2015 FQXi essay contest:
Trick or Truth: The Mysterious Connection between Physics and Mathematics

Physics Suffers from Unwarranted Interpretations

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Abstract

Some seemingly mysterious interpretations of mathematics by physicists are just unwarranted. When the ancient mathematicians attributed abstract notions like number and shape to physical objects, they didn't distinguish these notions from real objects and didn't accept fictitious numbers, numbers in excess of the pebbles of abacus. The progress of science stagnated until with Renaissance mathematicians ignored the restriction to countable elements of reality. By liberating mathematics they paved the way for calculus and complex calculus; these then boosted physics and technology.
Later on, free evolution of mathematics was proclaimed. Such freedom contradicts discovered laws of nature rather than invented ones. By means of clever restricted constructs, modern set theory promised rigorously avoiding the gap between Euclid's point that has no parts and Peirce's continuum every part of which has parts. Belonging inconsistencies in physics gave rise to suggest a more natural foundation of mathematics. It renders CH, ZFC, EPR, and Bell irrelevant.
Physics mainly suffers from bad habit to maximally generalize models and to interpret results immediately in an artificial mathematical domain as if they were automatically valid in reality too. Hilbert may be blamed for his denial of the *now* and for his strategy to formulate axioms and then to deduce physics. Let's instead reinstall obedience to natural restrictions on the results of calculations and reject arbitrarily enforced rigor that misled us into futile increasingly speculative theories. For instance: It is not warranted to ascribe singular points to reality; time symmetry is an artifact due to careless use of complex calculus; translation from fiction back to reality is a must.

1 Most basic clarifications

This essay shares at least partially positions [1] to [7] from which mathematical physics deviates. It offers non-speculative answers to notorious basic questions which are kept for relevant in practice. There was ample reason to look for ways out of inconsistencies between and within pillars of physics. Each pillar is based on presumably flawless mathematics and seemingly confirmed by tremendous effectiveness of its application [8]. May we trust in physics that is based on mathematics with an emphasis on symmetries? Can physics easily distinguish between just clever and true models?

Any mathematical model that ignores actual causal structures is at best clever but definitely not true. For instance, deduced theories of hearing are too artificial as to come close to physiological facts.

Moreover, they use Heaviside's trick which tempts to unwarrantedly interpret results of complex calculus as if they did immediately correspond to reality.

On the level of elementary physics, the distinction tends to be more difficult. For instance, continua can be treated as if they were built from discrete parts while sets may also be treated as continuous. Let's hope for consistency of all science as criterion of truth. Physics must be fully consistent with the premise of only one causally connected real world. In the end, merely one if any out of mutually excluding theories can prove correct.

Let's check the following tenets whether yes they are logically correct or no they aren't:

- *Mathematics is identical with science/physics*; the ancient Greek word mathema meant what is now called science.

<u>No</u>: Physics is an empirical science. The essence of mathematics is claimed to be its freedom [9].

- *Anything is* (a commensurable) *number*.

<u>No</u>: The Pythagoreans didn't expect incommensurables.

- *There is no motion* (Parmenides)

<u>No</u>: Zeno's paradoxes were fallaciously fabricated. They cannot really support this claim. Nevertheless, the philosophy of Parmenides, Zeno, and Spinoza still persists in the notion spacetime.

- *It is absurd to claim that bodies are composed of areas, areas of lines, and finally lines of points* (Spinoza).

<u>Yes and no</u>: Spinoza equated the physical notion "body" with the mathematical notion volume. The same confusion is still hidden in Dedekind's use of the word body in mathematics. A body in reality cannot at all be composed of abstract items.

- *A point is something that doesn't have parts* (Euclid) and *a continuum is something every part of which has parts* (C. S. Peirce).

<u>Yes and yes</u>: Euclid's point, line, etc. are idealized and therefore as unreal as also is limitless divisibility in mathematics (Anaxagoras). Just set theory claims that a continuum consists of points.

- *There is no largest* (approachable by counting) *number*.

<u>Yes</u>: Archimedes argued that any (natural) number has a successor n+1. Infinity is not a quantity but an ideal quality.

- *The relations smaller than, equal to, and larger do not apply to infinite quantities.*

<u>Yes:</u> Galileo referred to scales of variables without upper limit, not to concrete values and of course also not to Cantor's transfinite cardinal numbers.

Only those were correct who didn't ignore that mathematics deals ℚwith notions at a higher level of abstraction than physics which is bound to observed or at least assumed reality.

