What is the Point of Reality?

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Introduction

Since the beginning of science, the most important aspects of a physical theory have not been its successes, but its failures, and of all the theoretical failures none is greater than the failures related to our inability to adequately define the real point.

What is a physical point? We can talk about a point particle theory, but the very concept of a point particle, carrying a charge and something called spin, is seriously contradictory.[1] We can talk about a black hole theory, but only if we are willing to admit that physical law ultimately has no meaning at its foundation.[2] We can talk about the Big Bang theory, but only at some time after the postulated bang has expanded the initial point into a non-point.[3] In short, the enigma of the point concept for modern man, as well as for the ancient Greeks, is that, while it is everything, it is also nothing. For us, just as for them, the point is everything, because everything lies nascent within it, in perfect unity, but it is also nothing, as long as it lies undifferentiated.

Ironically enough, however, it is very clear that the physical entities of the universe consist of a collection of discrete points, a highly organized, countable, collection of elementary points, normally consisting of photons, electrons, neutrinos, quarks and gluons, together with their anti-particles. Nevertheless, to the dismay of the human intellect, all these points are observed to act as if they were not points at all, but the traveling waves of a continuous, unbroken medium, not the traveling waves of something substantial, but waves of nothing, points that are yet waves with wavelengths, amplitudes and frequencies that can be measured.

Man's long struggle to understand the enigma of physical points is reflected in the age-old dilemma of trying to "square the circle," and to cope with the "incommensurability" of the diagonal within the square. The ancient Greeks were perplexed by it; the mathematicians of the 17th and 18th Centuries were fascinated by it, while the scientists of the 19th and 20th Centuries found useful ways to work around it.

Indeed, the astounding technology of modern civilization in the 21st Century is in a lot of ways a testament to the cleverness of man's intellectual efforts to reconcile these two seemingly incompatible faces of nature. Yet, after all that has been said and done, we still fail to fully understand how nature *seamlessly* integrates the discrete with the continuous.[4]

For instance, our greatest scientists, from Newton and Einstein to Heisenberg and Feynman, to their successors, who occupy their predecessors' chairs at the world's most prestigious universities, have not been able to explain how light, which is undoubtedly composed of particles called photons, can yet behave in every way as if it were waves of an unbroken continuum. And, conversely, neither have they been able to explain why gravity, which behaves as if its effect stems from the fact that mass is embedded in an unbroken continuum of space-time, cannot also be attributed to the properties of a discrete particle of some kind.

Consequently, we are left with two fundamental theories of nature, one for gravity that is based on a continuous concept of reality, and the other for matter that is based on a discrete concept of reality, and the trouble with the much-vaunted successes of these two physical theories is that they are fundamentally incompatible with each other.[5]

The Problem with Algebra and Physics

In 1916, decades before the completion of quantum theory and the standard model of particle physics, Einstein wrote to his friend about his personal struggle with the discrete versus continuous challenge: "The [continuum] should be banned from the theory, as a supplementary construction, not justified by the essence of the problem, [a construction] which corresponds to nothing 'real.' But we still lack the mathematical structure, unfortunately. How much have I already plagued myself in this way…the continuum is more ample than the things to be described…."[6]

Nothing much has changed since Einstein's day in this regard, except that, with the advent of new observations, the situation has been greatly exasperated. We now know that the "too great" of the continuum that so plagued him, has lead the advocates of particle theory to the most absurd and embarrassing result in the history of physical calculations,[7] and it has led the advocates of non-particle theory to conclude that no universal truth of nature can be discovered.[8]

While Einstein sought an algebraic physics, a science based on discrete numbers, as opposed to a geometric physics, a science based on the continuum of geometric magnitudes, he was perplexed, because he couldn't find the mathematical structure that would enable him to pursue it. Others carried on, after him, seeking to find this structure in Lie groups, and more recently in Octonions, but, to this day, although the Lie algebras of Lie groups have worked out well for quantum mechanics in some ways, no algebra of numbers has been found that leads to a discrete theory of gravity.[9]

While Einstein confessed that he plagued himself over this problem, he certainly was not alone. Today, the most serious controversies in theoretical physics are directly attributable to our unsuccessful efforts to resolve this incompatibility of the discrete and continuous,[10] and, yet, the modern intellectual affliction is no more severe for today's theoretical physicists than that which tormented the ancient mathematicians and geometers, so many centuries ago.

