The Role of Mathematical Thought in Defining Physical Reality

Dan Bruiger
March 2015

Abstract

One reason for the "unreasonable effectiveness of mathematics" is that it is never compared to nature itself, which is ambiguous, but to well-defined idealized versions (models). Math is consistent with nature in unfamiliar situations because it is consistent within itself.

PART ONE: the Map and the Territory

Nature and mathematics are divided by the same categorical gulf that has long plagued the relationship between the physical and the mental. Just as there is no mind without body, there is no mathematics without mathematicians. Mathematics and physics are alike cognitive activities undertaken by intelligent organisms. The relationship of one to the other—and of each to nature—must be considered in their context as embodied cognition. We moderns understand the universe through mathematical models, which are idealized constructs that correspond only approximately to reality. It is easy enough to mistake the model for the reality—the map for the territory—when one has learned to think of nature as literally consisting of such idealizations. This mistake parallels the naïve realism of ordinary cognition, in which we normally take our perception as a transparent window on the world. One can then even conclude that the universe is literally mathematics.

It is perfectly reasonable for physicists to believe in the guidance of mathematics. Yet, if physics is a form of cognition, then it is also reasonable to believe in the guidance of cognitive theory, evolutionary psychology, and theoretical epistemology (the nascent science of possible cognitive systems). Such things are not a part of physics as we know it. The Scientific Revolution redefined natural philosophy as "first-order" science: study of the external world in strictly objectivist terms. Focus on the object excluded focus on the subject.

Cognition in general is a form of map-making. So, then, are mathematics and physics. The map, however, is not the territory. At best it represents it selectively, symbolically, and adequately for specific purposes. In the case of ordinary cognition, these purposes are bequeathed by evolutionary history. Math and physics may be driven by other purposes as well, better described perhaps by sociology and anthropology. In any case, the map corresponds only grossly to the territory. There is always simplification and streamlining involved.

Science tends to mask the real complexity of the world when it *presumes* simplicity or prefers tidy systems to messy facts. The complexity and messiness of nature are the signs of its reality, which (contra Plato) lies in its very "imperfection." We vaunt the ability of mathematics to exhaustively represent natural systems, but the reality of nature lies precisely in its ability to resist such exhaustion.

Mathematical laws generally describe ideal things and circumstances that do *not* occur in nature, and that could not even be stated without idealization, isolation, and experimental control. (Ellis 2002, p90-94) To single out a causal relationship, one must know how the process would occur in the absence of other influences. In nature, however, there are always multiple influences, some of which are disregarded as irrelevant.

Mathematical modeling is essentially simulation. It is effective to the degree that simplification and idealization are—and limited by the degree they are not. Equations may be isomorphic to each other, and to the model systems they define, but neither model nor equation is strictly isomorphic to the real processes or systems it represents. A theory, equation, or program may correspond *at points* with natural reality, in the way that the integers correspond to points on the real number continuum. But aspects of reality may lie outside the simulation, just as there are real numbers that are not integers. If nature were literally generated by an algorithm, it would not manifest infinitely varied detail. No physical quantities would be truly random numbers.

Unlike the truths of mathematics, physical knowledge is contingent. Even on the macro scale it is necessarily statistical, involving *some* uncertainty. Physical knowledge pertains to *phenomena*, in Bohr's sense (rather than to the unambiguous realities preferred by Einstein). However perfectly differential equations may describe idealized systems, they correspond only imperfectly to real systems.

Mathematics is a human creation shaped by the needs and cumulative experience of a living organism interacting with a real world. The entities and operations of arithmetic, for example, generalize and idealize the most basic properties of physical things and actions presented in ordinary experience (a view early propounded by J. S. Mill). These include the integrity of "objects" (integers), various possible groupings (sets) and operations (such as addition and multiplication). Not only are things, collections, and actions abstracted, but the abstractions are further abstracted as well, in abstract algebra, for example, and in the notion of formal axiomatic system—or the more informal notion, simply, of 'system.'

The cognitive advantages of abstraction, idealization, and generalization are obvious; they help to categorize experience and anticipate future experience in similar, if not identical situations. They establish the basis for prediction, control, and formulas of action. What is not so obvious is why such formulas often work in situations that are not, at first blush, familiar. Evidently nature has its own order consistency, apart from the human need to perceive it. If nature were totally chaotic, there would be little point in trying to discern patterns or formulate laws. On the other hand, if it were perfectly ordered, it would simply not be real. We live in the ambiguity between. This may be one reason why science has taken such refuge in mathematics as the sheer embodiment of reason and order, even as the ontological basis of nature. The alternative is uncertainty and perhaps an inexplicable universe.