2 Liberation of mathematics from restrictions

Ancient mathematics had culminated in Euclid's still valid consequent abstractions and definitions. More practical methods stood outside because of naïve objections: namely exhaustion by Eudoxos, infinitesimals by Archimedes, and adequality (approximate equality) by Diophantus, who also had already allowed $\mathbb{Q}+$.

It took millenia until a first step towards more fertile mathematics was made; instead of just the pebbles of abacus, fictive numbers were accepted too: the zero, negative and imaginary numbers. Before Renaissance, it was common opinion that numbers must be positive because length, area, and volume resulting from geometrical constructions necessarily all have to be positive. In 1494, Pacioli ignored this opinion and published the elsewhere already known concept of negative numbers. In 1545 Cardano still called negative numbers fictitious ones. Bombelli's Algebra dealt in 1572 with $\sqrt{-1}$, something that was even less understandable in terms of reality. Success of algebra and infinite series went along with liberation of mathematics from its former restriction. Work by Galileo, Kepler, and Cavalieri resumed the mentioned ancient approaches. This altogether paved the way for calculus by those like Galileo's pupil Torricelli, Barrow's student Newton, Leibniz, Johann Bernoulli, and de l'Hospital who wrote in 1696 the first textbook on it.

The invention of the number line from -∞ via 0 to +∞ is credited to John Wallis (1616 - 1703), the same mathematician who used the symbol ∞ for the first time. Although it was seemingly not new at all since yard sticks were already in use in ancient Egypt, the infinite to both sides line was distinguished by its arbitrarily chosen reference point zero, and this innovation proved important in mathematics while misleading in physics.

3 Negative and imaginary numbers in physics

Use of negative and imaginary numbers not just in mathematics but also in physics requires careful interpretation. Such necessity tends to be increasingly ignored or even denied with the questionable argument that most general algebraic solutions are best suited as to deal with concrete geometry and physical reality because they cover any particular case too.

In [10] Descartes strived for as little arbitrariness as possible and arrived at coordinates with indeed maximal mathematical flexibility but the need to arbitrarily choose a point zero. According to Lanczos [11], Descartes had pondered about just positive coordinates. Much later, Fourier was likewise pondering about an option of unilateral integration from zero to infinity. Today, mathematicians consider the beauty of mathematical inventions not limited by the real world.

When Georg Cantor proclaimed the freedom of mathematics as its true essence, he defended his own speculations. Not every play with numbers is reasonable. Nonetheless, Cantor was right in so far that mathematics itself doesn't depend on restrictions by application. Only physics can decide whether or not the attribution of being negative or imaginary is indispensible for correct description of a physical object.

In 1787 Georg Christoph Lichtenberg attributed the positive sign to so-called glass electricity and the negative sign to the oppositely charged resin. Later definition made the elementary charge, the electron, something negative, and glass electricity merely a lack of electrons.

Beyond such confusing choices, there are cases where engineers need operating with locally negative values of differential quantities, e.g. with negative resistance, negative damping, and the like; while virtually no object and no basic physical quantity has a counterpart with opposite sign. Squared velocity is obviously always positive. Non-negative, so called absolute quantities include not just absolute temperature but virtually everything that is measured on a linear, not logarithmic scale: distance, frequency, pressure, mass, age, the number of coins in my pocket, just anything. On the other hand relative measures always refer to an arbitrarily chosen reference point above their natural zero. They can therefore change their sign.

When we operate with negative frequency, negative mass, negative probability, or the like, we always to do so within artificial constructs, e.g. in complex domain, not with genuinely negative quantities. The three Cartesian coordinates of space do necessarily likewise have an arbitrarily chosen point of reference and arbitrary directions. That's why they include negative values.

No matter how natural our common notion of time seems to be, time in this sense can clearly not be a measurable natural physical quantity since it refers to an arbitrarily chosen event at t=0 between positive time after that event and negative before it. In order to get rid of this arbitrariness, it has been recommended [12] to distinguish between elapsed and future time. The scales of elapsed and future time have a common zero between them: the now, which is the only natural reference. The now and the conventional time scale are steadily moving relative to each other. This radical change of perspective avoids the awkward practice of relocating the time window in real-time Fourier analysis of a signal. Choosing the now as natural reference may be regarded as a logical necessity rather than just a trick. Only elapsed time can be measured and attributed to what already happened. It is always positive.

