Trick or Truth: The Mysterious Connection Between Numbers and Motion and Geometry

Introduction

How can we understand Wigner's "unreasonable effectiveness of mathematics" in physics, other than as a deep, mysterious and unexpected unity, which is observed between mathematics and physics? We can start by recognizing that there is an obvious and straight forward connection between numbers and geometrical magnitudes, after all. When the former are properly understood, as rational numbers and this connection is extended to time, as well as space, the mystery disappears.

What are Numbers?

In our modern science of mathematics, the answer to this question is complicated. We have real numbers, complex numbers, quaternion numbers and octonion numbers, recognized as the members of the only four normed division algebras known to exist. [1] Physicists have used these four algebras, with varying degrees of success, to bring the science of theoretical physics to its present state. [2]

Nevertheless, at an elementary level, numbers count things, and given two such numbers, one greater than the other, there is always another number, greater than them both. [3] In counting things, it's possible that the things counted are parts of a whole, where we use two numbers, related to each other. One number counts the total number of parts into which the whole is divided, while the other counts those parts of the whole that are under consideration. The relation between the two numbers is expressed as a ratio, where m/n = 1, when m = n.

We can write the set of all these rational numbers as a number "line," in the following manner:

n/m, …1/3, 1/2, 1/1, 2/1, 3/1, …,m/n

Notice that the ratios to the right of the whole number, 1/1, are the inverses of the ratios to the left of the whole number. The magnitude, m, of the ratios, relative to the whole, increases in both directions, in the following manner:

m, … 3, 2, 1, 2, 3, …, m

Traditionally, mathematicians assign symbols of direction, such as polarity symbols, '+' and ' –', to distinguish between those numbers on the left and those numbers on the right:

-m, … -3, -2, 1,+2, +3, …, +m

But, as is well known, the algebraic use of these numbers requires a negative unit, -1, so it was gradually, but eventually added to the list of numbers, even though it makes no sense and cannot be reasonably placed in the number "line" of rational numbers above:

-m, … -3, -2, ***-1***, 1, +2, +3, …, +m

For this reason, mathematicians originally called it an "imaginary" number, and it has complicated the field of mathematics tremendously, and it has complicated the field of physics even more. [4]

To avoid this complication, perhaps we can use the rational numbers themselves, where their inverse magnitudes easily distinguish them as belonging to the "positive" or "negative" side of 1/1, a whole number which is neither "less than one" or "more than one," thus, eliminating the need for polarity symbols all together, and the subsequent requirement of a negative unit that complicates algebraic operations.

However, in order to make this work, we will have to consider the fractions on the left of 1/1, as the inverses of the "fractions" on the right of 1/1. Fortunately, this is easily accomplished by regarding the *denominator* of the fraction as the number below the division symbol (vinculum), "/", for the fractions on the left of the whole number, 1/1, and the number above the vinculum, as the *denominator*, for the fractions to the right of the whole number.

This view of elementary numbers may seem bizarre, at first, but it also may be the start to "the new way of looking at things," that the notable mathematician, Sir Michael Atiyah, is thinking we need, given the current prospect that mother nature's reality might be composed of a "vastly complicated mathematical structure," inherent in string theory. [5]

The Motion of a Two-Face Clock

To understand this unorthodox view of elementary numbers, we need only consider a clock, with two faces, back-to-back. One face will represent the fractions, where the denominator is below the vinculum, those ordered to the left of the whole number, or the unit number, while the opposite face of the clock represents the inverse of these numbers, those ordered to the right of the unit number, where the denominator of the fraction is above the vinculum.

Of course, the first challenge will be to find a way to order these numbers on the clock face so that they repeat, *ad infinitum*, as do the 12 (or 24) numbers on our clocks, which represent the 24 hour revolutions of the earth on its axis. In other words, we need a repeatable physical connection, a periodic motion, which this set of rational numbers can represent.

One way we can do this is to consider that the *motion* of the clock hand, in the clockwise direction on one face, and in the counter-clockwise direction on the opposite face, is actually the combination of two motions. One motion is the bidirectional movement of the clock's escapement, while the other is the unidirectional movement of the clock's escapement wheel, driving the clock hand. [6]

In this analog, as the clock hand moves, the numerator, representing the escapement, repeats, or oscillates, over 1 unit, a number of times that is equal to a corresponding increase in the denominator, representing the escapement wheel. In the simplest case, the numerator oscillates between 0 and 1 continuously, while the denominator increases simultaneously from 0 to 1 to 2. In other words, as the clock hand moves over the units on the face, the numbers in the rational number that corresponds to its motion indicate the number of units transited by the motion. The number in the numerator indicates that there is no net change as it increases and then decreases by one unit, alternately, while the number in the denominator indicates a two-unit increase, associated with the net-zero change of the numerator: Thus, the rational number, “1/2,” corresponds to this motion.

