginary unit. The definition is ambiguous. Galois showed how to study these "invented numbers" by studying the ambiguity in their definition. This is known as a Galois group. Considering the totality of these invented numbers gives rise to what is called the absolute Galois group. It's an infinite group, and so these numbers are infinitely floppy. We can imagine painting these numbers, in the same way we painted the square above. This gives another contravariant relationship between symmetry and structure:



Galois group   number field

In Galois' final letter of 1832 he called this work *the theory of ambiguity.*[4] Closely related is the idea of *entropy.* We use entropy as a measure of information. But entropy is *uncertainty*, it is a lack of information. This was the insight of Shannon and Boltzmann: we cannot measure information, but we can measure the lack of it. Information is the negative of entropy:

entropy   information

This situation is wholly analogous to Kleinien geometry: group theory is a way of measuring ambiguity, just as entropy measures uncertainty. We evade having to define information, just as we evade having to define any geometric figure.

The beginning of, or impetus for, the age of reason is often identified with the great Lisbon earthquake of 1755.[5] It is a strange connection to make: we still don't know how to predict earthquakes, and moreover, don't even seem to be able to mitigate against such disasters. We can build stronger houses and bridges, but then we also put a nuclear reactor in the path of a giant tsunami. These kind of outlier events are exactly the weak point of statistics. Such approaches to reasoning also underlie modern artificial intelligence machines. Some amount of context, or training data, is placed in a "container" and then we fit a model to whatever is in the container. The search for artificial general intelligence involves somehow making the container big enough, or the model flexible enough, so that whatever is outside the container does not break the model. It sounds promising, but it hasn't worked out that way. The central problem of this research is the problem of overfitting. The more flexibility, or *capacity*, your model has, the less it is able to generalize to a wider context.[6]

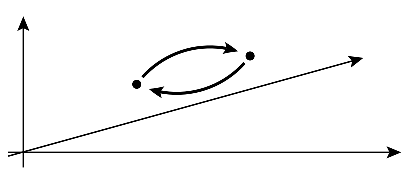
context   capacity

One of the first jobs I ever had was selling kitchen products. I had not much talent for sales, but distinctly remember the first lesson received. This lesson comes in the form of a paradox. I have a bottle of water to sell to you for $1. As a salesperson I must convince you that I am mistakenly parting ways with this good bottle in exchange for your $1. It is a paradoxical situation, in the same vein as the Epimenides paradox, named after the Cretan philosopher who said “All Cretans are liars.” The solution to this paradox is to see that value is relative to context. To poor me, the $1 is more valuable than the bottle of water, but to thirsty you, the bottle of water is more valuable than the $1. The lesson is that value is an illusion.

Language, number, geometry, information, value: these things we are discussing are *maps* or *models*, so of course they are ambiguous. The map is not the territory, and that's the whole point of making the map, to forget stuff about the territory. Otherwise the map will not fit in your pocket.

This is a reasonable criticism to make. But even so, why is the interesting thing about these maps, not what is on the map, but what is left out of the map?

My square painting on the wall looks symmetric, but if you look closer you would find some slight imperfections that destroy this symmetry. This is *epistemic* symmetry. It is symmetric by virtue of ignorance. What would *real* symmetry, *ontic* symmetry, look like? Is there such a thing? It is difficult to imagine. This would be a state of reality that could be re-arranged somehow but you end up back with the same state of reality. It sounds like a Zen koan: what stays the same, while changing?

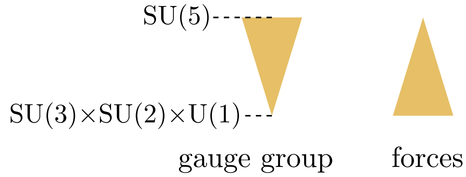


Quantum physics crosses over the epistemic-ontic divide. I claim that the transition from classical to quantum physics is the transition from epistemic to ontic. Before the discovery of atoms, before Max Planck posited irreducible quanta, it was not even clear that there was a *number* of anything. Maybe all the stuff in the world could have turned out to be continuous, without any minimum increment. Counting, the theory of number, would have remained epistemic. Three sheep is only epistemically three: these sheep are not the same. Even the taxonomy of “sheep” is epistemic. However, in quantum physics identical particles do exhibit ontic numbers. Because of this, identical Bosons exhibit ontic symmetry: when you swap these around, you get back to the same state of reality. We know this because this is how lasers work: reality falls into a highly symmetric state. This ontic symmetry is exactly the symmetry of numbering. If I claim that “I have three things” then this theory is ambiguous in six ways. Street buskers make money from this theory by hiding a ball under one of three shells.

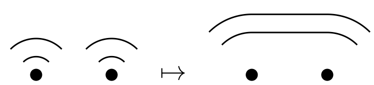
I can generate epistemic randomness by rolling a dice. It is epistemic because presumably if we got really good at it, we could predict what number the dice would end up at, destroying the randomness. Is there such a thing as ontic randomness? It is difficult to imagine. Such a happening would have no cause. Quantum physics does appear to exhibit ontic randomness in the outcomes of measurements. This ambiguity of outcome is in direct parallel to the ambiguity of symmetry. For some people, talk of ontic randomness causes much gnashing of teeth, whereas the appearence of ontic symmetry does not. I would suggest that these are both equally deserving of gnash.

