Reality is Digital, with a Quantum-Mechanical Phase

Michael P. Bradley
Research Fellow, Electricity Department (Watt Balance), BIPM
(Bureau International des Poids et Mesures)
92312 Sèvres Cedex France
&
Associate Professor, Dept. of Physics & Engineering Physics
U. of Saskatchewan, Saskatoon, SK S7N 5E2 Canada

Introduction

What is the fundamental nature of reality ? After the development of the atomic theory of matter and nearly 100 years of quantum mechanics, we know that all objects in our world are made up of discrete units, namely atoms and molecules. Experiments in atomic, nuclear and particle physics have shown that this discreteness (i.e. digital character) is replicated at shorter and shorter distance scales, to dazzlingly short ranges and almost unimaginably high energies. Thus, despite the superficial appearance of continuity at the macroscopic length scales on which we live and breathe, at its core reality is discrete in character, that is to say, digital. However the fact that this discreteness is correctly described by quantum mechanics means that it is a special kind of "digitalness" which we can express in the following way: we live in a world made of digital objects with the added complexity of a continuous quantum-mechanical *phase*.

Posing the Problem

To address this question in detail, we first need to define what we mean by "digital" (or "discrete") versus "analog" (or "continuous"). Digital or discrete objects and entities are *countable* ones (like the fingers—or digits—on our hands). They may be infinite in number, but they are always denumerable. Continuous entities are non-countable, and in this way they are like the real numbers; there are always more real numbers (in fact, infinitely many more) to be found between any two real numbers on a number line, whereas for countable digital entities like the integers this is not the case. An example of a continuous entity is a classical fluid, which can be divided into smaller and smaller volumes without changing or compromising its essential nature. In practice we know that such fluids do not exist, since as we divide down the volume we will eventually reach a point where the volume considered contains only a single discrete fluid molecule, which will then no longer exhibit fluid-like behavior at that length scale. The fact that atoms and molecules are almost unimaginably tiny compared to Man is the main reason why the view of matter as continuous is such a good approximation, and is the main reason why the inherent digital nature of reality was obscured for such a long time.

What I shall argue in this essay is that reality is at its core digital or discrete, but that this is a special kind of "non-classical" discreteness in which *phase factors* play a role. This "non-classical" discreteness could only have been developed in the context of the continuous mathematics which served physics so well for several hundred years before the development of quantum mechanics. History is important in this discussion and I shall therefore begin with a historical discussion to show where the bias towards a continuous description of reality came from. I will then show how the development of simple atomic theory (first proposed by chemists) led eventually to quantum mechanics and the correct digital description of realty. Finally I shall give some miscellaneous examples drawn from various other areas of science.

Historical Background: from the Countable to the Continuous

"Is reality digital or analog ?" must have been a confusing question since the earliest humans took time out from their daily hunting and gathering to contemplate the night sky, the stars, and the incredible beauty and seemingly unlimited complexity of the world around them. The earliest mathematics involved discrete or digital (literally "finger-based") counting of objects like sheep, cooking pots, and haystacks, and more profoundly the stars. Perhaps a little later, once building evolved beyond the simplest thatched huts, continuous mathematics was first developed in the form of geometry, which however remained utterly disconnected from the mathematics of countable objects until Descartes realized in the 17th century that an ordered triplet of numbers could uniquely describe a position in space [1]. From this was born *analytic geometry*, probably the most foundational achievement in the applied mathematics of its day. The realization that algebraic equations could describe continuous curves in space led to great developments in both algebra and geometry, and also permitted the logical development of the calculus. These mathematical tools constitute the backbone of most of the important applied mathematical and physical science from Descartes' time up until the present. In the playbook of modern theoretical physics, algebraic and geometrical thinking hold about equal weight, and one is often turned to when the other gives no further insight.

