Galilei, Gold, Ren: votes for ultimate realism

by Eckard Blumschein

Abstract: *Maybe Galileo Galilei's infinity is not as outdated as mathematicians are trained to believe, and we may hope for ultimate realism in physics? Thomas Gold raised an ignored while reasonable objection to a premature but accepted theory. Tianying Ren performed direct measurement that refutes seemingly flawless tenets, which were so far confirmed in an abundance of ingenious experiments. Restriction to elapsed time complements Ren's work. Apt restrictions might avoid ambiguity due to arbitrariness in general.*

Science is subject to some general fundamentals that deserve absolute priority. At first, theory has to obey reality, not the other way round. While G. Cantor claimed 'the essence of mathematics is just its freedom', physics does not let room for mysticism or mere speculation. Accordingly, the traditional concept of causality is indispensable. Science must be a puzzle whose elements do or at least will fit together with no contradictions and no arbitrariness. Let's consider reality an open system and distrust any final condition. Unfortunately, such attitude is at odds with prominent doctrines including conjectured general symmetry [1].

The chance to get common recognition for refutation of established guesswork is comparatively high in biophysics. Moreover, results of evolution still outperform models of hearing being based on the same mathematics that is also fundamental to the physics of particles. Therefore this essay prefers to discuss a topic chosen from still ongoing research in physics of cochlea before addressing the more general question how inapt formal use of rigorous mathematics might be to blame for various cases of presumably elusive interpretation and fallacious experimental confirmation.

Schroedinger's cat is seen to reflect the arbitrarily chosen algebraic consideration of the continuum as a set of single points. Non-arbitrary mathematics could resume allegedly outdated reasoning and describe physical reality more adequately. This includes that one has to discard unrealistic parts from general solutions of differential equations. Physics benefits a lot from various tools based on negative and imaginary numbers. However, do these really offer additional degrees of freedom in reality?

1. Mutually contradicting claimed impossibilities: Common sense has won

In 1844 Georg Simon Ohm argued: 'A component missing in the spectrum cannot be heard.' He wrongly ascribed pitch directly and exclusively to frequency analysis. His opinion was accepted as long as no explanation was available to the strange phenomenon of an audible missing fundamental. For many decades, mainstream physicists ascribed pitch just to the frequency analysis in cochlea. Looking in vain for delay lines in midbrain, they concluded from so called tonotopy that the spectrum is preserved. Those who favored temporal features, including autocorrelation, were blamed wrong. However, evidence against autocorrelation models [2] was inconclusive in so far as it did not take into account a theorem by Wiener, according to which autocorrelation is equivalent to the spectrum of a spectrum. Seebeck's reply to Ohm was correct: The ear decides what it hears. In 1948 Thomas Gold argued that with realistic data of attenuation the passive hydro-mechanical long-wave model of cochlea cannot work [3]. Leading experts ignored this. They gave preference to the argument that so called accumulated phase indicates a genuine traveling wave (TW) on basilar membrane (BM) [4]. They felt confirmed by precise experimental evidence obviously showing a TW, and they trusted in a well-tweaked passive model by Lighthill [5]. Experts of signal processing accordingly implemented passive filter bank models with transmission line structure.

The discovery of stimulated acoustic emissions in 1978 and subsequent work by Brownell [6] revealed activity inside cochlea and proved Gold correct, at least in that a passive cochlea would not work. Most experts are nonetheless trying to maintain their belief in a genuine traveling wave on BM. They argue that the hair cells may amplify such TW [7].

Several researchers managed to utter doubts to be read in prestigious journals, except for JASA: Some animals have a hearing organ behaving as does a TW on BM while they do not have a BM at all. Others exhibit an acoustic fovea that would reflect a genuine wave. Even proponents of the TW admit that measured latencies do not agree with prediction by Zwislocki [8] and that longitudinal coupling cannot convey the acoustic energy from base to apex [7]. Recent investigations [9] indicate: High frequencies may directly stimulate hair cells.