At around the same time that Galois was cavorting around Paris, on the other side of the English channel Michael Faraday was pondering a mysterious “electrotonic potential”. He never defined exactly what this was, but this intuition of Faraday’s was later recognized by James Clark Maxwell as the electromagnetic vector potential. This is ambiguity in the theory of electromagnitism. Quantum physics shows that this is ontic ambiguity, or $U(1)$ gauge freedom.[7]

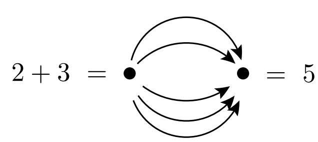
Not only are the so-called fundamental particles defined using ambiguity, or *Lie groups*, it is conjectured that there is a unification of forces that occurs at high energy scales. No one knows what the ultimate group is in this case, or even if it is a group. For example, one such unified theory[8] uses SU(5) :



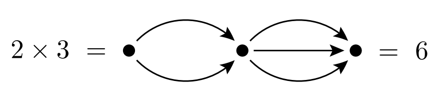
Huygens’ principle[9] describes how wavefronts propagate:



It can also be seen as a statement about linearity, or a kind of distributivity. Most simply, we can see this in path counting. If there are two ways to reach my destination, and three other ways to reach my destination, then we just add these paths together to find the total:



If I am travelling via an intermediate location, and there are two ways to get there, and then to complete my journey there are three ways, we multiply these numbers to find the total:



These rules are also the rules for manipulating amplitudes in quantum physics. Once we allow multiple starting and ending points we find linear algebra pops out.[10] You can even see the wavefronts of Huygens’ principle by fooling around with parentheses in the rule for distributivity:

$$A(B + C) = A(B) + A(C).$$

This is the pivotal idea behind linear algebra, path integrals, Feynman diagrams, etc. It is fundamental to mathematics. Distributivity is how numbers count things,

$$(1 + 1 + 1)a = a + a + a.$$

The same kind of equation is used to define linear operators,

$$f(a + b) = f(a) + f(b).$$

The wavefronts of Huygens' principle are maps. Quantum physics is just this: ontic waves, or ontic maps.The maps or tools of mathematics allow us to navigate the world, just like any map does. The conclusion of quantum physics is that these maps are real. Reality is maps mapping maps.[11]

This helps explain the great efficacy of mathematics in physics. And what is mathematics? We don't know; Gödel's second incompleteness theorem states that we can't even write down the rules for mathematics. If mathematics is real, as quantum physics seems to be telling us, then we should not be so surprised that the result is also incomplete. Put most bluntly, the axioms we choose for mathematics have no cause, as we find the happenings of reality have no cause.

Consider the following experiment. A photon is incident on a half-silvered mirror. We then measure if it was transmitted or reflected. What does physics tell us about the outcome of this experiment, what happens? It does not tell us what happens. But physics is not entirely mute in this case. Physics says this: that it is *not possible to say what happens.* It is a meta-statement about the experiment, a statement about *descriptions* of the experiment.

In 1948 Norbert Wiener was propelled to fame after the publication of his book "Cybernetics". At the time, there was growing interest in the new computing technology: what were these machines going to be capable of? This was the beginnings of the information technology revolution.[12] Now we are at the dawn of the quantum computing era, and there are similar questions to ask. For example, what can I do with quantum bits, *qubits*, that I cannot do with ordinary bits? This is a subtle question, with plenty of fine-print still to be determined, but we can also approach this naively.

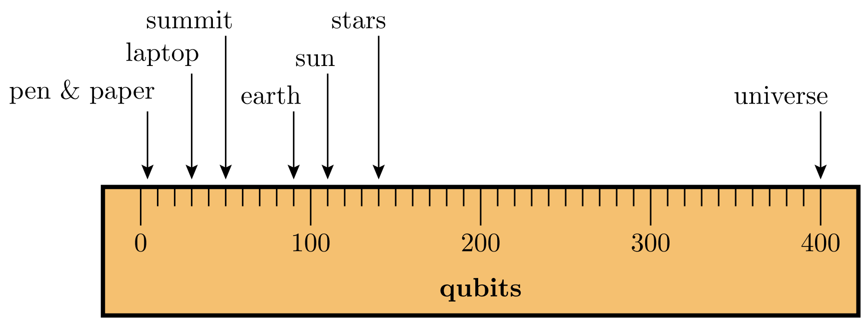
On a good day, given a pen and some paper, I can perform some calculations that model 4 qubits. More qubits than that and I reach for my laptop. I've managed to crash my laptop so many times by running qubit simulations, but if I'm careful I can simulate up to about 30 qubits. Beyond this we need some big iron. The Summit supercomputer uses 10 megawatts of power and covers an area of about two basketball courts. Using this machine it would be possible to simulate about 50 qubits. This is close to the current state of the art: the Google Sycamore processor has 54 qubits.

