Predictable and un-Predictable in the Universe and the Mind
FQXi 2020 Essay Contest Entry
Michael P. Bradley

Undecidability and Predictability:

The idea of a clockwork Universe goes back a long way, and it is ultimately rooted the human need for control. The first detailed astronomical observations of the stars and the phases of the moon were done with the intent of aiding navigation, and predicting the seasons and the tides. This had great and immediate relevance for the harvests, and for travel upon the seas. In an unforgiving natural world, these detailed observations gave some measure of control, and hence some predictability. Although it comes from later era, we may even connect it with Archimedes' famous boast "Give me a lever long enough, and a place to stand, and I shall move the Earth". While not quite the same statement, the general idea, and the underlying human need to assert control, is strikingly similar. The basic idea is that the human mind is paramount, and that with a few well-codified laws, nothing in Nature can escape our understanding. In this worldview, understanding means direct, quantitative, mathematical predictability.

Astronomy as a Template- the Clockwork Universe:

The modern idea of the clockwork Universe can be traced to Laplace, and his "daemon" of supreme intelligence (although he did not call it by that name exactly). It does not take much to jump from Laplace's superintelligence to a modern computer, which is what modern commentators on the subject have done. Laplace was probably motivated by the increasing analytic tendency in mechanics which occurred during his lifetime, much of which was driven by his work. Laplace's work, as well as that of his contemporaries, is nothing more than a reformulation of Newton's mechanics, but with an analytic focus, rather a geometrical one as Newton had. It constituted a mathematization of the subject. While a geometric approach encourages holistic thinking, since causes and effects are intertwined, an analytical approach promotes a separation of cause and effect; this can be as simple as the two being on opposite sides of an equation, whereas there is no such separation geometrically. Analytic work also lends itself to numerical computation, which can be algorithmatized and hence automated. Hence the clockwork concept.

The early and mid 19th century can be regarded as the apotheosis of the clockwork Universe concept, which was more or less explicitly or implicitly accepted by all physicists and mathematicians (the distinction between the two being less pronounced in those days than in our present time). Cracks in the picture began to appear in the late 19th century, with the development of statistical mechanics in an attempt to explain experimental results in thermodynamics, and even more importantly, the stubborn tendency for events to be irreversible in real life, despite the reversibility of the "governing equations" of physics. However the greatest shocks to the clockwork worldview would come a bit later, after the turn of the century.

Undecidability and Non-Computability in Finite-state systems:

After more than two centuries of great successes of analytic predictions in celestial mechanics and many other areas of physics, the 20th century brought two major surprises, one in the form of quantum mechanics, and the other in the work of Gödel and Turing. For its part, quantum mechanics seemed to require some inherent uncertainty in its formulation. However this is actually a bit misleading, as the predictions of quantum mechanics are quite definite, and the theory deterministic, up until the moment of wavefunction collapse, or measurement. In any case, because quantum mechanics largely replicates the predictions of Newton's laws when scaled to the macroscopic realm (a consequence of Ehrenfest's Theorem), in fact neither the inherent uncertainty in quantum mechanics, nor the Measurement Problem, really posed any serious threat to the clockwork Universe idea.

A seemingly more serious challenge to the clockwork notion came from Axiomatic Mathematics. In their quest to solve problems posed by Hilbert, Gödel and Turing, respectively showed that axiomatic mathematical systems can nonetheless have inconsistencies, and that not every well-defined computation can be expected to yield an answer in a finite number of steps. Turing's and Gödel's results are in many ways more shocking than quantum mechanics because they did not rely on any new laws of physics, applicable in a special realm (e.g. the realm of the impossibly microscopic in the case of quantum mechanics). Unless you are a fanatical reductionist (as, unfortunately, many 20th century physicists have been), it is not difficult to imagine, or concede, that somewhat different laws may apply at vastly different length scales, e.g. that Quantum Mechanics is the right physics for atomic and molecular systems but not very relevant for biology or the cosmos, while General Relativity is relevant for astrophysics, but has nothing much useful to say about what atoms are doing. Ordinary people believe this kind of thing implicitly, but physicists have trouble accepting it. This has been due to the remarkable success of various "Unifications" in physics, starting with Maxwell's unification of Electricity and Magnetism, and continuing on with the Electroweak Unification of Glashow, Salam, and Weinberg. These were significant achievements to be sure; but their success has tended to put blinkers on physicists, who assume such unifications must continue "hasta la victoria final". Such hopes and dreams have driven the string theory program since its inception, without much concrete to show for the effort. This makes more sense when one realizes that the hope itself may have been flawed; no amount of genius and effort can make up for barking up the wrong tree.