4 Point of reference, abstraction, and relativity

Literature neglected a trifle: Leibniz as well as Newton fitted calculus to their belief in a pre-established timescale, the point zero of which is obviously not pre-established but RELATES to an event that must be arbitrarily chosen. The definite integral F(a,b) omits the points zero. Alternatively, it may be written as the difference between the antiderivatives F(b) – F(a) at upper border b and lower border a, respectively. While an integral depends on two borders, each derivative only belongs to a single location. Mutual symmetry between differentiation and its inverse cannot be reached because they are operations in opposite directions.

Differentiation provides a higher level of abstraction, a loss of integration constants. That's why differential equations are more universal than concrete integral pictures of reality and why laws alone can never completely rule the world.

Performing a first integration let's say of acceleration as a function of time yields velocity as a function of time with reference to an unknown constant component. This is behind Newton's first law of motion. The subsequent integration of velocity does likewise only provide a RELATIVE quantity, the distance between two locations. The operation of integration in mathematics corresponds to deduction from general to special in logics. RELATIONS tend to be more abstract while ABSOLUTE measures are usually more concrete. Of course, as illustrated in Fig. 1 of [13], reality and any abstracted from it theory remain categorically distinct from each other.

While Leibniz argued in favor of understanding space as merely distances between locations, i.e., as RELATIONS, Clarke on behalf of Newton kept space and time for being ABSOLUTE, being substances. Leibniz and Newton merely agreed on that acceleration is an absolute quality. Let's illustrate Newton's mistake with the metaphor of an unlimited to both sides box [14]. Only if there is a preferred point of reference, it is possible to attribute a position to it. In space, such point is usually missing. Newton believed having demonstrated with his bucket experiment that space is ABSOLUTE. His background was in theology, alchemy, and the old fluentist view of moving indivisibles.

Leibniz criticized Newton's ABSOLUTE space as too restricting (to God). When he replaced fluxions by the derivative dx/dt, he made calculus more attractive by pragmatically calculating with fictitious infinitesimal quantities. Neither Newton nor Leibniz realized that the rotation of the bucket defined a point of reference. For the same reasons Michelson's 1881/87 null result was not understood but kept for at odds with the Sagnac effect [15].

The infinitesimals were attacked by Berkeley: *They are neither finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them the ghost of departed quantities?* Leibniz didn't simply rename as *monads* what was called *non-quanti* by Galileo or *indivisibles* by Cavalieri. His system of a pre-established divine harmony distinguished three levels of infinity. The highest level belongs to God. It corresponds to the so called ABSOLUTE infinity, the quality of having no limit. The second level is for instance attributed to the whole space and to eternity. So we may call it undecided between the ABSOLUTE quality and a RELATIVE quantity of being larger than every nameable one.

Euler and others rejected the monads and mocked their proponents as *Freygeister* (free thinkers). Euler's successor Lagrange made aware of a discrepancy: On one hand, mathematics and physics benefit from using RELATIVELY infinite and infinitesimal quantities, on the other hand, there is a serious objection: The term *infinite quantity* (in the sense of a particular value, not of a variable without upper limit) is self-contradictory if infinity is an ABSOLUTE quality. While Lagrange tried and failed to eliminate the infinitely small by means of a series expansion, the majority of mathematicians simply ignored the apparent imperfection.

The most convincing meaning of being continuous is being endlessly divisible; every part of a continuum has parts, as defined by C. S. Peirce. Even Leibniz thought so although he certainly felt the mismatch with his discrete monads which he fitted to all human experience so far: items of reality cannot endlessly be split. He called continuity a labyrinth in which human reason often gets lost and considered infinitesimals and imaginary number as fictions with *a fundamentum in re* (the Latin word res means object). Calculus benefits from trick of ignoring this mismatch and calculating *AS IF* there were atoms of numbers.

In ancient time, Greek aristocrats dealt with geometry which stimulated more abstract and strict reasoning while natural numbers were naively attributed to sets of real items. Leibniz offered a way out of the dilemma; he referred to the lowest of his three levels of infinity, the RELATIVE infinite, when he meant: the rules of algebra hold for infinity too. This statement contradicts to rules like *something infinite plus any quantity is still infinite*, that are valid for the ABSOLUTE quality of being infinite. Already Galileo [1] meant: the relations $<, =, >$ don't apply for variables without an upper limit.