As David Hestenes describes it in his book, *New Foundations for Classical Mechanics*, Euclid was careful to keep calculations and proofs of discrete numbers separate from those of geometry, revealing his recognition of the fact that, since geometric magnitudes exist for which there are no numbers, discrete numbers are unsuited for describing reality completely.[11] In other words, like Einstein, Euclid recognized the foundational problem, wherein the continuum is more ample than the things that can be described with numbers. Worse yet, man's effort to understand reality is confronted with still another major obstacle: The magnitude limitation of Euclid's numbers is accompanied by a dimensional limitation of his geometric magnitudes, which, unlike numbers that can be raised to any power, are limited to no more than the three-power observed in physical dimensions.[12]

Ultimately, however, all concepts of numbers, magnitudes and dimensions depend upon the adequate definition of the point, and the task of adequately defining a point is almost as difficult as adequately defining God. We can symbolize it and refer to it, but understanding it is another matter entirely. The student of the Greek language begins his scholarly career firm in the knowledge that *en arche en ho logos* means "In the beginning was the word"; yet, for Dr. Faustus, the interpretation of the meaning of this enigmatic sentence from the *Gospel of John* was only a clarification of his growing disillusionment with language. He wore himself out trying to explain that verse and finally gave up.[13]

While the language of mathematics is much more precise than other languages, the success of mankind in explaining how the zero-dimensional point can be the origination of all things is no more satisfying than Dr. Faustus' ruminations on the Greek word *logos*. What is needed is an understanding of a mathematical point that can be differentiated into a line, a plane, and a volume. However, just as Dr. Faustus settles on a word extracted from his own vocabulary, realizing that it is inadequate and only justifiable from his own point of view, so also theoretical physicists have settled for a definition of the physical point that they acknowledge to be inaccurate, one that is only justified, if at all, by their own needs.[14]

For most modern mathematicians, the answer lies in the use of symbols and logic. For them, the drama of defining real points played out to a satisfactory conclusion, when the curtain came down in the final act of the play staring Dedekind, Cantor and Hilbert, who defeated their antagonist, Kronecker.[15] But, actually, decades before this real number debate erupted in the latter part of the 19th Century, Sir William R. Hamilton lamented the philosophical conundrums, which inevitably arise from permitting the practical utility of algebra to guide its logical and symbolic development, without proper regard to philosophical constraints.[16] Indeed, he felt that the trouble with algebra is that it does not have the same consistent philosophical foundation that geometry has, and that this can be remedied by recognizing that algebra should be founded on an entirely different set of principles than that of geometry. In his essay, "Algebra as the Science of Pure Time," [17] he warned that basing algebra on "forms of logic and a symmetrical system of expressions, with useful rules that depend on them," instead of an intuitive understanding of nature, was unwise.

Hamilton's proposed alternative to formulaic algebra was based on a system of continuous progression, the "flowing" quantity of time, rather than on a system of geometric magnitudes, the points and lines of which are formed and fixed. His approach was to replace the "bounded notion of magnitude," and the operation of increasing and diminishing it, with the "original and comprehensive thought of order in progression."

Unfortunately, Hamilton soon abandoned this effort to re-found algebra on "order in progression," and his essay on this is little remembered today. However, it's important to note that we now know that the fundamental dynamic of the physical universe is that <u>both</u> space and time are continuously flowing (expanding, increasing) in all physical dimensions, outside the limits of gravity: The fact is, modern observations show that space is continually expanding in three, geometric, dimensions, as time increases in one, non-geometric, dimension, carrying each galaxy away from all other galaxies.[18] Could this be a clue that Hamilton was on to something fundamental, given the mysterious connection between mathematics and nature?