Covering the entire planet Earth with Summit supercomputers we could fit about one trillion of these. Then we could simulate around 90 qubits. We would also need about 10 billion nuclear reactors to power these supercomputers.

Completely surrounding the sun with a giant sphere, called a Dyson sphere, we could use the resulting solar energy to power 10^{19} Summit's. Assuming the calculation would finish before the sun burnt out in 5 billion years, this gets us to 110 qubits.

If we surround every star in the entire universe with Dyson spheres, and use those to power supercomputers we could simulate up to about 140 qubits. This completely disregards the time it would take to run such a computation. Every part of this multi-star supercomputer needs to communicate with every other part, and the speed of light puts severe restrictions on how long this would take.

Suppose we somehow managed to juice all the entropy out of every particle in the universe. This is the absolute maximum amount of information that could be stored using all the (classical) matter of the entire universe. Assuming this maximum capability[13] we could simulate about 400 qubits.



Just about any reference on quantum physics will say something about how the quantum realm is about the physics of really small things: atoms, photons, and things like that. Those things "down there":

This change in perspective, the qubit ruler, suggests that we have this entirely wrong. We are the small thing "down there" and the atoms, the quantum realm is the big thing. This flip in perspective is summarized by the seemingly ridiculous slogan: atoms are big!

Popularizers of physics, and science in general, like to make celebratory claims of the great successes of physics. They often cite quantum electrodynamics, that most well-tested of theories, and to how many decimal points we have verified this theory. If not this example, some other claim along the lines of "things happen because physics, and we have mostly figured this out except for some interesting details that the experts are hard at work on."

Such confident statements are often made by happy sounding loud-mouths. I would also be loud and confidant if I believed these things. But this is not the conclusion that I draw from the evidence. The conclusion I find is that the world is profoundly mysterious. Physical law and mathematics itself, is not only incomplete, but objectively so. We can measure this incompleteness, but that does not change what it is. Far from "things happen because physics", we don't know why any thing happens. Physics is mute on this exact subject. It is without a leg to stand on.

This is the flip. If you understand that the meaning of a word *is* the totality of the context that word appears in, then you have been flipped. In Kleinian geometry, a line is not what it carries with, but the ambiguity left behind. The value of $1 is not "$1". The value is all the things you can get in exchange for it. Value is possibility.

These things, these objects, are only the address of the meaning, a location on a map. They are not the meaning itself. This is not even particularly new understanding. No doubt the ancient philosophers, both eastern and western, already understood this. I would not bother with writing this essay, except this needs repeating in these days of bedazzlement. Numbers! This is how to create engaging content about shoes: promise to enumerate the ten best (or worst!) shoes ever. Just thinking about such a list makes me want to buy some shoes.

Consider this quote from a recent popular science book: "We are physical beings made up of large collections of particles governed by nature's laws. Everything we do and everything we think amounts to motions of those particles."[14] This is reductionist philosophy. We don't even know which way is up, how can we propose to "reduce" anything?

We look at $1 and see value. It's not value, it's an illusion. We see a word and think it means something. This is a category error. So too, when it is claimed that "physics predicts what happens." No, this is a category error. It may have been true with classical physics, but not quantum.

Language is a kind of artificial intelligence. We use it to contain meaning in the same way that artificial intelligence tries to contain intelligence. Language is fragile for the same reasons: it is brittle to changes in context.

We do not live in the age of reason, we live in the age of control. Reason is something else. This is the flip.

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Notes

[1] From the linguist Dan Sperber who was criticising the standard approach to meaning [13].
[2] See [14].
[3] This is Klein's Erlangen program [8].
[4] "For some time my main thinking was directed towards the application to transcendental analysis of the theory of ambiguity. It was concerned with seeing a priori in relations between transcendental quantities or functions what exchanges one could make, what quantities one could substitute for the given quantities, without the relation ceasing to hold. That makes immediately recognizable the impossibility of many expressions that one could look for. But I do not have the time, and my ideas are not yet well enough developed in this area, which is immense." Galois, 1832 [4]
[5] See for example [11].
[6] I am describing what is known as statistical learning theory [15]. The *capacity* of a model measures how many data points it can distinguish. For example, the degree of a polynomial is one less than how many points can be distinguished by that polynomial.
[7] See [16] for a historical overview of the roots of gauge theory in the work of Faraday, Kelvin and Maxwell.
[8] See [3].
[9] A fuller account of this principle of distributivity would include Fermat's principle of least distance, the calculus of variations of Newton and Bernoulli, the Hamilton-Jacobi equation, and many more ideas such as dynamic programing of Dijkstra and Bellman [1, 6].
[10] This is known as graphical linear algebra [12].
[11] The mathematics of category theory is all about "maps mapping maps" [2, 10].
[12] See [5].
[13] There is at most 10^{122} bits of entropy in the classical universe [9].
[14] Quotation from [7].

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