Ren measured that, in contrast to the theory of a genuine TW on BM, which predicts an asymmetrical wave, the slow wave is pretty symmetrical [10], and it does not travel backward [11]. Ren's results are highly unwelcome because recognition of the TW as a matter of fact has become a Nobel price awarded belief that resists correction. JASA published supporting evidence for backward TWs. While author Wei Dong refused discussing her paper in public, Ren's diversified direct measurements provided strong arguments, which are not based on possibly wrong assumptions.

The phase accumulation argument tacitly presumed that TW and spectral decomposition are the same phenomenon. Most likely the outer hair cells do not amplify a genuine TW but the other way round: They are involved in a rather localized wave-like active process that locally amplifies resonant and perhaps largely radial motions of liquid covering the BM. The visible TW is an epiphenomenon. As already did Ohm, v. Békésy could not imagine his interpretation a premature conclusion. Uttering This is impossible they meant I cannot believe it. While even in science, such guess can be justified and valuable for a while, at best one out of mutually excluding possibilities can be correct.

2. A constructive complement to the refutation by Ren

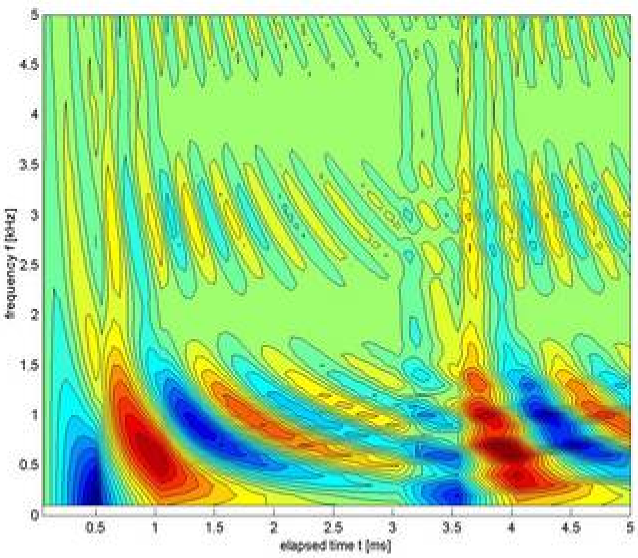
The refutation of a genuine TW on BM is in particular unwelcome to those who adapted filter banks with transmission line structure to psychophysical data. They have to admit that such delay lines fail to correctly model the delay between beginning motion of the incus and measurable response on BM. While this delay is missing in case of strong rarefaction clicks, as to be seen e. g. in Fig. 1 of [12] or Fig. 3 of [13], it may amount several periods at the threshold of hearing; cf. e.g. Fig. 2 of [14].

Spectrograms do not presuppose the questionable delay-line structure. Isn't this an advantage? Not yet. Filter banks with delay-line structure and implemented gammatone or gammachirp behavior are nonetheless more realistic models of cochlea. Hardware filters offer natural realism: Hardware is always causal, and it directly outputs filtered real functions of time, not magnitude and phase. Therefore a frequency vs. time plot of the superimposed outputs for all frequency bands fits well to measured motion of BM. A representation in this style is called a cochleagram. Of course, a feasible number of frequency bands cannot compare with about 30,000 auditory nerve fibers projecting to about 3,000 accurately graduated inner hair cells.

The alternative style of representation just displays magnitude as third dimension in the frequency vs. time plane. It omits the phase of complex spectral components. This resembles the interpretation of wave function by Born. Heisenberg used the expression 'real part' for the last time in [15]. Neglect of phase is roughly justified by Ohm's law of acoustics: The ear is highly phase-deaf for stationary signals.

Usual spectrograms are lacking realism. Do not blame their magnitude-style of representation for that. It is almost equivalent to cochleagram-style. The main reason is: Arbitrary windowing affects all software that is based on computationally efficient algorithms like short time fast FT and MDCT.