Order in Space/Time Progression

Clearly, if we consider flowing space as the metric of one of the orthogonal sides of the unit triangle, while flowing time as the metric of the other, then the numeric ratio of the two sides is 1:1, which is not an irrational number at all, but a unit speed, $\Delta s/\Delta t = 1/1 = 1$. Consequently, since the continuous magnitudes of space cannot be measured without flowing time, and the continuous progression of time cannot be measured without changing locations in space, this suggests the consideration of an algebra based on "the order in progression" of <u>*both*</u> space and time, instead of the familiar algebra based on the traditional "bounded notion of magnitude."

Indeed, this author suggests the consideration of a new algebra, based on the geometric *progression* of space/time; That is to say, assuming an eternally expanding three-dimensional space over time continuum, it appears that one may logically select a point in that expansion and subsequently identify a function that produces at once a sequence of numbers with an associated set of continuous geometric magnitudes that can form the basis for a new algebra of multi-dimensional numbers, isomorphic to the 0D reals, the 1D complexes, the 2D quaternions and the 3D octonians, but without having to resort to the use of *ad hoc* concepts such as logical sets and imaginary numbers. Nevertheless, there exists a significant caveat to this new and unique approach: It is only possible if the concept of a point in the space/time expansion can be defined, so as not to conflict with the definition of motion.

The reason that a more consistent definition of a point is necessary is due to the fact that a point must have no spatial extent; that is, by definition, a physical point in a continuous progression must have zero dimensions, at the moment of its selection, or when time $t = 0$, in the expansion, if it is to correspond to the origin of an expansion. Clearly, however, any selected point in the space/time expansion immediately expands at <u>*any*</u> time $t > 0$, in all three dimensions, giving it 1D, 2D and 3D magnitudes, with arithmetic and geometric properties. Therefore, the first questions to be answered in constructing a more intuitive algebra based on "order in progression" are, "How is it possible to select a point in a 3D expansion, such that it has no spatial extent at time $t = 0$?" and, "How is it possible that an instant in increasing time can even exist (i.e. have zero duration)?"

As usual, the ancient Greeks had much to say about these questions. In his FQXI essay, "Time for a Change," [19] Peter Lynds explains the paradox: The definition of a point and the definition of motion are mutually exclusive. Lynds asserts that this ancient argument of the Greeks still stands, in spite of the practicality of the modern notion of limits, first used in the infinitesimal calculus.

Redefining Points

Consequently, to avoid this inherent paradox in the logic of the definition of a point in the context of motion, a new definition of point is proposed that exploits the inherent "direction" property of dimension. On this basis, since each geometric dimension includes two "directions," relative to a point of origin (its positive and negative "directions" we may say), we are free to incorporate a change of "direction" in the definition of a point in the context of motion, in order to define it in time *and* space in such a way that it is logically consistent with the concept of motion. The word "direction" is here placed in quotes to distinguish it from the ordinary sense of direction, defined by coordinates in a coordinate system. Of course, the set of directions in three dimensions is much more ample than the set of "directions" in three dimensions. In fact, the former is an infinite or continuous set, while the latter is limited to the discrete number, $2^3 = 8$.

To understand the fundamentals of the new concept, let us think of the point, the line, the area and the volume, not in terms of one, two, three and four points, and distances between them, as geometric forms are normally described, but in terms of a single point, the origin of all, generating all these forms with corresponding numbers, the way many of the ancients did. For them, everything had to have a middle: The point was the middle of the line, dividing it into two equal parts, polarizing it we might say. It was also the middle of the circle, drawn from its center, and the middle of a volume extending out from its origin.

Traditionally, the task of specifying any direction in these continuous spaces has been approached by incorporating the power of logic and symbols to define a set of irrational numbers that can be added to the set of rational numbers, expanding the set of real numbers, to include any geometric magnitude, and also by defining the correspondence between certain algebraic and geometric operations, as imaginary numbers in higher dimensions. Then, the consternation of many philosophers notwithstanding, combinations of these two sets of numbers, the real numbers and the imaginary numbers, can be used to calculate vectors in higher dimensional vector spaces.[1]

The problem with this traditional approach is that only the original algebra of points maintains all the properties of a division algebra. Each time we move up a dimension we must sacrifice another of the three important properties, the distributive, commutative and associative properties of zero-dimensional algebra.[20]

So, in a sense, our intuition, based on the "bounded notion of magnitude," wants to keep our understanding of algebra confined to the zero-dimensional space of points, while it wants to keep our understanding of geometry confined to the three-dimensional space of volume. Of course, mankind has refused to sit still for these restrictions, and has devised many ways to try to get around them, none of which has ever completely succeeded. Nature still has the last laugh.