5 Enforced self-consistency of mathematics may fail

Cauchy and many others didn't bother to reveal the hidden reason behind the problem of how to reconcile the notion point with the completeness of a continuum. Instead they provided technically correct epsilontics. Bolzano followed Leibniz, in principle, when he introduced the notion Menge (set) of points as a mathematical term. Replacing monads by points, he overlooked that Euclid's strict definition of a point contradicts his expression *all points of a line*. Hence he arrived at paradoxes [16]. According to C. S. Peirce, locations of points on a line are mere potentialities and are therefore unquantifiable. Georg Cantor came up with a naïve idea. He fabricated an infinite quantity *omega* in order to count transfinite cardinalities in excess of it. His $\aleph_0$ denotes numbers that can be one-by-one aligned to the natural scale, as already shown by Galileo. His $\aleph_1$ makes still sense as the continuum of all real numbers. Further alephs did not yet find any useful interpretation so far. Cantor's originally naïve set theory has been appreciated as providing urgently desired rigor although its uncorrectable logical inconsistency cannot be denied [9]. Hilbert even called point set theory including transfinite cardinal numbers *Cantor's paradise*. ZFC and all that replace Cantor's naivety by cleverly choosing axioms. ZFC managed enforcing self-consistency within the self-made paradise by means of a somewhat brutal and superficial elimination of paradoxes. However, physics does obviously not at all need any kind of point set theory, non-standard analysis and the like. On the contrary, foundation of mathematics on point sets still gives rise to basic reality-related questions.

Dedekind had honestly confessed hesitating for years to publish a similar view as Cantor's because of lacking evidence for it and little relevance of the matter. Asking for the meaning of numbers [17] he nonetheless touched the unsolved key problem.

Brouwer rejected set theory and denied the validity of the *tertium non datur* (= excluded middle) at ∞. Unfortunately, physics did also not benefit much from Brouwer's intuitionism. He failed making mathematics more self-consistent because he considered the *Ur-Intuition* of counting pebbles as the ultimate basis of all mathematical doing.

Weyl called Hilbert a rat-piper because Hilbert declared that any problem in mathematics and physics can be solved. Hilbert was proved wrong although he managed getting rid of contradictions just by definitions and arbitrarily tweaked axioms. Does mathematics really need such maneuvers?

Not just the paradox of Buridan's donkey challenges to find more natural basics. Engineers [18], [19] got aware of a logical imperfection too: One-sided Laplace transformation requires integration within $\mathbb{R}+$ (only positive time). Setting the lower border of integration not at zero but at a tiny negative value is dirty rather than just inappropriate mathematics.

6 Truly real numbers render CH, ZFC, Hilbert space, and EPR irrelevant

In order to allow discovered rather than invented laws of nature, the free evolution of mathematics should be more natural in the sense of being comprehensively self-consistent.

Dedekind fell back into the very old ancient notion of pebble-like numbers. Of course, there is no "pebble zero" on the abacus; ZFC required a correction by adding "$0 \in U$" to the axiom of infinity (U is a set with the property: $A \in U \Longrightarrow A \cup \{A\} \in U$).

Before Dedekind, numbers were still well understood as measures according to Euclid's definition. Measure fits to infinite divisibility of continuum if irrational length is allowed. More specifically speaking, measure is the upper limit to the length of the vector directed from zero to the point under consideration. Dedekind truncated Euclid's measure to a pebble; hence the reference to zero and therewith the unilateral direction got lost. His pebble-like cut is actually a knife rather than a cut. It corresponds to a symmetrically to both sides extended nested interval of infinitesimal width. That's why topology cannot perform a symmetrical cut. There is no problem with that width as long as the backward extension of the interval keeps clear from zero. However, correct treatment of cases like a step or a Dirac delta at zero requires more natural measure-based numbers like 0+ instead of 0.

Already Leibniz had realized that once the location ZERO and the reference point ONE on a line were chosen, any second measure is either rational or irrational. In physics and numerical mathematics, there seems to be no reason for such distinction at all. Any irrational number can be approximated by a rational one as closely as desired; nesting intervals converge to zero. By definition, 1-0.999… equals to zero.

How can physics suffer from such seemingly reasonable definition? An implicitly given limit point on the Peirce-continuous line always differs from any approximation. The notion *all points* of such a line is nonsensical when interpreted as a fixed set of points. In other words, a continuum cannot be made more or less smooth by adding or removing points. The property being continuous is as absolute as is the property being infinite. One cannot eat the cake and have it. The distinction between open and closed intervals isn't warranted for the Peirce-continuum of properly understood truly real numbers.

7 From ideal model back to reality

Naively one could wonder:

- Isn’t any number unique? Isn’t it distinct from its neighbors?