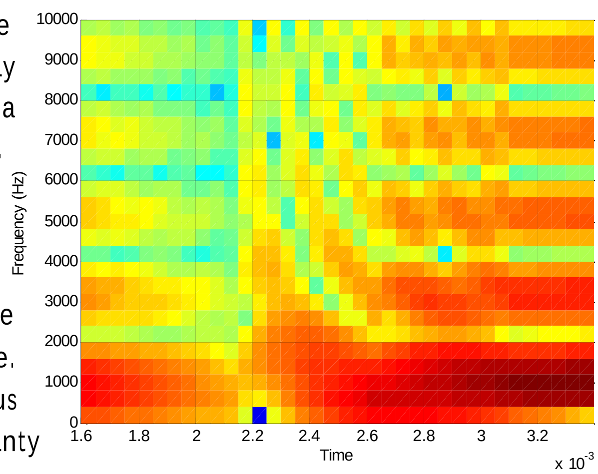
Fig. 1 shows a spectrogram with details that demonstrate the possibility to interpret the TW as an epiphenomenon instead of resorting to a transmission-line structure. Its unique realistic features are related to decisive peculiarities:



- one-sided, gradually fading window of time, instead of an arbitrarily wide and arbitrarily hopping one
- based on many elementary real-valued cosine transforms (CTs) instead of Fourier transforms (FTs).
- represented in cochleagram style, magnitude style by means of Hilbert transform optionally available.

Fig. 2 shows, for the same stimulus, a magnitude style spectrogram obtained with a program spectrogram of MATLAB version 6.2 and extreme overlap.

Time values are wrong. New versions therefore exclude to set Noverlap = Window - 1 = 63, the only way to get a spectrogram based on arbitrary windows a bit similar to the epiphenomenal TW and to Fig. 1. Other combinations of parameters yield either only horizontal or only vertical lines. Horizontal lines of constant frequency belong to a window of time with large width Δt. Conversely one may choose a large bandwidth $\Delta\omega$ and get vertical lines of constant time. The product $\Delta\omega\ \Delta t$ always exceeds $1/2\pi$. This notorious trade-off obviously resembles Heisenberg's uncertainty relation: $\Delta p\ \Delta q\ /h > 1/2\pi$.



Does uncertainty really exclude the possibility to combine good time resolution and good frequency resolution as stressed by Karrenberg [16]? Let's ask first our ears and then Fig. 1.

If compared with physiology, deduction from laws of nature has proven the less successful approach to realism in biophysics. Biological solutions evolved via trial and error, always guided by advantages for survival. They achieved seemingly impossible performance with clever combinations of simple means, and they are distinguished by efficiency, simplicity, and robustness. The human ear is able to hear astonishing small differences of frequency and of time down to the order of one Hz and some ten microseconds, respectively. Human ears outperform the usual spectrogram with respect to $\Delta\omega\ \Delta t$ by orders of size. Therefore it seemed legitimate to look for fundamental differences:

Physics ascribes past as well as future events to an event-related scale, necessarily referring to an arbitrarily chosen point zero. While so called ideal filters behave non-causal, this is ultimately impossible to reality. The ear cannot and does not use arbitrarily chosen windows of time and their awkward relocation. It has no synchronization to our common arbitrarily agreed timescale, and it cannot hear future signals. An eye can also not foresee anything. The only available point of reference to our senses is the very moment. No physical process is influenced from its future state. The ear tells us: Spectral analysis only needs the real-valued cosine components of FT.

Well, CT could not be applied to the unphysical pure sine function. However, the dead point is irrelevant for a running frequency analysis as well as for bicyclists while calculation of real-valued radial wave numbers k_r is known to suffer from poor convergence [17].

For a spectral analysis as shown in Fig. 1, the product $\Delta\omega\ \Delta t$ can be made as small as desired because in this case there is no limit to the frequency resolution of CTs for every elementary pulse or step. Their values are a priori known. Time resolution is given by the assumed sample rate.

