Ghostly Dimensions

Niklaus Buehlmann

I) Introduction

When we assume that reality is analog, we implicitly say, that we can measure things. This means, that we can map quantities of existing objects such as „length" or „area" onto the axis of real numbers, where we have an ordering relation: „greater than" , „ smaller than" or „equal".

Reality in this context is our everyday world, but equipped with the most modern measurement instruments that mankind has ever developped. These instruments allow us the best possible mapping of lengths and areas onto the axis of real numbers. This has the advantage, that we can compare the lengths and areas and e.g. make statements of the following kinds:

a) „This line is longer or equal or smaller than that line

or

b) This area is larger or equal or smaller than that area

Such statements however, should have the following two properties:

1) They should be in line with our everyday experience and not contradict it.

2) Equipped with the most modern measurement intruments we should be able to decide which relation („larger" or „equal" or „smaller") is true for the lengths and areas of two objects of our everyday world.

   If 1) is not satisfied, then our model of an analog reality is not adequate.

   If 2) is not satisfied, then the model of an analog reality is useless, because there is no mapping onto the axis of real numbers for such fundamental quantities as length and area. ([1] Theorem 167 pg. 94)

If 1) or 2) were not satisfied, the consequences could be far reaching, as the length is a fundamental quantity in each coordinate system.

In this paper we will show in chapter II a simple counterintuitive example for the lengths of two straight lines, where we cannot decide which one is longer and in chapter III another counterintuitive example of two squares, where we cannot decide which one has the larger area. In each case a ghostly dimension, which is beyond our limits of observability is the reason.

II) An Example of Lengths of two „Straight Lines"

In Fig.1

R*Pi is the length of the big semicircle. A short calculation shows, that

R*Pi is also the total length of the two semicircles with radius R/2

R*Pi is also the total length of the four (blue) smaller semicircles with radius R/4

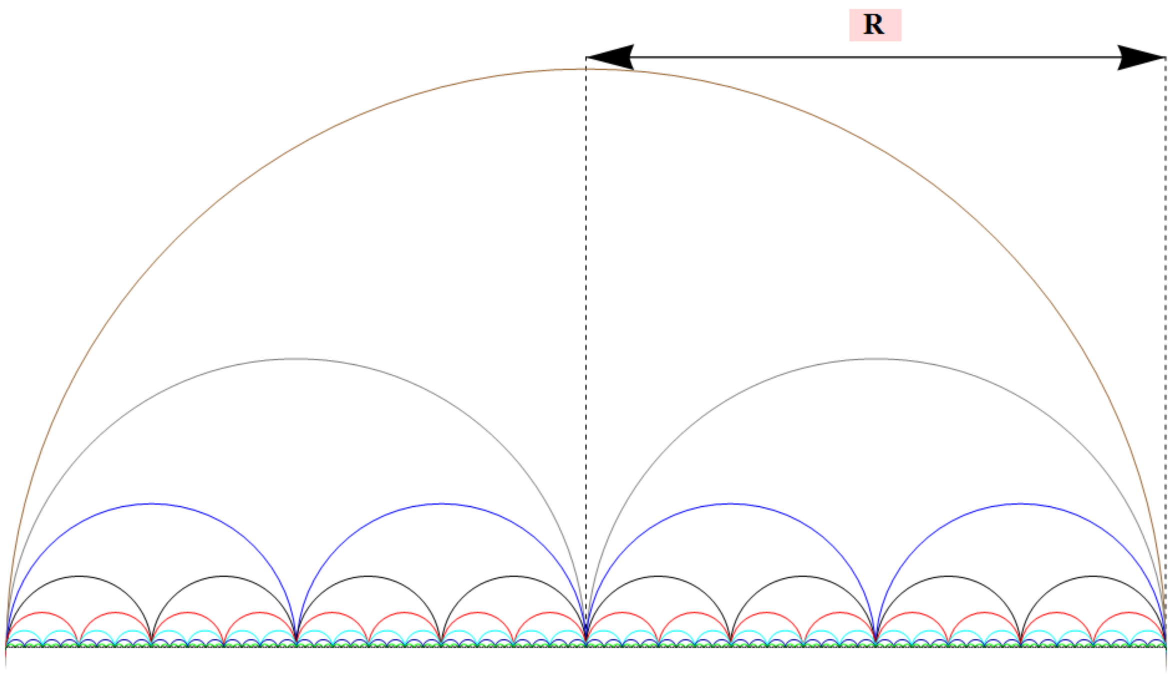
or is also the total length of the 8 black semicircles with the radius R/8

or is also the total length of the 16 semicircles of radius R/16 etc.