Inapplicability of Turing Machine Concepts to Physical Systems:

Despite its profound implications for mathematics, Turing's results have little relevance to physics. This is because Turing machines are finite-state machines, whereas the world is not. The possible states that a physical system can assume (e.g. a list of all particle positions and velocities, and internal excitations) are arbitrarily fine-grained. In a simple "old quantum theory" quantization of physical systems, particles are confined to 6-dimensional (6D) phase-space boxes with 6D volume

$$V = \Delta x \Delta p_x \Delta y \Delta p_y \Delta z \Delta p_z \sim \hbar^3$$

This would at first seem to finite-ize the state space and thereby make it countably (rather than uncountably) infinite, and thereby be a satisfactory solution. However this kind of argument completely ignores the quantum phase, which is a continuous variable. Introduction of the phase shatters any illusion that the Universe is a finite-state machine.

There is a countable infinity of Turing machines. Therefore Turing's result is surprising; intuitively we might have expected that all possible halting problems could have been predictable *a priori*, but they are not. But the space of all possible Turing machines is much, much, much smaller than the state space of the real physical world, in Cantor-ian terms. We should not in any sense be surprised that real physical systems do not oblige us there. An additional element missing from Turing machines, and a major problem if we wish to apply Turing machine concepts to physics, is the complete lack of a time variable. This completely decouples Turing's models from the physical world, and should shatter any illusions we might have regarding their applicability to physical systems.

Importance of Quantum Mechanics:

First of all, it is important to realize that quantum calculations almost always replicate semiclassical calculations. While randomness and uncertainty are important in quantum mechanics, overall their significance has been massively overstated. In practice the success of quantum mechanics came because it matched or confirmed semiclassical calculations. Only in a few signature cases (e.g. the electron magnetic moment, the Lamb shift) did quantum mechanics (and its intellectual progeny, quantum field theory) really contribute something new. Engineers realize this and continue to quite happily design everything we use in modern life (including semiconductor devices) by almost totally ignoring quantum results, to great effect. Physicists have not for the most part grasped the significance of this, but this fact needs to be taken seriously.

Statistical Mechanics and Chaos

One of the great achievements of statistical mechanics and theoretical condensed matter physics in general is the achievement of fairly accurate and quantitative predictions about matter in various states (and phase changes between these states) *without having a complete description of the motion of all the atoms*. A similar situation is found in many domains- e.g. astrophysical studies of galactic clustering, simulations of fluids, particle-in-cell (PIC) simulations of plasmas, and many other examples. In all these cases, detailed and quantitatively accurate predictions and calculations of physical systems proceed without detailed knowledge of the exact particle motion.

To return to the matter at hand, the clockwork Universe concept sustained even more severe body blows when advances in brute-force computing (better clockworks, essentially) led to the observation (first formulated by Edward Lorenz) that not all classical physical systems can be predicted accurately, because of their exquisite sensitivity to initial conditions. This result holds true for classical physical systems, i.e. even in the absence of quantum mechanics, and even when the basic governing equations are known.