Meanwhile, turning the clock 180 degrees, to view the opposite clock face, the direction of the clock hand’s motion is now in the counter-clockwise direction. The two-unit increase of the denominator is now shown above the vinculum, while the one-unit oscillation is shown below it, indicating the reciprocal nature of this motion. Thus, the rational number corresponding to it is, “2/1.”

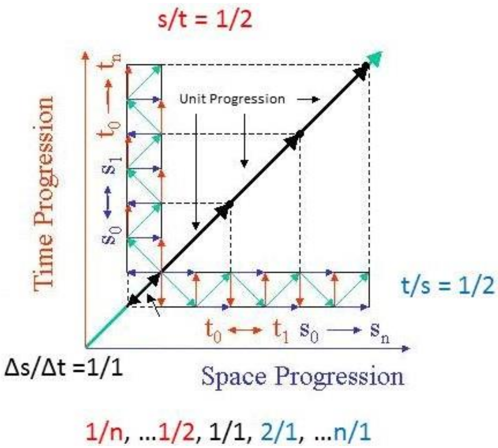

Figure 1.

Each of these two rational numbers, then, symbolize two changing, reciprocal quantities, one of which is double the other, due to a constant direction reversal, after a one-unit change, in one of them. Since space and time are reciprocal magnitudes, in the equation of motion, v = s/t, figure 1 shows how this motion can be plotted graphically.

In the first square, in the lower left corner, a unit increase in space corresponds to a unit increase in its reciprocal, time. Therefore, the net result is motion along the diagonal. The rational number equivalent for this motion is “1/1,” meaning that there is a one-unit increase in the numerator per one-unit increase in the denominator.

However, in the second unit, which would normally be identical to the first unit, the possibility of a direction reversal occurring in one of the two reciprocal magnitudes is shown.

If the direction reversals occur in the increasing space component, the result is the rising plot shown as the vertical oscillation. If the direction reversals occur in the reciprocal time component, the result is the rightward extending plot shown as the horizontal oscillation.

In each case, the two-unit increase of the reciprocal component of the motion that does not oscillate is double the oscillating unit’s magnitude. In one case, the corresponding rational number is “1/2,” while in the other case, the rational number is “2/1,” wherein the denominator is above the vinculum, which, as the graph shows, is only distinguished in its direction, relative to the unit increase, represented by the number, “1/1.”

Of course, instead of changing the number, by turning the numerator/denominator upside down, we can exchange the space and time terms, so that, on one side of the unit number, s/t =1/2, while on the other side, t/s = 1/2.

Focus on the Motion

Now, given that we have these numbers that don't require polarity signs, what can we do with them? Do they too have this "deep and mysterious connection" with physics that is so much wondered at? Actually, we can see that they are an expression of motion itself, so, right off the bat, we suspect that the answer is yes. Still, it might be hard to see just how it would be worthwhile to investigate physics with such numbers.

Newton's scientific research program, into the structure of the physical world, which continues with us today, "can be summarized," writes David Hestenes, "by the dictum: Focus on the forces." He goes on to write:

> This should be interpreted as the admonition to study the motions of physical objects and find forces of interaction sufficient to determine those motions. The aim is to classify the kinds of forces and so develop a classification of particles according to the kinds of interactions in which they participate. [7]

Clearly, the sense of "the deep and mysterious connection" between mathematics and physics has emerged from this research program of Newton's. However, it has been little noticed that an engineer, Xavior Borg, has recently discovered that the standard units of measure, the SI system of units of measure, can all be expressed with units of motion, or dimensions of space and time only! [8]

This means that mathematical equations used in Newton's program of theoretical physics can now be viewed in terms of the dimensions of motion and the dimensions of inverse motion, or energy. Some important examples are:

$E = mc^2$ becomes: $t/s = (t/s)^3 \times (s/t)^2$;

$F = ma$ becomes: $t/s^2 = (t/s)^3 \times (s/t^2)$;

$p = mv$ becomes: $t^2/s^2 = (t/s)^3 \times (s/t)$;

Borg has a long list of SI units converted to space and time dimensions available on his website. [9]

More to the point, however, the fact that these physical units can be expressed in terms of the dimensions of motion and its inverse, energy, implies that there is something we don't understand about motion and energy. Given the motion equation, $v = \Delta s/\Delta t$, we see that the equation requires no object. Motion is simply defined as a change in space over a change in time. Of course, we experience the continuous change in time, but only when we observe the distant galaxies do we see nature's continuous change in space.