We see that these sequences of semicircles converge uniformly to the straight line of the length 2R which is the diameter oft he big circle.

After a few steps, these sequences of semicircles can no longer be distinguished from the diameter of the big circle even if we use the most sophisticated high tech instruments. However we know, that each oft he above sequence of semicircles has a total length Pi*R, whereas the diameter, which cannot be distinguished from them, has the length 2*R. Hence properties 1) and 2) of chapter I) are not satisfied . The reason is that the semicircles go into a second dimension which is beyond our limit of measurability: a ghostly dimension, which leads to practical incomparabilities of lengths.

Fig.1



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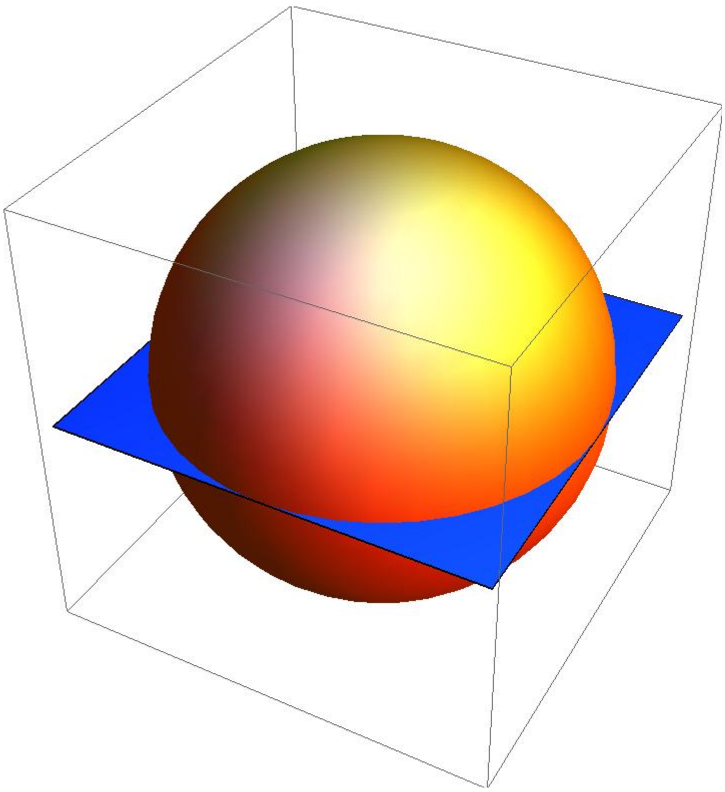
III) An Example with areas

We start with Fig. 2:

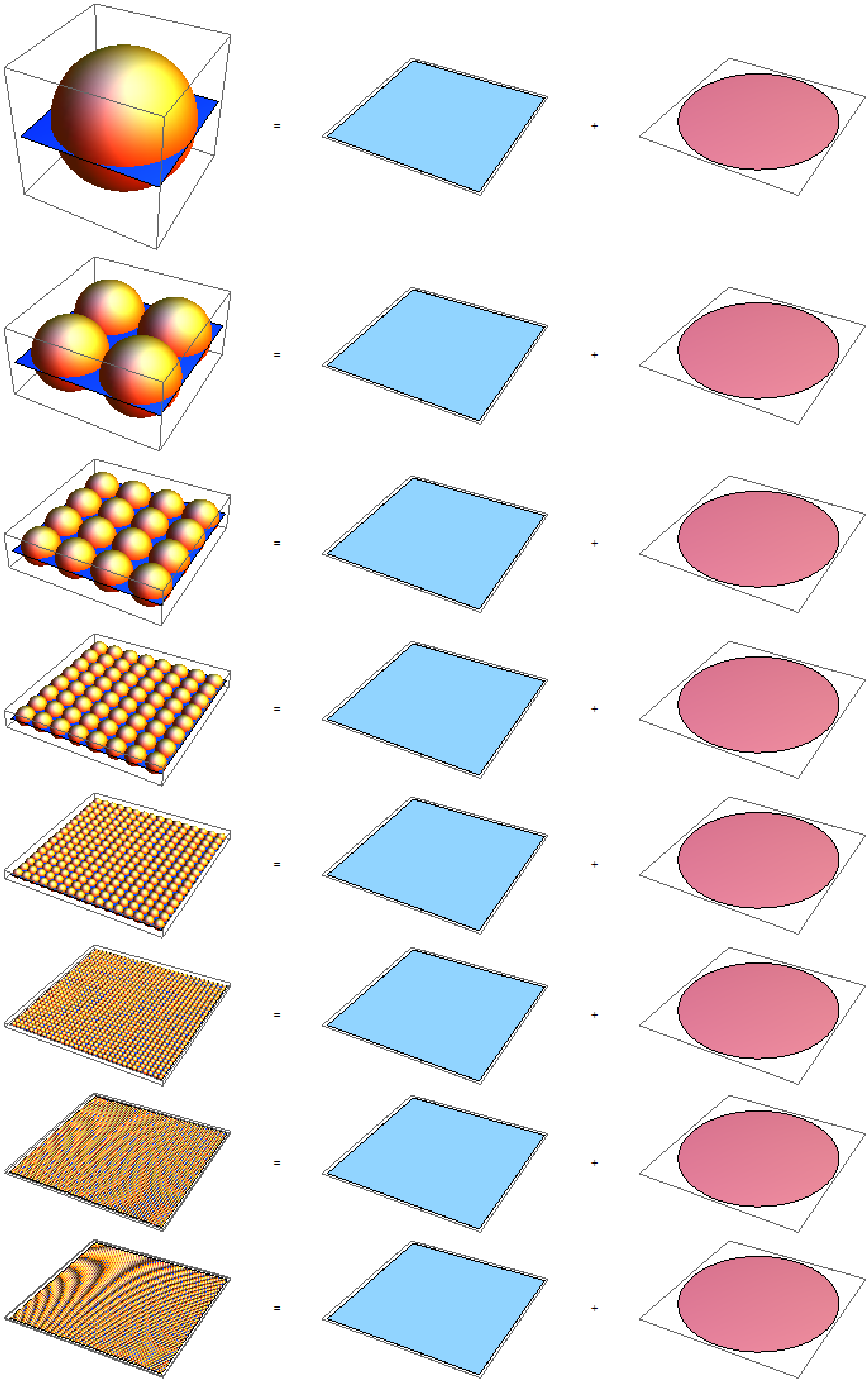


Here we consider only the set of the blue square with sidelength 2*R and the upper semisphere. Their total area is: 4*R^2 + 2 Pi* R^2 –Pi*R^2= 4*R^2+Pi*R^2.

This can also be interpreted as the total area oft he blue square plus the area of the inscribed circle within the blue square.

Now we perform a similar process as in chapter II), but with squares and semispheres instead of semicircles and can see immediately that the total area of all members of the sequence remains the same.

This can be visualized by Fig. 3:



After few iterations the squares of the left side of the equations remain indisdinguisible from the square on the rigth side, even if we are equipped with the most modern instruments.

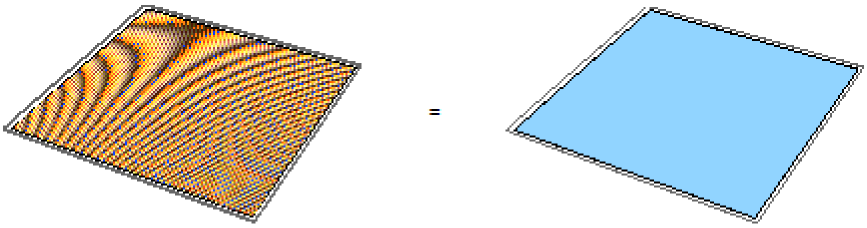
IV) Conclusion

III) and IV) show paradoxical examples where two lengths and two areas cannot be compared. One cannot decide, whether the first length is smaller than the second one or vice versa. The same is true for the areas. This paradoxical behaviour is caused by a ghostly dimension which remains below our level of observability.

But if we have an equation, which is symbolized as in the last line of Fig. 3, we can conclude, that a digital model of reality is preferable to an analog one, as the digital model of reality delivers only the trivial equation of Fig.4 and it does not claim, that all squares of the real world could be measured. Here the meaning of „equality" in the digital reality is simply „indistinguishability".

The digital model of reality at least does not run into such paradoxes. We simply have:

Fig. 4



This is compatible with our experience, in contrast to the equations of Fig.3 whose ghostly dimension is below the level of observability.

References:

[1] E.Landau, Grundlagen der Analysis, Chelsea Publishing Company, New York, 4th edition, 1965