A large part of the development of physics in the 18th and 19th centuries consisted of the development of the correct differential equations to describe natural phenomena. These equations are normally developed by consideration of a differential element of small but finite size acted upon by various forces; the proper differential equation for the final continuous object (beam, cord under tension, etc.) is then obtained by the mathematical process of taking the limit as the size of the object is reduced to zero. Mathematically this process is valid, provided certain conditions are met. (This mathematical assertion itself was originally controversial and was strongly disputed by Bishop Berkeley [2]). However, regardless of its mathematical status, physically this idea is simply wrong. As we reduce the size of a differential element of, say, an iron beam, we would ideally like to proceed to the mathematical limit of an element of zero size; but in practice there is no physical meaning in considering a differential element of the beam smaller than the size of a single iron atom (which is around 2.5 Angstrom units, that is 2.5×10^{-10} metres, in diameter). Below this length scale, there is no object or entity with the properties of iron. Thus this process of taking a mathematical *limit* of the infinitely small is not physically valid, but it is nonetheless sufficiently accurate that it allowed physicists of the classical period (from the time of Newton up until about 1900) to make enormous progress. In fact this process of using continuous differential equations to describe physical systems is also the basis of most modern engineering, which has been notably successful in the development of bridges which stay up, buildings which do not fall down, airplanes which fly, and so forth.

It is easy to see now why, against this imposing and largely successful backdrop of essentially continuous mathematics, the development of atomic, and later quantum-mechanical ideas, was so difficult to absorb. The idea of the atom was an old one but like many such ideas could not be advanced beyond simplistic philosophical arguments for many centuries. It was from chemistry and in particular the observation of fixed combining weights in chemical reactions that modern ideas about discretized matter first gained experimental support.

Atomistic Theory and the Return to the Discrete

After the development of atomistic theory in the context of fixed chemical combining weight ratios [3], and the observation of discrete line spectra of the elements which could not properly be explained by classical physics, physicists were forced to develop quantum mechanics. Atomistic theory had already been a controversial and hotly-debated subject in physics, with famous scientists lining up on both sides for and against. Indeed it is said that the suicide of Ludwig Boltzmann, one of the founders of the modern kinetic theory of gases based on the atomic hypothesis, was at least in part due to the negative reception his ideas received in some quarters. This tragedy is made even sadder by the fact that single atoms can be routinely imaged today by scanning tunneling microscopes and single ions can be directly observed as glowing spots of light in ion traps; thus there is no longer even the slightest possibility of a doubt of their existence. Given more than two centuries of successful development of physics based on differential equations and an underlying assumption about the continuous nature of matter, it was very difficult for physicists to accept that matter could at its most fundamental level be discrete. Even worse, they had no very clear mathematical base upon which to build such a theory of discrete matter. It was Werner Heisenberg who took the first big, bold step of introducing complex amplitudes in his matrix mechanics (developed with Pascual Jordan) [4], which amazingly, gave a good description of the Balmer spectrum of the hydrogen atom. But this mathematics, while seemingly correct, was so different from all theoretical physics which had come before as to be utterly baffling. Following Heisenberg and Jordan, a large part of the development of quantum mechanics consisted of methods for obtaining discrete quantized quantities from differential equations which described continuous fields. Erwin Schrödinger in particular was the first to truly synthesize all this, after being spurred on by Debye's remark that to talk about quantum mechanical waves (i.e. de Broglie waves [5]) without having a wave equation was "childish" [6]. Schrödinger was able to show that a suitable wave equation subject to appropriate boundary and phase conditions could give quantized results which corresponded exactly to the discrete formulation developed earlier by Heisenberg and Jordan [7].

So in a strange way the understanding of the world as either digital or analog had completed a full cycle; a great deal of effort had been expended to demonstrate how a continuous wavefunction which had its origin in the continuous (i.e. *analog*) formulation of mechanics in terms of differential equations could be appropriately treated to give discrete results which corresponded to the actual discrete (i.e. *digital*) nature of reality. If atoms and photons were bigger, so that humans could see them and count them directly, we would never have developed the Schrödinger equation, because there would have been no need, and the expressing of atomic quantities in terms of continuous wavefunctions might have seemed very strange indeed, a kind of mathematical backflip which, while giving the correct results in the end, took very a tortuous route on the way to getting there.

Quantum Mechanics

Schrödinger's formulation of discrete phenomena in terms of waves introduced very clearly a central idea which will affect our entire thinking about the question of the discrete versus continuous nature of reality. The idea is that of the quantum mechanical *phase*. Phases of waves and the importance of relative phase shifts in wave interference phenomena were well understood from at least century of developments in optics, and could be easily visualized in the context of, for example, ripples on a pond. Mathematically the phase factors get encoded in the form of complex numbers via Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$. Complex numbers (or "c-numbers" as Paul Dirac unpoetically called them [8]) had already been introduced into discrete quantum mechanics by Heisenberg and Jordan, but Schrödinger's formulation made their physical significance clear. The role that the phase of a wavefunction plays in its life and ultimate death (in the form of wavefunction collapse) is enormously dominated by this phase and the shifts that it undergoes.