In contrast to ordinary spectrograms, neither the ear nor the real-valued analysis shown in Fig. 1 does require to arbitrarily choose the width of a time window and the rate of its relocation. Even in comparison to any filter bank, their higher resolutions reveal more details. As a model of cochlea, Fig. 1 does not yet consider the build-up of above mentioned local resonance depending on sound pressure level.

In principle, Fig. 1 could also be calculated via many subsequent applications of complex FT with appropriate one-sided windows each, because FT is equivalent to CT. However, such superposition of one-sided functions of time would be new and a detour.

3. How to cope with what is behind Cantor's paradise?

If someone shifts a sinusoidal function by a phase angle with respect to an arbitrarily chosen point of reference, this does not cause a change in the reality described by it. The use of positive as well as negative values of time can merely be redundant if compared to using elapsed time without sign. Moreover, any function of natural numbers can be mapped to a corresponding one of any real number and vice versa. One does not need Cantor's concept of ordinal and cardinal numbers for that insight. Already Galilei considered bijection between natural and squared numbers. He concluded that the relations smaller, equal and larger are not valid for infinite quantities but merely for finite ones. This incomparability in terms of size is called fourth logical possibility. Galilei shared the old notion of infinity as the inexhaustible quality of numbers to have no limit. Georg Cantor claimed having 'proved' wrong or at least outdated the wisdom of Galilei, Gauss, Leibniz, Newton and many others by introducing his naïve set theory, which ignored definitions by Euclid and Peirce, respectively: 'The whole is larger than any part of it', 'a point has no parts', and 'every part of a continuum has parts'. Their genuine continuum cannot be resolved into single points. Cantor believed and defined that one can have both an uncountable entity of infinitely much of elements and each element of the many at a time. Fraenkel declared this definition of a set untenable [18]. Hilbert claimed that the axiomatic method rescued the naïve belief and wrote: 'Nobody might expel us from the paradise, which was created for us by Cantor.' He, Zermelo, and Fraenkel managed hiding the paradox between the axioms of extension and of infinity. Ebbinghaus [19] cautiously commented by quoting Lessing's Theological pamphlets: 'Given, there was a great useful mathematical truth, the inventor of which was guided to by an obvious fallacy; ... do I renounce the opportunity of using it?'

Teachers of analysis are still appreciating what Hilbert called Cantor's paradise because it seems to rigorously justify treating irrational numbers as if they were rational ones. Pragmatic users of mathematics did and do not need this questionable belief. It is an open secret that Cantor's paradise lacks a sound basis. Let's check whether set theory is really an opportunity to use. So far no $aleph_x$ with x in excess of 0 and 1 did find any use in physics. Likewise, we do not benefit from exclusion of a single number from a continuum. Not just intuitionism tells us that we fail to distinguish between two rational numbers if they differ from each other by a too small difference, and that real numbers could not at all be resolved into tangible points if they did constitute a Peirce continuum. According to Lavine [20] and Ebbinghaus [19] it is difficult to understand infinity and real numbers, respectively. Why? Before redefinition, infinity and continuity were qualities that can only be approached but never achieved with a finite number of discrete steps. Cantor called Galilei's infinity an 'infinitum aeternum increativum sive Absolutum' attributed to God, an inconsistent one, because it evades counting. He suggested an artificial infinitum creatum sive transfinitum. Dedekind's continuity leans on this mathematical infinity. The intention was to force the irrational numbers by definition into a body that obeys trichotomy, regardless of the lacking proof.

Because natural numbers are ideal repetitions of an ideal unity, any rational number precisely relates to this unity. For any chosen precision it is possible to decide whether a rational number x is smaller, equal to, or larger than y. However, one may not numerically pinpoint with absolute precision a location within a Peirce continuum of really real numbers. No single number, not even the neutral point zero can there be completely addressed, singled out, or excluded. Any single really real number in it is irreal and therefore irrelevant. Fréchet's compactness gets meaningless. The infinity of Galilei, Spinoza and others is a quality, not an Archimedean quantum: Infinity plus anything is infinity. Hermann Weyl compared the rationals with bones within the sauce of continuum. Being rational is a frozen property that gets lost with ideally continuous shift relative to the reference point.