But now, with the new information that space, like time, is not a static quantity, but a dynamic one, a new approach presents itself, where volume, and all the lower dimensions of area, line and point that volume contains, is simply generated by the continuous increase of time. That is to say, if we pick any point in the space/time expansion, given this new understanding of physical space, it immediately expands into some volume, containing the 1D magnitude of a continuous line in two "directions," the 2D magnitude of a continuous area in four "directions," and the 3D magnitude of a continuous volume in eight "directions." But, again, the question is, how can we identify a point in expanding space, at a given instant in time?

The Point of Three-Dimensional Oscillation

One way to do this is to consider the reverse phenomenon of contraction together with the phenomenon of expansion. While it seems impossible to identify a point in a continuous expansion, the point is unavoidable in a continuous contraction. Of course, this idea requires that we assume a polar reversal in all eight "directions" of the expanding volume, at some point in time. Subsequently, as the 3D contraction continues, the magnitude of the volume inevitably diminishes to zero at some future point in time, depending on the magnitude of the volume and the speed of the contraction.[2]

However, in the case where we specify a unit expansion, followed by a unit contraction, in such a 3D oscillation, the decrease in volume is equal to the previous increase in volume, which immediately raises the cosmological question of space/time conservation. "How can space increase, or decrease, over time, without violating the law of conservation?"

In a unit 3D oscillation, just as any passage of time, $t > 0$, generates a finite volume of 3D space, in the outward "direction," during expansion from a given point, any passage of time, $t > 1$, decreases a volume, by some finite amount, in the inward "direction," during the contraction toward the point. Clearly, this change in the volume of space, in either "direction," constitutes a violation of the law of conservation, unless there exists a simultaneous change in some inverse quantity.[3]

In the case of energy conservation, for example, a change in potential energy is always accompanied by a corresponding change in kinetic or electromagnetic energy and vice-versa, under normal conditions. This Law of Physical Conservation is central to our understanding of nature. It's embodied in the principles of symmetry, as shown by Noether's theorem.[21]

However, the only known quantity that is the inverse of space is the quantity of time, in the equation of motion, and the domain of this reciprocal relationship is normally restricted to the one-dimensional case.[4] Nevertheless, now that our idea of space must change from a static concept to a dynamic one, the general concept of 3D space expanding in time compels us to consider the concept of the inverse of this change, i.e. time contracting three-dimensionally, during a 1D space interval, to comply with the law of conservation.[5]

Assuming that such an inverse quantity exists, a space/time <--> time/space oscillation, or the smooth and constant change from one 3D quantity to its inverse 3D quantity, would have to occur at the zero point of their common origin. This is analogous to the changing position of a pendulum, swinging toward the plumb line. The position of the pendulum ultimately can be thought of as a swinging segment of its path, an arc, which, when it reaches the plumb line, constantly transforms into its inverse, on the other side of the plumb line. As one portion of the arc diminishes, its inverse increases, on the other side of the plumb line.

Likewise, in the 3D case of oscillation, any diminishing of the volume, during contraction, has to be transformed into the inverse volume, at its point of origin, until the entire volume has been inverted, whereupon the motion reverses.[6] It is precisely this point of transition, located at the boundary where an instantaneous change in inward and outward "directions" occurs between 3D inverses, which can be defined as a point of no spatial extent, thus avoiding the logical contradiction between its definition and the definition of motion.