The motions of the graph depicted in figure 1, like a mechanical clock's motion, are analogs of one-dimensional motion that are symbolically represented by the inverse numbers 1/2 and 2/1, which are derived from the unit number, 1/1, a one- dimensional ratio, comprised of two, one-dimensional numbers.

Nevertheless, given Borg's multi-dimensional units of space and time, perhaps it would be worthwhile to go beyond Newton's program of research and seek to develop a classification of multi-dimensional motions, that might correspond to known multi-dimensional geometries, even though no objects, forces or interactions are relevant, at this point.

One approach, currently under investigation, defines and classifies new multi-dimensional units of motion, according to new, multi-dimensional numbers. This classification follows the structure of the Greek tetraktys, or the familiar binomial expansion, which forms the basis of Clifford algebras. [10]

Here, however, fundamental units of multi-dimensional numbers are defined, where the numbers 1, 2, 3 and 4 of the tetraktys are raised to exponential powers, from 0 to 3, restoring the dimensional correspondence to fundamental geometrical units, which is lost when employing the tetraktys dimensions of 1, 2, 4 and 8, as we currently do, in the normed division algebras of our four number systems. [11]

In each case, the idea is to equate multi-dimensional mathematical units, or numbers, with corresponding multi-dimensional geometrical units of measure, which can be physically generated from multi-dimensional motion, as herein defined.

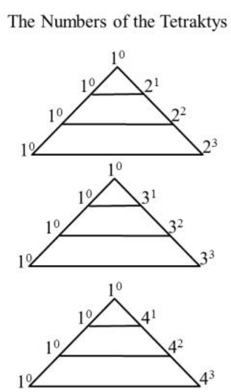

Figure 2. The four Numbers of the Tetraktys

The first number is 1. When it is raised to the power of 0, it is a number that corresponds to the geometrical point, at the top of the tetraktys. When it is raised to the power of 1, it is a number corresponding to the geometric line of the tetraktys. Raised to the power of 2, it corresponds to area and to volume when it is raised to the power of 3. These are numerical units that correspond to familiar geometrical units of points, lines, squares and cubes.

The next number is 2. It also corresponds to a point, at the power of 0, a line at the power of

1, an area at the power of 2 and a volume at the power of 3, but instead of these correspondences arising from a change in the dimensions of abstract numerical units, they arise in connection with a change in the dimensions of motion, which cause a change in position. However, since this motion necessarily involves the motion of an object's location, a unit increase in position can only be affected in one available dimension at a time.

The next number is 3. It also generates units corresponding to points, lines, areas and volumes, when

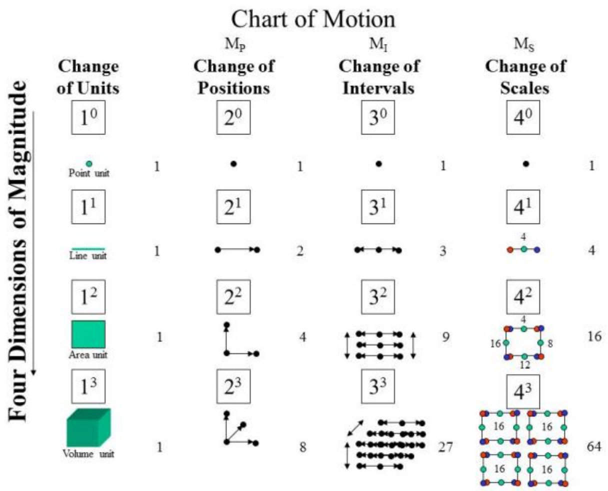

Figure 3. The Chart of Motion

raised to 0, 1, 2 and 3 powers, but these changes in the dimensions of motion correspond to the dimensions of intervals, rather than to changes of position. An example would be a 0D point, simultaneously stretched in two opposite directions to form a 1D line; a 2D line stretched in two opposite directions to form a 2D area, and a 2D area stretched in two opposite directions to from a 3D cube.