Physics benefits from the ideal concept of discrete numbers, quantity and linearity as well as from the likewise ideal concept of continuum, quality and non-linearity. In contrast to the algebraic view, unbiased reasoning tells us: Both concepts mutually complement and also exclude each other. Transition from one realm to the other and return are performed, for instance, via trigonometric or exponential functions in the kernel of integral transforms, typically mediated by Cauchy sequences.

Given we prefer a continuous model and agree that no signal can propagate with a velocity v in excess of c. Is there really an absolutely empty space, which does exactly demand v=c? Also, why not ignoring mathematical 'correctness' and write with a grin: |sign(x)|=1 without an exception for x=0?

Mathematics did not just at will attribute the unjustified value 0 to sign(0). It also puts useless solution into integral tables. For the integral (3) in [21], tables do not just give the values 1/2 and 0 for arguments below and beyond π, respectively. They also give for π a singular value ¼ with which inverse transform would not correctly return the original function. Ignore it.

Every child can cut a piece of paper into two symmetrical pieces. Topology is unable to do so. Algebra demands a neutral zero between positive and negative numbers. This is only correct for integer as well as rational numbers that represent countable physical distances.

How to deal with the neutral zero when all real numbers (IR=R written in blackboard bold) are separated into positive IR+ and negative (IR-) numbers? Mathematicians offer four variants: zero to IR+, to IR-, separately, or as you like. Is there really not the only convincing answer? Why not admit the impossibility to justify and to perform focusing on a single element of a continuum? Wittgenstein wisely argued: Infinitely many laws are equivalent to absolute lawlessness.

Terhardt [22] also criticized what Aseltine [23] put under his chapter heading 'Troubles at the Origin'. If negative values of x do not at all exist, one must not make sure the complete inclusion of $\delta(0)$ by starting a one-sided integral transform a tiny bit left from zero.

Schroedinger's cat shows: Those who interpret mathematics did still not yet learn a more humble attitude from Buridan's ass: Any point out of a continuum, not just zero, is not at all realistic. Only rational numbers can be related by finite linear operations to a not zero-dimensional reference unit. If reality was exactly expressed by rational numbers, the ass would suffer starvation. The message of Buridan's ass is the antithesis 'panta rhei' (there is no standstill) to the belief of Pythagoreans that numbers are the ultimate reality. David Hilbert's credo was: Any mathematical problem has a solution. Obviously he was not such an absolute finitist as are those who consider this wrong.

Albert Einstein uttered disapproval of Hilbert's attack on Luitzen Brouwer. He nonetheless shared Hilbert's view and called the controversy on fundamentals of mathematics pointless. When Einstein questioned the completeness of the quantum-mechanical description, he meant finished discrete values, not completeness in the sense of equivalence.

Hilbert's pupil John von Neumann introduced Hilbert-space into quantum physics. Just a few years later and perhaps as a reaction to [24] he confessed: 'I do not absolutely believe in Hilbert-space any more'. Why? He explained to Birkhoff:

'Hilbert-space (as far as quantum mechanical things are concerned) was obtained by generalizing Euclidean space, footing on the principle of conserving the validity of all formal rules. This is very clear if you consider the axiomatic geometric definition of Hilbert-space where one simply takes Weyl's axiom for a unitary Euclidean space, drops the condition on the existence of a finite linear basis, and replaced it by a minimum of topological assumptions (completeness and separability). Thus Hilbert-space is the straightforward generalization of Euclidean space if one considers the vectors as the essential notions. Now we begin to believe that it is not the vectors which matter but the lattice of all linear (closed) subspaces.'

Why was no convincing escape from this trouble for the rest of v. Neumann's life? Maybe, Fig. 1 can illustrate a synthesis between the theses of Simplicius and Pythagoras because it shows how infinity and equality mutually exclude and also complement each other. This dilemma is a plausible cause of the uncertainty affecting pairs of conjugate variables. It can be more easily understood with CT transform than with the more involved FT. While $i^4 = 1$, the CT of a CT immediately returns the original function. This may switch back and forth between discrete and continuous functions.