Two Interpretations of Number

Hestenes tells us that it was Clifford who first realized the significance of the fact that there are two interpretations of number possible, but, for all we know, Clifford's insight might have come from his careful study of Hamilton, because the important distinction between the two interpretations of number, which Clifford makes, corresponds exactly with the distinction Hamilton made much earlier between the usual practice of treating numbers as the "bounded notion of magnitude," the notion of "how many," or "how much," of something, and his proposal to treat them more generally, as the relation between the different quantities needed to describe "order in progression." Hestenes cites the example of his unit bi-vector, **i**, which quantitatively interpreted, is a measure of directed area, but when operationally interpreted, specifies a rotation in two dimensions.[22]

On this basis, assuming that the two most fundamental quantities of the physical universe are expanding space and time, and that the only known relationship between them is motion, a reciprocal relation, if space and time are continuously expanding, then the rational number, s^3/t, specifies a 3D expansion, operationally, but constitutes a measure of 3D volume, quantitatively.

Clearly, then, if we assume that the space and time of reality is composed of discrete units, we could, following Hestenes, operationally interpret $s/t = 2/1$, as specifying the unit expansion of length in its two "directions," in one unit of time, interpret $s^2/t = 2^2/1$, as specifying the unit expansion of area, in its four "directions," in one unit of time, and interpret $s^3/t = 2^3/1$, as specifying the unit expansion of volume, in its eight "directions," in one unit of time. But of course, this will not serve to describe reality completely, because nature expands as the continuous magnitudes of lines, circles and spheres, not as discrete magnitudes of lengths, squares and cubes, and the continuous magnitudes in the length of the line, the area of the circle and the volume of the ball are much more ample than what these discrete numbers describe. Yet, that is why Hestenes (and everyone else in the modern world) resorts to the logic of sets and the operation of the 2D motion of rotation, in order to describe areas and volumes and the geometric and physical relations built upon them.[23]

Regardless, however, assuming that the inverse of 3D spatial volume actually exists, as intuition strongly suggests, we could theoretically use the motion of 3D oscillation, instead of 2D rotation, to describe these geometric and physical entities. If so, then this means that the continuous 3D expansion/contraction operation can incorporate only *one* newly defined point, the common origin between the inverse volumes. It no longer appears appropriate to define a continuous magnitude in terms of an infinite set of points. To be consistent with this line of thought, we must define the continuous expansion/contraction in terms of only one point, the one point lying between polar "directions" of a given geometric magnitude. A proposal to accomplish this is based on the geometric construction shown in figure 1 below.

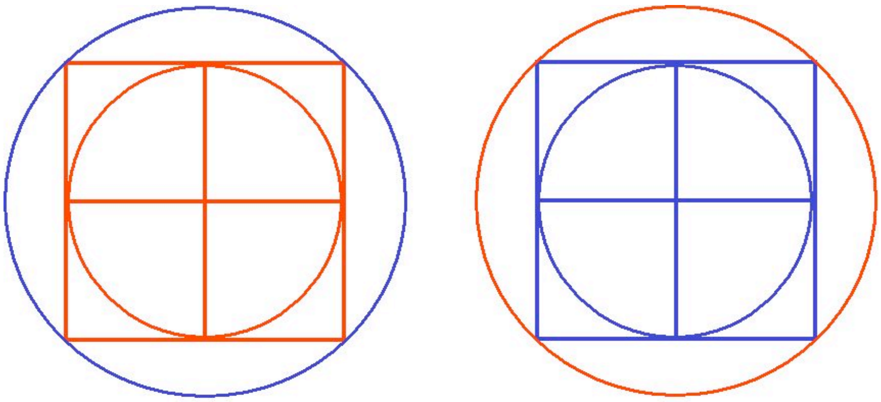

Figure 1. Negative (left) and Positive (right) Units of Discrete and Continuous Magnitudes

In figure 1 above, the inner and outer circles, within and without the two, 2x2 stacks of four unit squares, constitute the orthogonal view of two sets of two continuous volumes. Each is completely determined by the two, inverse, discrete volumes. The quantitative interpretation of $2^3 = 8$ cubic spatial units, generated from the origin, by the operational interpretation of $s^3/t = 2^3/1$, or one unit of 3D expansion of space/time, is a negative quantity. The inverse of this set, located to its right, looks exactly the same, but its two continuous volumes correspond to two continuous *temporal* volumes. These temporal volumes are determined by the discrete temporal volume, the quantitative interpretation of $2^3 = 8$ cubic temporal units, generated by the operational interpretation of $t^3/s = 2^3/1$, or one unit of 3D expansion of time/space, a positive quantity.