Finally, we come to the last number of the tetraktys, the number 4. Again, the change of the dimensions of this number corresponds to the geometrical changes of dimension, the dimensions of a geometrical point, line, area and volume, but here the unit change occurs as a result of changes in the dimensions of scale, rather than a change of the dimensions of position, or a change of the dimensions of interval.

It's more difficult to explain, but the 0D scalar "point" consists of a 0D entity represented by a balance between inverse unit magnitudes, where $4^0/4^0 = 1/1$. Consider it symbolically, as a physical barbell, where 4 raised to the power of 1 is equivalent to an 8-pound barbell, with 4-pound weights on each end ($4^1/4^1 = 4/4$). When 4 is raised to the power of 2, four of these 8-pound weights are formed, which are able to bound an area, by connecting them end-to-end, two-dimensionally ($(4^2/4^2) = (16/16) = (4/4)+(4/4)+(4/4)+(4/4)$)

When 4 is raised to the power of 3, sixty four of these weights are formed, which can be configured into 16 of the 8-pound barbells, making it possible to bound a cube with them, by connecting them end-to-end, four times, in three-dimensions.

$(4^3/4^3) = 64/64 = ([(4/4)+(4/4)+(4/4)+(4/4)] + [(4/4)+(4/4)+(4/4)+(4/4)] + [(4/4)+(4/4)+(4/4)+(4/4)] + [(4/4)+(4/4)+(4/4)+(4/4)])$

With this much understood, we see that each set of these multi-dimensional numbers, $1^{0\text{-}3}$, $2^{0\text{-}3}$, $3^{0\text{-}3}$, $4^{0\text{-}3}$, are the numbers that form the tetraktys, and they correspond to a unique geometrical entity of the same number of dimensions, created by the corresponding class of motion, or change of space, over time, in different ways.

In other words, each of these different classifications of motion creates multi-dimensional geometrical units of space over time, as points, lines, areas and volumes, which have their equivalents in multi-dimensional numbers of corresponding dimensions, and each in its own way: By way of a change of position, by way of a change of interval and by way of a change of scale. A chart of these relationships is provided in figure 3.

But what about rotational motion, why isn't it included in the Chart of Motion? Given that rotational motion is observed in the heavens and on earth, one would think that it is surely a fundamental motion. Indeed, understanding rotational motion, from the swinging pendulum to the orbits of the planets around the sun and the stars around the galaxies, is the foundation of Newton's research program.

Nevertheless, it's not included in the Chart of Motion, because it's not found in the tetraktys. It is essentially change of position motion that undergoes a constant change of direction, but change of direction, while important to understand in many respects, does not create a corresponding geometrical unit, independently of change of position motion.

The difficulty of comprehending the iconoclastic nature of this conclusion is hard to overestimate, because rotational motion is the foundation of our vaunted science and technology, [12]but at the same time, this conclusion goes to the root of our trouble with theoretical physics today.

Again, the consequence of adopting the imaginary negative unit, while extremely useful, may have been mankind's undoing, when it comes to understanding the mysterious connection between mathematics and physics, in the search for reality. Newton's program begins with particles, seeking to classify them, according to the forces involved in their interactions. However, now we know that particles themselves ultimately have dimensions of space and time, or motion, and force is just a quantity of motion. Consequently, we need a new program of research that classifies combinations of motion, or motions, and the relationships between them, as observed.

Presumably, such a research program would start with the implications of the charts of figures 1 and 2. The dimensions of the motion of figure 1 are unspecified, but if we assume that it is three-dimensional motion, then the oscillating unit would be a ball, expanding and contracting, while the reciprocal component would be expanding continuously, in three dimensions. Since the magnitude of the original motion, before oscillation begins, depends on the relative magnitudes of the space and time units, a natural candidate for them would be derived from the speed of light.

Taking these magnitudes for the space and time units of the graph in figure 1, this is the "speed" represented by the diagonal line, labeled "Unit Progression," in the chart of figure 1.

Now, given the onset of oscillation of the space component in figure 1, producing the increasing vertical time line, where each space cycle requires two units of time to complete, the frequency, f, of this entity is then one cycle for every two units of time.

But frequency, with dimensions 1/t, or cycles per unit of time, is a concept of rotation, normally expressed in terms of 2π radians per second, mathematically equivalent to the motion of waves. Even if f could be converted to velocity, why would we want to do so, when using the changing sine and cosine of the rotation angle, and the concept of angular momentum, underlying the wave equation, are simply indispensable to modern physics and engineering?