4. Natural restriction avoids arbitrary choice and misinterpretation from the very beginning

CT belongs to positive arguments. Introducing analytic geometry, Descartes hesitated to use negative as well as positive xyz. He strived for as little arbitrariness as possible. Disdain of a natural origin like r=0 implies the need to arbitrarily choose a point of reference. Virtually all basic physical quantities, including time-span, primarily extend in just one direction. Negative values of a number of items, of mass, of distance, of energy, of probability, etc. are not immediately reasonable. When Gauss in 1831 attributed reality to imaginary numbers, he referred to the already accepted negative numbers: 'They can only be applied when the counted number A has a counterpart B with A+B=0. Strictly speaking this is only valid if one does not count items but relations between two of them.' Georg Ch. Lichtenberg, a professor of physics in Goettingen, coined in 1787 the names positive (glass) and negative (resin) electricity. He did not yet know that the elementary electric charge is negative. Quantities like pressure of air, temperature, etc. also have natural points of reference. Nonetheless use of their agreed zeroes tends to be more appropriate.

Space and distance do not have a direction. The natural zeros of radius and elapsed time, makes these quantities directed ones. Positive elapsed time is the distance of a past event seen from now. This perspective does not change if we move to the next higher level of abstraction. Anticipated elapsed time is still positive. The time that has elapsed since a hypothetical Big Bang would also not be arbitrary. However, it is not qualified as a reference because it is unknown and it lets the question unanswered what was before.

There was wrong common consensus that frequencies cannot be negative while time can. Negative frequency was considered unphysical in the sense of impossible in reality. Actually, negative elapsed time is unreal, and negative frequency is a reasonable consequence of complex FT. Many engineers and physicists did not realize that the restriction for the original domain to measurable values of a real-valued function of either elapsed time or frequency necessarily implies functions of positive as well as negative fictitious frequencies or elapsed time, respectively, in complex domain. They were trained and familiar with time domain as original domain and frequency domain as the complex one. For convenience they introduced complex functions immediately there. Later on they also introduced complex functions of time, so called analytic signals. Eventually they forgot or even denied the obligation of return to reality.

Heisenberg as well as Schroedinger lost the usual link between reality and time domain when they moved from usual to the Hamiltonian point of view. This would have required interpreting the complex wave function as corresponding in reality to a one-sided function of time, cf. [21], [25]. Failure to do so necessarily implied apparent symmetry.

Hermann Weyl admitted in 1932 [26]: 'The problem of the proton and the electron is discussed in connection with the symmetry properties of the quantum laws with respect to the interchange of left and right, past and future, and positive and negative electricity. At present no acceptable solution is in sight.'

Schulman [27] wrote: 'Where is the frontier of physics?' … 'My vote is for 10^{-6} cm. Two of the greatest puzzles of our age have their origin at the interface between the macroscopic and the microscopic worlds.' … 'time symmetric microscopic laws acquire a manifest asymmetry at larger scales.'

Judge yourself. The limited scope of this essay does not allow for important conclusions to be further specified. Instead, the essay will go on focusing on general ideas concerning non-arbitrary mathematics and ultimate realism: Taking the user's point of view, it considers the operation integration the primary one and ending at zero. Primary does mean, there is no obstacle to repeatedly calculate an integral of a singular influence.