Like the polar opposites reflected in a mirror, the magnitudes of these two sets of cubes and balls, inverses of one another, are the basis of the proposition that a multi-dimensional division algebra can be found, up to the three dimensions of Euclidean geometry.[7] This scalar $\mathbf{R}^3$ algebra would have all the distributive, commutative and associative properties of **R** algebra, with a common point of origin, constituting a single point of instantaneous transition, from one "direction" to the opposite "direction," in the operational interpretation of its numbers, and the ratio of outer radii constituting the group identity element, in the quantitative interpretation of its numbers.

The radius of the inner circle, equal to the length of one of the sides of the cube, corresponds to the unit of time (space) in the expansion/contraction cycle, while the diameter of the inner circle corresponds to the unit of space (time). Thus, the speed of the oscillation is $\Delta s/\Delta t = 1/1 = 1$, or unit speed, even though the length of the diameter is twice that of the radius. This is because the 1D diameter simultaneously extends/collapses in its two "directions" relative to the origin, as time (space) progresses.

As the contraction reaches the zero point of the origin, both the inner and outer radii vanish simultaneously, even though the length of the outer radius is larger than the inner. This is because the outer radius is not an independent variable, but depends on the inner radius. If the inner radius has any magnitude at all, so does the outer, but if the inner radius has no magnitude, then neither does the outer. This is important to note, because the outer radius, and the outer diameter, are irrational magnitudes that are dependent on the rational magnitude of the inner radius.

Discretization of the Continuum

Now, it's clear that, if the inner radius, corresponding to the unit time (space) magnitude of the oscillation, is regarded as a bounded magnitude, it is infinitely divisible, which we can denote as a rational number, n/n = 1, where n = 1, 2, 3, …∞. However, the geometric construction of figure 1 is valid for any sub-division of the inner radius, 1/n, no matter how small. Therefore, an irrational magnitude, $(1/n)(2^{1/2})$, equal to the outer radius, is associated with every rational magnitude of 1/n, and since we know that the set of rational numbers in the unit radius is infinite, it follows that the set of irrational numbers in the non-unit radius is infinite also.

Now, what is more germane to the issue at hand is that the correspondence between the inner and outer radii works both ways. As Hestenes notes, a lack of this two way correspondence between a number n and any geometric magnitude x, hindered the development of mathematics and science for centuries, until the time of Descarte, who first began to denote geometric magnitudes of lines as x, for algebraic purposes.[24]

However, in this case, we can assign a number 1 to the inner radius that corresponds to any magnitude x of the outer radius, so the correspondence between number and magnitude works both ways; that is, unlike Euclid, we can assign the number 1 to any geometric magnitude, by first setting the outer radius to the desired magnitude and then assigning the number 1 to the inner radius. Subsequently, for every element x in the infinite set of irrational magnitudes in a given geometric entity, there is a corresponding element 1/n in the infinite set of rational numbers that is a sub-division of its corresponding unit.

On this basis, the size of the 3D oscillation, as measured in terms of space, or time, can equal any irrational geometric magnitude x in the continuum, and that magnitude will always correspond to an algebra of discrete numbers. With this approach to the "discretization" of the continuum, the task then becomes one of selecting the appropriate unit of time and space, the appropriate metrics, that will produce the correct physical results.[8]