That is a good question, but the answer is good too: We want to look at things differently to see if the reality of nature can be expressed without the "vastly complicated mathematical structure" of modern science. [13] And now we see, from a study of the tetraktys that rotational motion, so crucial to our modern science, is not even a proper class of motion, and moreover, it is clear that our use of it depends on the *ad hoc* invention of an imaginary, negative unit.

Nevertheless, and notwithstanding the understandable consternation this observation might engender, the chart of figure 3 shows that there are other options. Indeed, if we regard the natural 3D motion of interval (2^3 type motion), rather than trying to employ 3D motion of position through rotation (think of Lie groups and Lie algebras,) it appears that our task would be greatly simplified.

Using the interval concept of motion, the oscillations of figure 1 simply represent the expanding/contracting radius of a 3D ball. This means that the unit space volume goes from zero to unit value and back to zero, in two picoseconds, but the number corresponding to the cubic value of the three-dimensional interval motion (2^3=8) is incompatible with the numerical equation for the volume of a ball; that is to say, nature doesn't expand/contract in cubes.

However, we can easily quantify this oscillation for volume by recognizing that, while the number, 2^3 equals a 2x2x2 stack of 8, 1-unit cubes, and the unit volume of the ball equals 4π/3, which, although it is an irrational number, it is a ball that just fits into the 8-unit stack. This means that the ratio of one of the one-eighth volumes to the unit volume is 1:8, which also means that the ratio of the radii of the two volumes is 1:2.

As it turns out, then, the cube of the radius of this smaller volume is equal to the ratio of the two volumes:

$V_1 = (4\pi/3)$, $V_2 = V_1/8$

$V_2 = (4\pi/3)r^3$

$r^3 = (V_2/V_1) = 1/8$

$r = (1/8)^{1/3} = .5$

In other words, the *radius* of the unit volume (1) is the *diameter* of the 1/8 volume (1), and its *radius* is therefore half of its *diameter* (1/2). This is fortunate, because it allows us to map the expanding/contracting volume to the equivalent of 2π radians of rotation.

This is also important to show, if for no other reason that it takes two, 2π revolutions to complete one cycle of quantum spin, or 4π radians of rotation, to get a valid solution to the wave equation. Currently, this mathematical requirement has no satisfactory physical interpretation, until we understand that quantum spin might not be a case of 1D rotation, in 3D space, but rather a 3D oscillation of 3D space, as depicted in figure 4 below:

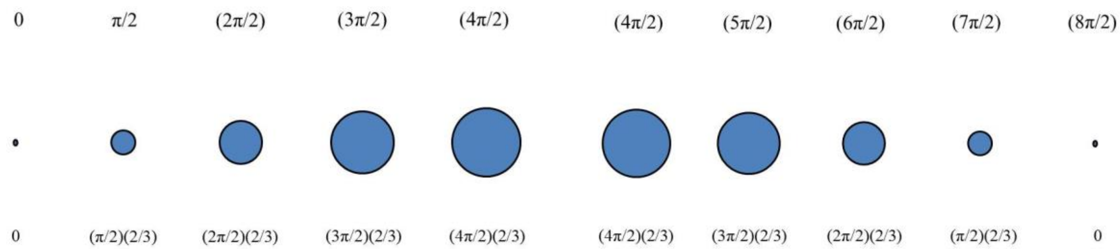

Figure 4. As (π/2)/3 = .523598… = $V_1/8$, two of these volume quantities, or V = (π/2)(2/3), are the equivalent of an increase, or decrease, of one, π/2, rotation (90^o)

As we see in figure 4, in the equivalent of one revolution of 2π radians, the 3D oscillation has fully expanded, which is one-half of its cycle. Contracting to the starting point at zero in the second half of its cycle, requires the equivalent of a second, 2π revolution.

Thus, we can conclude that, while Roger Penrose's enthusiasm for the beauty of "mathematical reality,"[14] contrasts with Sir Michael Atiyah's misgivings that, in the end, the beauty we find may only be "in the eye of the beholder,"[15] it is clear that there is a "new way of looking at things," which may be promising.

There is no doubt that the complex number works well, in many unforeseen ways, but the fact that we can easily understand the relationship between space and time, or motion, in another way, using rational numbers, as described above, that leads to gratifying results, lends credence to Sir Atiyah's observation that we really do need a less intricate, less complicated alternative, that "perhaps we use all this mathematics, because we got it and there is nothing else we can do." [16]

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