If mathematics generalizes and abstracts the most basic aspects of nature's own self-consistency, (and is correspondingly consistent within itself), then it is less surprising to find it consistent with parts of natural reality newly encountered. And this may help to explain the importance given to consistency within mathematics: without it, the usefulness of the map is compromised.

Cognitive connections between experience and world (map and territory) are established through natural selection. Their validity is attested by the fact that we are here to tell the story. We have conceived the notion of logical necessity, and would prefer that our cognitive connections were endorsed by it. But if such connections are contingent and historical in an evolutionary sense, and not logical in some a priori sense, then logic itself must be an evolutionary product. An evolutionary general theory of intelligence might help account for the astonishing effectiveness of mathematics, which would then be merely one development of a broader capacity to model, abstract, and generalize.

Mathematical laws succinctly express empirical generalizations, based on observation and experiment; they are algorithmic compressions of data. Such inductive generalizations cannot be proven, however, only disproven. This reflects not only a logical distinction (following Hume and Popper) but also a psychological disparity between the certainty of deductive proof and the mere plausibility offered by evidence, upon which much of actual science depends, as in everyday life. In contrast, theorems of mathematics can be rigorously proven because they are propositions within formal systems, not empirical assertions. The power of necessity ("governing power") that natural laws may appear to possess derives from translating inductive generalizations into theorems of a deductive system. There are always alternatives, however, to any proposed explanation of a physical phenomenon.

PART TWO: Math and Physics

Math is far more abstract than physics, since it deals exclusively with the most general properties of things. These generalizations, however, are then raised to the status of axioms, to be taken on faith, or logical propositions, to be decided by reasoning alone. Unlike mathematical concepts, which have been deliberately severed from particular reality, the generalizations of physics (laws) cannot be separated from the details upon which they depend (data). They are not logical but empirical propositions, to be decided by observation or experiment, not by deductive proof.

The connection between physics and mathematics as *disciplines* is historical, having developed within recorded history. The connection between mathematics as a set of truths and natural reality as we perceive it, or as we discover it scientifically, is a different question. As I have indicated, I believe that connection is also historical (in evolutionary time). The "pre-established harmony" between logico-mathematical truth and physical reality was not set by God but by natural adaptation of the human species, which eventually parlayed a capacity for abstraction into what we now call mathematics and logic. An aspect of that capacity is its proclivity to hypostasize its own inventions as external realities: hence mathematical Platonism.

Euclid formalized geometry, which hitherto had been an informal and pragmatic set of observations about the world. He transformed what had been essentially an inductive enterprise into the deductive paradigm that has become the scientific ideal. In so doing, he made a qualitative leap from empirical generalizations to analytic truths. Hilbert's program in late 19^{th} century called for the formalization of mathematics and physics as bodies of knowledge. These are significantly different tasks, however. Mathematics is already a deductive system; the task of proving its completeness and consistency met with frustration, as demonstrated by Gödel. On the other hand, to formalize mechanics, for example, meant first turning it into a deductive system, regarding its concepts as pure products of definition.

The ubiquity of analysis in terms of mathematical models reflects the cultural influence of the machine and the pragmatic effectiveness of mathematics applied to artificially defined systems: technology and other "nomological machines" (Cartwright 1999)—namely, mathematical models and experimental set-ups. In modern times, these influences merge in the computer, the universal machine, serving both as a powerful new tool and as a new metaphor for reality itself.

Mathematics is then projected back upon nature, as a theoretical lens through which to view the world, also influencing the way experiments are constructed. The expectation that nature will behave mathematically is then something of a tautology. Is it a miracle that an equation, which redefines a natural process in its own terms, happen to fit experimental set-ups that similarly redefine nature? Nature can certainly be described mathematically, but only to the degree and in the ways that it resembles a machine. Rather circularly, this resemblance comes of a particular mechanistic way of looking.

There is a pragmatic side to the use of mathematics even within theory. Science selects for study phenomena that lend themselves to mathematical expression, particularly linear relationships. Classical dynamics was framed in terms of differentiable smooth curves, applicable to idealized situations. There is a tendency to embrace what works mathematically, whether or not it makes physical sense (for example, action at a distance). The concept of prior probability begins with a treatment of what can be approached mathematically: precisely defined regular artifacts (for example, the coin with two sides, the die with six equal faces).