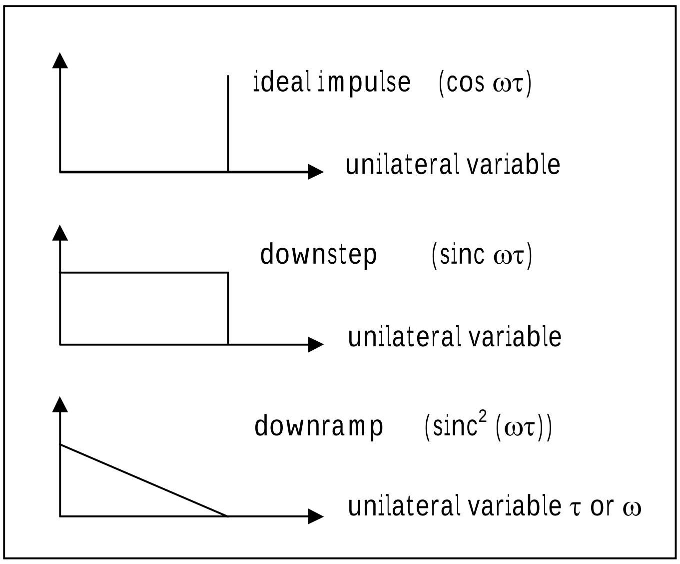


Fig. 3 shows subsequent always converging integrals in IR+. The downramp is the integral of a downstep, which on its part is the integral of the ideal impulse. CTs are given in parentheses. Sinusoidal and exponential functions are not subject to the restricted reduction to a basic singularity. Therefore they alone are unfit to describe real processes.

Directed does mean, integration is not a symmetrical relation between two summands but it continuously adds an infinitesimal increment to the sum. Incidentally, this aspect reveals time as an integral quantity. Differential equations are not the primary relations in physics but they arose by stripping off the link to reality and hence they opened the door for ambiguity.

Practice is a good touchstone for whether or not mathematics is ultimately realistic. Two more examples for still lacking application are the theory of generalized functions with unrestricted possibility of derivation by Laurent Schwartz and the complex cepstrum.

In other cases, practice points to mistakes that are not immediately obvious:

- Having 'measured' propagation of signals faster than light, Nimtz [28] himself admitted that this is impossible, but he failed to reveal the fallacy in his measurement.
- Do not trust in the nearly symmetrical bell-shaped and too wide functions of time 'measured' by Gompf et al. [29]. While these rely on approved single electron counting, they deviate from direct measurement with streak camera and also from what was to be expected.

There is a mounting variety of cases that obviously or at least possibly relate to lacking awareness of the one-sidedness of real time.

Sometimes, strictly speaking impossible models are nonetheless excellent approximations, for instance the Gauss pulse, which theoretically extends from minus infinity to plus infinity. So called N-wave in the near field of a blast is the opposite extreme.

Complex quantities compactly express relations between two quantities. Are they indispensable in quantum physics? Fig. 1 gives rise to doubt the latter. It shows a hyperbola of uncertainty for time and frequency, a conjugated pair of variables. We may replace the FT of a bilateral function by the CT of a unilateral one without loss of realism because only the unilateral function is not redundant. We may substitute time by position q divided by c. Circular frequency $\omega = 2\pi$ f parallels momentum p and equals energy E divided by Planck's factor h. Radius r and wave number k_r constitute another pair of positive conjugate quantities. Functions of the corresponding coordinates in IR have support only in the positive half of the original domain but complex values for positive and negative argument in complex domain. They all are subject to uncertainty for a plausible reason: FT as well as CT and their inverse mediate between quantization (n=1, 2, 3, ...) and continuity. For instance with INT=integral, t=elapsed time, and a singularity at τ=T:

INT_2 {($2/\pi$) [INT_1 downstep@T cos($\omega\tau$) dτ = sinc (Tω)] cos (Tω)} dω = downstep@T

discrete function of elapsed time τ → continuous f(ω)→ discrete f(τ)

The canonical (Born wrote 'verschaffte') quantization condition pq-qp = h / 2 π i made quantum mechanics more involved and beautiful colorful. Non-commuting variables belong to the arbitrarily chosen bilateral consideration.

Correct interpretation of solutions obtained from not aptly restricted to physical reality but arbitrarily generalized and therefore ambiguous mathematics is possible but it requires extreme care as to be ultimately realistic.