While any two usual rational numbers do indeed span a distance between them, a truly real number behaves like Cantor’s dust; it is not a pebble but the distance between two points and may therefore vanish. The truly real numbers constitute what Weil called a sauce. It is impossible to numerically address any irrational number. Zermelo’s axiom of choice may attribute discreteness merely to rational numbers, not to truly real ones.

- Aren’t there singularities?

Singular points are ideal i.e. unphysical mathematical notions as also are for instance line, area, volume, sine function, continuum, and infinity. So called singularity functions were refined in [12], [20]. There is no logical justification for ascribing physical reality to such arbitrary fictions.

- Doesn’t it make a difference if an atom is missing in a lattice?

In a Peirce-continuum there are no preferred points. If points were stolen from it, this could not even be noticed. Middle point of earth, point charge, line current and the like are fictive elements of continuous models. A lattice is different; it has a discrete structure. A lattice of Cartesian coordinates can definitely not be a natural structure of space because its position and orientation must be arbitrarily chosen.

Science and technology benefit a lot from using fictions. Engineering is simplified by calculating the magnetic field around an electrical current as if *AS IF* the current was concentrated in the fictitious middle of the conductor. Likewise, so called point charge is just a model that partially behaves *AS IF* it was physically real. The unwarranted idea of physical singularities in space continues letting room for wild speculations.

Checks may help to reveal unwarranted in physics generalizations:

- Infinite values, for instance infinite field strength at r=0, indicate missing restrictions.
- What depends on an arbitrary choice cannot be true.
- Elapsed time steadily grows. Restriction to retarded solutions is mostly reasonable.
- Perfect symmetry is rare in nature. It tends to indicate artifacts:

8 Artifacts due to careless use of tricks

Complex calculus is often mistaken as mysterious [21]. Why? Transformation into complex plane replaces differentiation d/dt of a function $\cos(\omega t)$ by multiplication with $i\omega$. Because any periodic function equals a sum of sinusoidal functions, this trick provides simple solutions to linear differential equations, even if these are otherwise difficult or not at all solvable. It offers use of a simple procedure without bothering about how it works. Careless omission of kept for trivial logical steps led to immediate interpretation of results in complex domain as if they already did describe reality.

There is an as simple as compelling argument against the wrong belief that $i\omega t$ may be interpreted in terms of reality: The minus sign in the argument of $e^{-i\omega t}$ was arbitrarily chosen. In contrast to scientists of other disciplines who continued agreeing on the positive sign, EEs decided to prefer the negative sign in order to avoid negative signs in the formulas for complex power. In other words, the attribution of anti-clockwise (mathematically positive) rotation in complex plane to multiplication with the complex unit is just a convention. Identities like $2\cos(\omega t) = 2\mathrm{ch}(i\omega t) = e^{\omega t} + e^{-\omega t}$ are not conventional and therefore still unambiguous. Already Bombelli understood that a+ib always appears together with its complex conjugate a-ib. Perhaps it is necessary to remind: Transformation into complex plane means splitting a real function into two components and then arbitrarily omitting one of them. After calculation in complex plane, inverse transform back into reality is a must. The arbitrary omission is equivalent to addition of redundancy. While it does not change the original data, it adds data about the arbitrarily chosen point of reference. These data are needed for the correct return to the domain of reality.

Even more care is required if one uses the trick by Heaviside to prepare a just unilaterally extended sinusoidal function of time for its transformation into complex plane by means of zero-valued continuation over the missing (future) part and then splitting it into an even and an odd component. Lacking awareness of the obligation to revert that preparation too led to the wrong interpretation of results as mirror-symmetric. This misinterpretation nurtured speculations up to susy and time travel.

Schrödinger was trained to Fourier-transform unilateral functions of time. He didn't pay attention to Heaviside's preparation which implies mirror-symmetrical complex functions of positive and negative frequency. Belonging redundancies must not be removed. They encode what makes the original function unilateral. Dirac argued: there are no negative frequencies in reality. However, when a non-negative function of frequency/energy was introduced in complex domain, this led to seemingly mysterious mirror symmetry of the so called analytic signal in time domain.

Pauli considered quantum theory the first discipline that cannot be formulated without the imaginary unit [22]. Of course, Hilbert space is quasi based on complex pebbles, and it's hard to swallow that Fourier transformation and Hermitian symmetry are just arbitrary extensions of cosine transformation and half matrices. While arbitrary extensions may provide elegant tools, they merely add redundancy, not truly essential data. Heisenberg's matrices instead of semi-matrices reflect the same artifact.

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