So the universe is discrete, with energies, charges and particles coming in identifiable chunks (e.g. every ^{133}Cs atom is the same, which is why atomic clocks based on ensembles of cesium atoms can provide such spectacularly precise timing). But the unexpected element in all this, the plot twist which the caveman counting stars on his fingers could never have imagined, is this idea of the phase. It is the phase which allows a real number to slip in and out of the imaginary plane, depending upon how the phase is rotated, and thus allows particles to appear and disappear (or rather to be more likely to be measured "here" rather than "there") depending upon the relative phases with which wavefunctions combine.

Quantum mechanics confirms to us that the universe is discrete, but with a twist. Energy levels and masses come in digital units but the phase is continuously variable. Most of the complexity in our wonderful universe come from this basic fact.

Philosophical Considerations

Quantum mechanical details aside, there is a fundamental philosophical reason why the universe must be at the end digital in character, rather than analog. The reasoning is similar to that which we used earlier when discussing the differential equations for the mechanical motion of an iron bar. If we imagine a block of material of a particular substance, we know that we will measure certain definable properties; its color, transparency, magnetic properties, etc. If we then subdivide the block the properties persist, but how finely can we divide the block into partitions and retain these unique properties ? If matter were continuous, the subdivision could be done on an arbitrarily fine level, but this clearly cannot hold to arbitrarily small distance scales; at some very small length scale there would not be sufficient information contained within the tiny cube to describe uniquely the properties of the macroscopic block; that is to say if the block is too small it will contain insufficient degrees of freedom to describe and hence "store" the macroscopic properties. Thus there must be some smallest unit which carries the properties of the material. In practice this unit turns out to be the atom or the molecule.

There are two complications regarding this argument; the first is that additional properties can emerge when large numbers of fundamental units are brought together [9] [10]; this does not change the basic argument, however. The second is that the fundamental units are in fact *divisible*. The argument I gave above is essentially that of Democritus, which we all learn in school; when I first heard it I gave it little weight, since it seemed obvious. (Such is the bias of children of the atomic age.) In fact the argument is more profound than that. We are usually also told that the word "Atom" comes from the phrase "a tomos" or "un-cuttable". At this point nuclear and particle physicists love to tell us that this argument was thoroughly wrong since we know atoms are (*relatively* easily) broken apart into protons, neutrons, and electrons and that protons and neutrons at least can be further divided into quarks using those modern beasts of big science, the particle accelerators (so far the electron has not yielded to such subdivision and remains, for the moment, "un-cuttable"). So in the light of this evidence (clearly not available to Democritus) the name "atom" seems almost laughable. But I think that in our rush to be know-it-alls and debunkers we have perhaps got Democritus wrong here. I am no scholar of ancient Greek but I suspect that what Democritus may have meant was not that atoms were indivisible but rather that if they were cut up any smaller you would lose the essence of the matter being considered. That is, I don't think the atomic theory hinges on the idea that atoms cannot be split, but rather on the central observation that the atom, once split, loses its identity as the basic constituent of a particular substance. The "Atom" is not "un-cuttable" as in "you *cannot* cut it" but "un-cuttable" as in "you *had better not* cut it or you will lose the essence of the thing". Which is in fact what we now know to be true.

Information

There is a clear connection with information theory in all of this. Somehow the properties of the atoms of nature must encode the information necessary for the properties which the atoms exhibit. This argument may even lead to a reasonable size estimate for an atom (using quantum mechanical arguments not available to Democritus).

Thus reality is digital, but not classically digital, in the sense that the quantum mechanical version of "digital" allows the inclusion of a phase factor. It is this phase factor which is responsible for much of the richness of the physical world. Physical quantities come in discrete units but with a variable phase factor which affects how they interact and how they are measured. We can argue that while on the one hand a truly continuous world would require too much information to describe, a classically discrete world would on the other hand not exhibit sufficient richness; thus we can see that a quantum digital world in which phase is present strikes exactly the right balance.