References

[1] Wigner EP: Events, laws, and invariance principles. Nobel Lecture, 1963

[2] Kaernbach C; Demany L: Psychophysical evidence against the autocorrelation theory of auditory temporal processing. *JASA (=J Acoust Soc Am)* 104 (4) 2298-2306 (1998)

[3] Gold T: Hearing. II. The physical basis of the action of the cochlea. *Proc R Soc Lond B Biol Sci.* 135: 492–498 (1948)

[4] von Békésy G: The variation of phase along the basilar membrane with sinusoidal vibration. *J Acoust Soc Am* 19: 452-460 (1947)

[5] Lighthill J: Energy flow in the cochlea. *J Fluid Mech* 106:149–213. 20 (1981)

[6] Brownell WE, Bader CR, et al.: *Science* 227: 194-196, (1985)

[7] Robles L; Ruggero M: Mechanics of the Mammalian Cochlea. *Physiological Reviews*, Vol. 81, No. 3, July 2001, pp. 1305-1352. See also physrev.physiology.org/cgi/content/full/81/3/1305.

[8] Zwislocki JJ: Theorie der Schneckenmechanik. *Acta Oto-Laryngol Suppl* 72: 1-76 (1948).

[9] Rabbitt RD, Clifford S, Breneman KD, Farrell B, Brownell WE: Power efficiency of outer hair cell somatic electromotility. *PLoS Comput Biol* 2009 Jul 5(7):e1000444. Epub 2009 Jul 24.

[10] Ren T: Longitudinal pattern of basilar membrane vibration. *PNAS* 99:17101-17106 (2002).

[11] Ren T: Reverse propagation of sound in the gerbil cochlea. *Nat Neurosci* 7, 333-334 (2004).

[12] Recio A; Rich N; Narayan S; Ruggero M: Basilar membrane response to clicks at the base of the chinchilla cochlea. *JASA* 103: (4) 1973 (1998)

[13] Lin T; Guinan J: Auditory-nerve click responses. *JASA* 107: No 5, Pt 1: 2615-2630 (2000)

[14] Guinan J; Cooper N: Efferent effects on basilar-membrane click response. *JASA* 124: 3 (2008)

[15] Heisenberg W: On the Quantum Reinterpretation of Kinematical and Mechanical Relations. *Zeitschrift fuer Physik* 33: 879 (1925)

[16] Karrenberg U: An interactive introduction to signal processing. Berlin: Springer 2002

[17] Williams E: Fourier Acoustics. New York: Academic Press 1999

[18] Fraenkel A: Einleitung in die Mengenlehre. 2nd. Edition, Berlin: Springer 1923

[19] Ebbinghaus HD et al.: Numbers. New York: Springer 1991

[20] Lavine S: Understanding the Infinite. Harvard University Press 1998

[21] Blumschein E: Adaptation of Spectral Analysis to Reality. IEEE Workshop on Signal Propagation on Interconnects, Berlin 2006, 173-176; Amended home.arcor.de/eckard.blumschein.M283.html

[22] Terhardt E: http://www.mmk.ei.tum.de/persons/ter/top/laplace.html

[23] Aseltine J: Transform Method in Linear System Analysis. New York: McGrawHill 1958

[24] Einstein A; Podolski; Rosen N: Can Quantum Mechanical Description of Physical Reality be Considered Complete? *Phys Rev* 47: 777-780 (1935)

[25] Blumschein E: A still valid argument by Ritz. home.arcor.de/eckard.blumschein.M290.html
Let's benefit from special mathematics for elapsed time. http://fqxi.org/community/forum/topic/369

[26] Weyl H: Gruppentheorie und Quantenmechanik. Preface to 2nd edition, Leipzig: Hirzel 1931, VII

[27] Schulman, LS: Time's arrow and quantum measurement. Cambridge: Univ. Press 1997, p. XV

[28] Nimtz G: Superluminal signal velocity. *Annalen der Physik*, Leipzig 7 (7-8) 618-628 (1998)

[29] Gompf B; Guenther R et al.: Resolving Sonoluminescence Pulse Width with Time-Correlated Single Photon Counting. *Phys Rev Letters* 79: 1405-1408 (1997)