Mathematical treatment depends entirely on artificially redefined and restricted situations. In fact, most real-life situations are far too messy to be treated mathematically. The ideals of simplicity in explanation (Occam's Razor) and elegance in mathematics are preferences with little empirical basis. They may reflect no more than prejudice deriving from human cognitive needs and conditioning. (Oreskes et. al. 1994) There is no a priori reason why the world should be simple or why simple explanations should be true more often than complicated ones. We prefer simple formulations because they are *manageable*, and choose to look at simplified aspects of nature because these are what we can master. However, complex systems involving feedback and systems sensitive to initial conditions seem to be far more prevalent in nature than formerly acknowledged. Their neglect before the invention of the digital computer reflects an historical bias toward idealizations that can be treated mathematically with paper and pencil: stable linear processes, closed reversible systems, and so forth. Some concepts, because they are mechanical, can lead to the construction of real machines, useful technology, and the successful manipulation of nature. As computation refines and extends our concepts of mechanism, and expands what is technically feasible, our view of nature must evolve accordingly.

The effectiveness of mathematics may be satisfying for psychological reasons as well as for the competence it affords. We are pleased to think that physical reality is rational and can generally be appropriated to human terms. We like to believe the world corresponds to our ideas and hopes. Technology richly confirms that faith.

PART THREE: The Pre-established Harmony

In a more secular age, Eugene Wigner would famously call Leibniz' pre-established harmony "the unreasonable effectiveness of mathematics." (Wigner 1960) The question remains: why does the world behave at all in ways that equations approximate, sometimes very well? Many classical systems, such as the solar system, behave like their mathematical descriptions to great accuracy. We can explain such macroscopic behavior as a massive statistical effect of scale. There is no such recourse in the case of quantum entities, which (as far as we know) are not composed of massive numbers of smaller things. Yet, they appear to behave according to mathematical principles and manifest the precise and simple integrity characterizing products of definition. The properties of truly fundamental particles and forces (if such exist) can only be accepted on faith, axiomatically, as though reality were ultimately a matter of our definitions. How to separate the fundamental particle's intrinsic discreteness from the discreteness it enjoys as a theoretical construct? Since a black hole is considered internally structureless, should it be regarded as an elementary particle?

An obvious explanation of the pre-established harmony is that there is order in the world, at least during the cosmic epoch in which live (Unger & Smolin 2015), and we choose aspects of this order to focus on that are readily expressible with the maths we have created. This is part of a larger adaptation of brain to world: thought and perception accord with external reality in a manner pre-established by evolution. It is not surprising that the world looks familiar, and no more surprising that a conceptual tool shaped by familiar experience should be useful. Yet math is often useful in realms far outside ordinary experience. It is the business of science to explore these realms, extending our cognitive powers. The salient question is why mathematical intuition should continue to apply to realms that are far beyond our ancestral and even present everyday experience. The "harmony" is based on limited experience, yet appears to apply very widely.

The correspondence of mathematics with physical reality recapitulates the correspondence of our perceptual models with the external world. In the case of perception, such modeling serves the creature and is endorsed by its evolutionary success. However, mathematical modeling has not been around long enough to prove its long-term evolutionary value. Our perceptual models are so ingrained that we take them for the world itself; taking *mathematics* for the world itself is the corresponding illusion on a different level. It suggests that physical reality *is* math. Or, it suggests that the domain of math is a higher version of reality, with its own sort of substantial being, which exists prior to and independent of physical reality. One may then think that math applies in unfamiliar domains because—like those physical domains—a parallel mathematical domain is simply there already, waiting for us.

Just as one rejects naïve realism, I believe one should reject this naïve idealism. The matter is more subtle. As cognitive strategies, there is some continuity between logico-mathematical principles and physical laws, with mathematics at the extreme of generality concerning the behavior of things. Physical laws are more conditional and restrictive, dependent on particulars. However, one absolute difference is that physics is always a function of empirical evidence, while mathematics is always a product of definition. Though gleaned from experience, its entities have been formally defined; they have the precision of definition, while science has only the precision of measurement. On the other hand, logical truths are devoid of new information for this very reason. The deduction is already latent in the premises. The solution to a computational problem is but a more explicit form of information already provided in the formulation of the problem. (Goldreich 2005) Mathematical is well defined, whereas nature is not.

To be mathematically treated, a natural entity or process must first be idealized and formally re-defined. It is then no longer a natural found thing, but a created artifact. It is this idealized artifact that is the actual object of theoretical, experimental, and mathematical treatment. Nature is ambiguous. A natural entity cannot be fully specified or exhaustively described without referring to broader contexts with which it is indefinitely and complexly interconnected. Hence, the significance of simple isolated systems, which limit the context. But such a system is not only physically isolated; it is precisely and only what we say it is, no longer ambiguous. This renders it qualitatively as different from reality as a mental image is from the thing it is an image of. The important point, however, is that such an artifact *can* be exhaustively treated with mathematics. However, there is no guarantee that it corresponds perfectly to the natural entity or process it models. In the case of our mental images, the closest the species comes to this guarantee is the fact of our survival to date—which may be no more than luck.