Analytic Prediction vs. Simulation

When we come to the issue of predictability we run up against the issue of understanding- roughly speaking we could say that prediction is the clearest proof of understanding, and this kind of understanding (predictive understanding) has been a goal of physics since at least the time of the Ancients. This kind of predictability is distinct from the ability to simulate- simulation is modelling a physical system in as fine a grain as possible, and then “seeing what happens”. Simulations do not really predict, just as real physical systems do not predict their future- they simply evolve. The fact that we have been able to split off small subsystems with tightly controlled physics and use them as “computers” should not mislead us. This computational approach does not scale; the Universe cannot compute itself, and there must be a large reduction in superfluous information for understanding to emerge. The recent rise to prominence of massively powerful computational simulations enabled by the vast increase in availability of low-cost, high-power computing has confused the issue somewhat. No-one, no matter how intelligent, can look at the results of a complex computer simulation (e.g. a multiphysics plasma or hydrodynamics simulation) and glean any particular understanding from it. In this sense simulations are truly numerical experiments. All that a simulation can do is provide access to degrees of freedom which are masked or inaccessible in the real physical system being modelled. But, ultimately, real understanding comes by identifying and throwing away irrelevancies, not by adding in extra superfluous degrees of freedom. As Richard Hamming said decades ago, the point of scientific computing is “insight, not numbers”.

From Prediction to Understanding: A Hierarchy of Information Compression

So we see that in the kinds of calculations and simulations of the types discussed above, we have the following hierarchy:

Actual Physical System (no information compression)
↓
Simulation (medium-to-low information compression)
↓
Analytic Calculation (high information compression)
↓
Fundamental Equations (extreme information compression)

Considering the above, we see there is a hierarchy of information compression, beginning at the actual phenomenon level, and ending at the level of a small set of fundamental equations. This is what Dirac referred to in his famous boast about all of Physics and most of Chemistry being solved by his eponymous equation. This also correlates well with Rolf Landauer’s idea of discarding information (and energy) in a computation. *Without such lossy compression, there can be no understanding.* Some superfluous information (in fact quite a lot) MUST be thrown away for understanding to result. This is the true meaning of computability and predictability in a physical system.

Predictability: Discarding Superfluous Information:

Thus we have seen that predictability requires an inherent loss of information-prediction is a kind of lossy coding. But, we must take care that the information that is lost is superfluous. For example, think of the extreme compression involved in modelling orbital dynamics using Newton’s laws; suddenly the behaviour of massive celestial bodies consisting of enormous numbers of atoms, in various states of matter (on the Earth, solid, liquid water, and a gas atmosphere) is condensed to a small set of equations that can be solved with good accuracy on a rudimentary computer. As a further proof, consider that many quite accurate astronomical calculations were done by hand up until the end of the 19th century, and simple computers orders of magnitude less powerful than even a basic smartphone of today were used to plan and calculate the orbital trajectories for the Apollo missions.

Timescales and Chaos:

Chaos threw a wrench into the above picture, showing that in certain respects it was too simple. Edward Lorenz showed that even a simple, stripped-down atmospheric model exhibited unpredictability; this unpredictability manifested itself as an extreme sensitivity to initial conditions. Thus we see that physical systems are in many cases quite predictable- unless they are not. This simple fact, revealed by Lorenz, contains much. One aspect of this is the question of timescale- something entirely ignored in finite mathematics, but absolutely crucial in physics. For a concrete example, which much predates Lorenz, consider the question of the ultimate stability of the the orbital motions of the planets in the solar system. Clearly they are stable on the timescale of thousands and millions of years, but what about on the 1-billion year timescale ? This question intrigued Laplace and many others, right up until the present day. In addition we need to realize that as beings with finite lifetimes ourselves, how would we ever be able to be sure of our result ? In this case predictability would fail us because of an inability to measure.

This simple example shows us that the question of predictability is in general further complicated by the fact that measurement is imperfect at both the small scale (atomic level, due to quantum mechanical measurement issues) as well as on the macroscale (where cosmologists make their untestable predictions). Thus, in the absence of proper measurement (which means an absence of perfect verification), it is not even entirely clear what perfect predictability would mean. We are once again back to the inherent issue of lossy compression of information, without which there can be no understanding. Roughly speaking, we can say that the Universe does not understand itself.