There is a connection here with Georg Cantor's concepts of countable numbers, set theory, and orders of infinity. Discrete integers are countable and infinite but only to order aleph-zero (i.e. $\aleph_0$); as such they do not have sufficient richness to describe the natural world. We require number sets of higher cardinality (i.e. higher "aleph number") to describe our physical reality.

Building Up from the Discrete to the Apparently Continuous

In the early days of quantum theory, once the discrete nature of electrons and nuclei had been established, there had to be a way of assembling these objects to make atoms; this was the *Aufbauprinzip* (German: "building-up principle") of Niels Bohr and Wolfgang Pauli. Similarly, there is a kind of building-up principle inherent in our discussion. The world is at its core digital, but we assemble discrete quarks into nucleons, nucleons and electrons into atoms, atoms into molecules and molecules into solids and liquids which appear to be continuous to us. At each level we can find a level of abstraction which is sufficient to describe most of the observed phenomena. This is probably most clearly seen in the different treatment by chemists and solid-state physicists of objects common to both their domains, i.e. liquids and solids composed of various materials. Philip Anderson [9] and Robert Laughlin [10] have been particularly clear on this. Chemists treat a solid as always being composed of the basic discrete units, while physicists often invoke (mainly for convenience rather than historical reasons) ideas of continuity which usually allow correct solutions but which are in fact clearly not exact. These assumptions of continuity break down in situations of complexity; a good modern example is the case of cuprate high-T_c superconductors [11]. Despite the existence of an apparently good theory (BCS theory [12]) for simple elemental superconductors, no proper theory of high-T_c superconductors has been developed along these lines, and in fact, most of the progress on high-T_c materials (including the discovery of new materials with ever higher T_c) has been made by inspired guesses using the chemical point of view, for it seems that the detailed structure of the copper-oxide planes of these superconductors plays a major role in the phenomenon. This is a powerful example of the importance of discrete units and their fundamental role in our world.

The development of these ideas has had a long history. It is striking that the development of counting ideas by early humans led to discrete mathematics which then later developed into continuous mathematics which was suitable for the types of physical problems accessible at the time. Only later as techniques to view the microscopic and ultimately the quantum world were developed, did an appreciation develop of the fact that at the most basic level the phenomena of the world are discrete in character. However a lasting legacy of the long development of continuous mathematics is the idea of complex numbers, without which the modern theory of discrete quantities with continuously variable phases could not have been developed.

Examples from Other Areas of Science

Biology and Neurology

Biological examples present another powerful example of the inherently digital nature of the physical world. It is well known by now that the detailed arrangement of the atoms and molecular units which make up nucleic acids and proteins are critical in their performance. Even the most basic aspect of chirality, i.e. *handedness* of biological molecules (discovered first in the metabolism of sugars by Pasteur [13]) shows that inherently digital arrangements of discrete forms are essential to life as we know it.

It is worthwhile also to consider the brain. The brain is an excellent example of these phenomena; in it we have billions of discrete units (neurons) assembled to make a whole. At some level the capacity of the brain is large but finite and countable; how then do we know that the apparent discreteness of the macroscopic world is not an artifact of the discreteness of the device with which we perceive things? I think that there are two answers to this, more or less. The first is that our brain is as much of physical object as anything else and thus its inherent discreteness is simply a manifestation of the discreteness of the word as a whole; this is a neat argument but not a very strong one. The second answer is that the discreteness of the basic processor is not a barrier to the consideration of hypothetically continuous entities; for example we need look no further than the great success of modern digital computers in calculating the continuous differential equations necessary for most of modern engineering, aerodynamics, etc. From this we see that inherent discreteness of the processor is not a barrier to understanding continuous systems; levels of approximation and progressive refinements can be used and have been used successfully since the 17th century. The ultimate surprise however was that at its core our apparently continuous analog reality had a digital character.

Fluid Mechanics

Fluid dynamics provides us with an enlightening example of this situation. We know of course that fluids are made up of individual molecules which interact with one another collisionally. This easily connects with the idea that fluids are materials in which molecules are adjacent to one another (as in a solid, and in contrast with a gas) but which do not have fixed relative positions (in contrast with solids).