Conclusion

To conclude, this sketch of a new approach in defining a spatial point of no extent, and a temporal point of no duration, suggests that the answer to the age-old question, "Is reality digital or analog?" is this: "No, reality is not digital *or* analog; It is both; It is a continuum that nevertheless can be digitized," as all physical evidence to date indicates, and as the findings of the new approach support. However, the major discovery that emerges from this approach is that describing real magnitudes as consisting of an infinite set of discrete *points* on a line, in an area, or in a volume, may be inappropriate in some cases. Sometimes, we should think in terms of an infinite set of discrete *segments* of a line, discrete *sub-areas* of a plane, and discrete *sub-volumes* of a ball, where *each* segment, sub-area and sub-volume, *ad infinitum*, can be referenced to only *one*, true, discrete point, in the middle of that segment, in the center of that sub-area or at the origin of that sub-volume, a single point of zero dimensions, or zero extent, the very point of reality.[9]

End Notes

[1] Hestenes advocates his Geometric Algebra, wherein the introduction of the "geometric product" greatly simplifies the manipulation of geometric forms in vector spaces, by augmenting the grammar of Clifford algebras with a number of new definitions and concepts. While his approach reveals a more intuitive understanding of the nature of imaginary numbers, it still must rely on the accepted definition of irrational numbers and rotation.
[2] No special mechanism is needed to explain the reversal of 3D "direction," at a given location in the expansion, just as no special mechanism is needed to explain the origin of quantum spin. The fact that it is a possibility is all that is needed, philosophically speaking.
[3] When we think of motion as a change of quantity, rather than a change of location, then any change in the length of the positive side of a line for instance, must be accompanied by a corresponding change in the length of the negative side. We usually think of this principle in terms of slide rule construction, or in terms of its incorporation in balance scales.
[4] In the 1D equation of motion, $v = \Delta s/\Delta t$, the change in space is specified by the changing location of an object. Therefore, this type of motion can only be one-dimensional, but since the equation itself only requires a change in space, and does not require an object to specify that change, then, strictly speaking, any change in a quantity of space over time constitutes motion, even though it doesn't involve a change in location.
[5] It's hard to imagine that the inverse of 3D space could be 3D time, because we only experience time as a sum of zero-dimensional moments, a 1D duration between 0D instants of time. Nevertheless, logically and mathematically, we can construct 3D time from inverse 3D motion.
[6] If we think of the position of the swinging pendulum as consisting of some segment of space, instead of a point of no extent, then once the distance from the pendulum to the plumb line is equal to that segment, the segment (that is the pendulum's position), in effect, crosses the plumb line starting at the beginning of the segment and progressing to the end of the segment, until it has completely crossed the plumb line. During this interval of time, we cannot specify the exact position of the pendulum, but the "direction" of the pendulum's motion is constantly changing from inward to outward, relative to the plumb line. The change of "direction" is instantaneous along the entire segment; that is, no portion of the segment is ***not*** moving, in one "direction" or the other, relative to the plumb line, during the interval of time it takes to complete the transformation. Another way to look at the same thing is to follow the projection of the pendulum's position on the plumb line. While the motion of the pendulum never stops, its projection on the plumb line must stop at some point, since it reverses the "direction" of its motion. When we analyze this point of "direction" reversal, we find that it is a point of no spatial extent and no temporal duration.
[7] In effect, the new algebra replaces the unit of 1, with the unit of the square root of 2, R. Then 1/R is the inverse of R/1 and the identity element of the group is R/R. The R group is a group of irrational numbers, isomorphic to the usual group of rational numbers, but unlike it, the R group actually consists of three concurrent groups, R, R^2 and R^3.
[8] Theoretical physicists use a combination of physical constants (G, $\hbar$,c, k_e, k_B) to determine the appropriate space and time metrics, known as Planck units, but an alternate approach uses the Rydberg and the speed of light constants (R_∞, c) to determine them. The difference is not only one of size, but one of theory, as probing the Planck length would require so much energy it would doom space-time, given the general relativity theory of gravity. In the new theory, there is no need to probe the unit space.
[9] The unit quantity that transitions to its inverse does so as a result of the motion instantaneously changing "directions" at the "direction" boundary. Since there is only one boundary point in a given unit of oscillating space or time, as defined here, it makes no sense to think in terms of the unit being infinitely divisible, since any division is tantamount to selecting a new unit. Instead, we should think of the unit as transformable into its inverse, no matter how small it might be, ad infinitum.

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