A deeper reason, then, for the mysterious correspondence is that mathematics is not actually compared to physical reality itself but to ersatz models, which are artifacts *designed* to be treated mathematically. One deductive system is thus commensurate with another because they are both products of definition. Mathematics applies effectively in "unfamiliar" situations that have been appropriately redefined in familiar terms. In that sense, the correspondence is actually between some part of mathematics and some other part that may not yet even have been explicitly developed.

Science assimilates the complexity of appearances to relatively simple models. It restricts its scope to simplified situations (both in theory and in experiment) with a minimum of factors in play. The virtue of such abstractions is that they can be treated mathematically, which means that parameters of particular interest can be isolated and predicted, though at the cost of omitting others. But perhaps not all the relevant factors of a theory have been included, or do not pick out clearly identifiable properties. *Defining* them as mathematical variables nevertheless gives the illusion of completeness and definiteness, possibly masking ambiguities present in the experimental or observational arrangement.

We cannot exhaustively know a natural reality, although we can exhaustively describe a theory of physics and list its elements and propositions. (To exhaustively model a real system would be equivalent to finding an algorithm to express a truly random sequence—a contradiction in terms.) There can be perfect isomorphism between *models*, as between other mathematical constructs, but never between the model and the inherently ambiguous natural phenomenon it represents. The discrepancy between the model and what it models is considered noise—until it becomes so blatant that a new model must be sought! In its extremity, the cognitive strategy to map the world mathematically becomes an unwarranted article of faith: the belief that the natural world literally *consists* of the conceptual artifacts of science. Galileo held mathematics to be the language of nature. More accurately, it is the syntax of science. As syntax can upstage semantics, producing jibberish, so mathematical formalism can dominate the interpretation of nature.

What if indefinitely many factors are involved in natural phenomena and we chose to consider them relevant? Would that mean that nature could not be treated mathematically, or only that more sophisticated mathematics is required? Perhaps the simplifications typical of classical physics, in particular linear equations, are no longer either sufficient or necessary for the progress of science. We are on the verge of glimpsing the underlying connectedness of everything to everything else, and computation now enables us to begin to engage the complexities of this wholeness. On the other hand, the computer reflects the organization of human thought, which (though shaped by nature) is now also projected back upon the natural world. An entirely different approach may be required to understand nature's *self*-organization. (Bruiger 2014)

Though it is convenient to treat mathematics platonically in some contexts, as it is to treat theoretical models as literal realities, it could prove equally useful to consider mathematics and science as genetically and culturally constructed maps of nature. Since nature includes us, such maps would reflect both the physical world *and* our adaptations to it, which include science and mathematics. Such an approach would be self-referential, like the rest of human consciousness. Scientists can no longer afford to focus exclusively on the physical world, as though they were situated outside it. Mathematicians can no longer afford to ignore the genetically inbuilt relation of math to physical reality (the real pre-established harmony) or its significance as a conscious human creation.

Some physicists believe that mathematical laws carry a power to fix the behavior of matter in advance of actual events. Some believe that fundamental physical constants should be derivable from theory. Some have faith that a final theory is possible and imminent. Some physicists hold that the world has a definite and finite information content, and some mathematicians advocate a discrete or finite mathematics, a "digital physics." Any of these scenarios, however, would imply that the world itself is effectively a deductive system—an artifact or simulation. There is a long tradition of such beliefs, stemming in part from our religious heritage. If nature is *real*, however, there will always be something unaccounted for in theory. No matter how confident we are in our equations, the nature of reality—and the reality of nature—is that it will always surprise us.

REFERENCES

Bruiger, Dan (2014) "An Argument for a Second-Order Cosmology" Academia.edu

Cartwright, Nancy (1999) *The Dappled World: A Study of the Boundaries of Science*, Cambridge University Press,

Ellis, Brian (2002) *The Philosophy of Nature: a guide to the new essentialism* McGill/Queens UP

Goldreich, Oded (2005) "Randomness and Computation", Dec [web archive]

Oreskes, Naomi; Shrader-Frechette, Kristin; & Belitz, Kenneth (1994) "Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences" *Science* v263, footnote 25, p645

Unger, Roberto M. & Smolin, Lee (2015) *The Singular Universe and the Reality of Time: a proposal in natural philosophy* Cambridge UP

Wigner, Eugene (1960) "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" Communications in Pure and Applied Mathematics, vol. 13, No. I