Turing, Gödel, and Physics:

We can state definitively that Turing's and Gödel's results have nothing to do with physics; they both deal with finite-state, countable infinities (e.g. $\aleph^0$), instead of the much greater magnitude of infinity associated with the continuum. Physics is not a formal propositional calculus, and the Universe is emphatically not a finite-state machine, nor will all our mathematical modelling make it so. The inherent imperfect unpredictability in physical systems (both classical and quantum) does not stem from an axiomatic mathematical structure; it stems from the overwhelming number of atoms which must be tracked. Because of the past unreasonable effectiveness of the reductionist approach in physics, physicists have become blinkered in their work. The Higgs boson, the biggest "discovery" in physics in our current era, shed no light on any other problem- the mass of the Higgs could have had any value and there would have been no real implication for theory, or change in other important physical constants. This somewhat shocking fact is a rather brutal indictment of extreme reductionism in science.

The late Philip Anderson was much closer to the truth when he realized that each level of description in science exhibits its own phenomena and requires its own new understanding and paradigms. This is far different from Dirac's boast about having solved all of physics and most of chemistry with his equation. Of course, to a man with a hammer, everything looks like a nail, and to a man with a successful fundamental equation, everything looks like an application thereof. However this wrong; it is untrue both epistemologically as well as ontologically. Slowly, perhaps, physicists are learning this; as large swaths of the theoretical community slowly burn themselves out in pursuit of various kinds of fundamental theories which have little connection to measurable quantities, the rest of the world (including large segments of experimental and applied physics) marches on. In retrospect, many of the most fundamental questions of our day may even look foolish or badly misguided to future generations. Fundamental physics, and in particular the philosophy of extreme reductionism which motivated it, and the ideas derived therefrom, may have somewhat run its course. Far from this being the end of science, it is simply a time in which the scientific mantle will be passed, and reductionism and extreme reductionists will no longer reign supreme.

Implications for Theories of Mind

In summary, the world is not a finite-state machine, and the human brain with its massive interconnectivity and part-chemical operation is certainly no Turing machine. Thus Turing's results (important for digital computers) have no bearing on the operations of the human mind. Similarly, a simple geometric argument shows that an ordinary computer chip, with an essentially 2D interconnect pattern, is orders of magnitude away from the interconnectivity of the human brain. The comparison is simple one of apples and oranges, and attempting to apply fundamental results from one domain to the other, is a likely a fool's errand.

Concluding Thoughts:

So let us summarize what we have learned. We have seen (through the work of Gödel and Turing) that axiomatic mathematics exhibits quite surprising phenomena of undecidability and uncomputability; these results were very important, because mathematics is entirely made human-made, and ought to have been more amenable to our understanding. Physics, on the other hand, has always exhibited unpredictability, as every experimentalist knows in his or her bones. The idea of perfect predictability should never even have entered our minds, but we as a community were so seduced by the effectiveness of mathematics in physics problems that we created a fantasy, constructed out of whole cloth, that the Universe should be, must be, and indeed has to be predictable. This is not a fact- it is an ideology. And, of course it is utterly incorrect, as daily experience tells us. This is not a deep phenomenon- it is an obvious one. What should have been more surprising is that the Universe would exhibit any predictability at all. This would have been the view (and a very natural one) of the Ancients. However, blinded as we are by about 300 years of post-Enlightenment success, and arrogant as we are with our many technologies, we cannot quite grasp this. Until some new emergent unpredicted phenomenon (like Covid19) comes along to stop us in our tracks and give us an object lesson in humility. Indeed, to paraphrase Hamlet, there are (many) more things in Heaven and Earth than are dreamt of in our (natural) philosophy. And we simply have not grasped it yet. Many scientists are profoundly ahistorical in their outlook, implicitly or explicitly believing in absolute truth, without historical contingency, but this is a wrong outlook, and a great weakness. History will show later generations that the current quest for unification and complete predictability was misguided, an error enabled by the unusual and ahistorical success of mathematical methods in the period which proceeded it. Clearly there is something deep in the human psyche at work here, something related to control, and security; for why else would predictability be so paramount, and put upon a shining pedestal ? Why should we mourn the demise of the dream of ultimate predictability ? Was not a predictable Universe a sterile one ? What exactly was the appeal ? Was not the excitement in the hunt, the chase for new phenomena ? What was this bizarre quest for mathematically precise prediction ? It was in many ways, a somewhat ludicrous pursuit.