However the atomistic description of fluids is not a very useful practical description and is not used except to determine certain bulk properties such as viscosity. Instead we use the Navier-Stokes equations based on continuous assumptions about fluid properties to derive the necessary behavior. These assumptions of continuity lead to various unresolved paradoxes (as witness the ongoing Clay Mathematics Prize problem addressing the ideas of turbulence).

Quantum Electrodynamics

Interesting aspects of the interplay between the discrete and the continuous can also be seen in the modern theory of quantum electrodynamics (QED) in which physics quantities such as the electron charge and spin magnetic moment are “dressed” values which differ from the bare values by the interaction with virtual photons. These interactions with discrete numbers of virtual photons created and absorbed can be enumerated and tabulated in a manner described diagrammatically by Feynman. Ultimately there are an infinite number of such processes of higher and higher order, each of which makes a correspondingly smaller contribution to the final quantity. Even a discrete quantity such as a single electron spin has embedded within it millions of processes (articulated by Feynman diagrams) which give rise to the final quantity.

Conclusion: A Special Kind of “Digitalness”

There is a historical arc to this narrative: discrete mathematics came first because discrete objects came most readily to hand (literally, in the form of ten fingers). Subsequent developments initiated by Descartes’ insight combining algebra and geometric reasoning led to an explosion of continuous mathematics which dominated much of physical science for about three centuries, and led to a belief in the continuity of matter, even though the equations themselves had been based on unphysical assumptions which could not be tested at the time. Discoveries in chemistry and later atomic physics led, first gradually, and then joltingly, to the realization that at its base matter is in fact not continuous, but quantized. This ultimately led physicists to quantum mechanics, a much more sophisticated version of the original atomic hypothesis of Democritus, which had been formulated long ago.

The development of quantum mechanics (tested in remarkably many ways experimentally) leaves little room for doubt that the universe is at its core discrete (i.e. digital) in character. However, this digital aspect has a novel element which could not have been understood without the development of continuous wave mechanics. This important element, which required the previous development of continuous mathematics to be fully understood, and complex numbers to be efficiently represented, gives a degree of freedom to the discrete physical world which would otherwise not be present.

Thus we are led to the conclusion that reality is digital and that the objects in it can be described by discrete sets of complex numbers, in the form of quantum-mechanical phases. It is from this phase information that most of the richness of the physical world emerges. The great arc of humanity’s understanding of the universe has come full circle, from original digital notions embedded in countable sets, to physics based on continuous mathematics and an underlying and unphysical assumption of the continuity of matter, and finally back again to an inherently digital version of reality with the crucial addition of a quantum-mechanical phase.

References

[1] R. Descartes, “La Géométrie” (1637).

[2] G. Berkeley, “The Analyst: A Discourse Addressed to an Infidel Mathematician” (1734).

[3] J. Dalton, “A New System of Chemical Philosophy”, (R. Bickerstaff, Manchester, 1808).

[4] W. Heisenberg, “Uber quantentheortische Umdeutung kinematischer und mechanischer Beziehungen”, Z. Phys., **33**, 879-893 (1925).

[5] L. de Broglie, “Recherches sur la théorie des quanta”, thesis, Paris, 1924.

[6] F. Bloch, “Reminiscences of Heisenberg and the early days of quantum mechanics”, Physics Today, December 1976.

[7] W. Schrödinger, “An Undulatory Theory of the Mechanics of Atoms and Molecules”, Phys. Rev. **28**, 1049-1070, 1926.

[8] P. Dirac, *Principles of Quantum Mechanics, 4th ed*., (Oxford University Press, Oxford, 1958).

[9] P.W. Anderson, “More is Different”, Science **177**, 393-396, (1972).

[10] R.B. Laughlin (2005). *A Different Universe: Reinventing Physics from the Bottom Down* (Basic Books, New York, 2005).

[11] J. G. Bednorz and K. A. Müller, "Possible high T_c superconductivity in the Ba−La−Cu−O system". Z. Physik, B **64** (1): 189–193 (1986).

[12] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, "Microscopic Theory of Superconductivity", Phys. Rev. **106**, 162 - 164 (1957).

[13] L. Pasteur, “Sur les relations qui peuvent exister entre la forme crystalline, la composition chimique et le sens de la polarisation rotatoire″, Annales de Chimie et de Physique **24**, 442-459 (1848).