3rd FQXi ESSAY CONTEST: Is Reality Digital or Analog?

Continuation Causes Superior but Unrealistic Ambiguity

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While there is no evident reason for believing that the world was anyhow made of discrete parts but overwhelming evidence for a rather continuous evolution, signal processing is superior if based on discrete values. This seeming contradiction gave rise to an investigation on how analog and digital approaches relate to each other and to reality. Three interrelated mathematical pillars of physics were found to suffer from unjustified generalization: Points instead of endpoints, once and twice redundant equivalences. A realistic interpretation of abstraction-made ambiguity sheds new light on quantization and apparent symmetries.

1 Realism of analog vs. merely continuous vs. digital models

Analog computing is outdated. It was immediately bound to the real behavior of lumped electric amplifiers, capacitors, and resistors. Its results could be wrong, e.g. due to ignored invariance laws. However they were always causal, unambiguous, and measurable, because analog computers mimic what primarily happens in real processes: temporal integration of superimposed influences up to the moment of consideration. In practical application, this realism of analog computing was rather a drawback: Analog computers modeled systems of differential equations (DEQs) by means of integrators; and it was difficult to correctly choose all belonging initial conditions.

Mathematically implemented continuous models are directly based on DEQs. Reduction of originally integral equations to DEQs is an abstraction with serious implications: The natural link to reality gets lost. One has to arbitrarily choose point of reference and direction of increase, e.g. t=0 and direction of time. DEQs have ambiguous general solutions. Usually, one considers only the realistic retarded particular solutions and ignores the redundant advanced ones. In other words: In order to obey causality and get rid of redundancy and ambiguity, the arrow of time must be added.

Virtual equivalence between discrete and continuous mathematics worried the formalists a lot. Related fundamentals of mathematics are questioned in appendices A and B. Strictly speaking there are neither absolutely continuous nor absolutely discrete signals, because any tangible thing is finite. In terms of usual spectral analysis, the frequency bandwidth is always finite, and accordingly there are neither ideal steps nor singularities in a belonging function of time. Conversely, finite width of a time window for analysis makes resulting spectral lines look bell-shaped. Even begin and end of an animal's life are continuous while irreversible processes.

Freeman Dyson asked whether life is analog or digital and got seemingly contradictory answers [1]. "Neither nor" meant: Nature is the object of abstraction; it must not be equated with its models. "Both" is also appealing: Some phenomena can better be described by continuous "analog" models, others by discrete models. Peas are preferably treated as if they were uncountable. Uncountable liquids are measured in terms of countable gallons.

Evolutionary selection led to combined solutions for sensory organs of animals. Lacking awareness of this fact caused decades of fierce disputes between those who explained auditory perception just as recognition of a discrete frequency pattern and those who trusted in evidence that supports a temporal code. Actually, it would be naïve explaining the function of brain like a digital computer with hardware and software. A better example of a discrete memory is the genetic code. Genetics can be seen a natural solution that provides almost eternal life combined with the chance for adaptation by means of death, birth, and stochastic repair not to an individual but to interbreeding species.

Analog signal processing has a decisive disadvantage: It cannot get rid of influences like temperature and noise that make the final results more or less imprecise. Digital signal processing is subject to the same disturbing noise. However, a sequence of discrete impulses can be reshaped as often as desired. Moreover, by means of suited coding in combination with synchronized tact signals, it can also be checked for errors and even repaired. This ensures the absolute accuracy and reliability of digital signal processing. Furthermore, digital computers are much better in position to implement advanced mathematical methods.

While Peirce still considered time and space continuous, quantum physicists are trying to derive from created mathematics a completely discrete structure of reality. They are hoping for a theory that will explain everything, for instance transversal electromagnetic waves as a summary effect of many tiny particles. Let's omit some questions concerning the possibility to realistically define and measure hypothetical smallest entities. Instead, we will see that both the foundations of mathematics and, to a larger extent, basic interpretations of implication for physics are not yet as mature as possible.

Mathematics can definitely not be the queen of science if it is subject to independent free creation. The elusive sentence "everything is number" has been ascribed to Pythagoras. It shows: Early philosophers equated ideals with reality. It also indicates the same attitude as uttered by G. Cantor: "The essence of mathematics is its freedom." Actually, decisive roots of mathematics are linked with philosophical doctrines. Theoreticians at the time of Descartes hesitated but decided to obey the usual concept of a time-line with no natural point zero but seamless extension from eternity ($-\infty$) to eternity ($+\infty$). Paving the way for a pragmatic use of linear DEQs, they simultaneously introduced the mentioned belonging ambiguity. Instead defining integrals that refer to zero and extend to b, they defined integrals from a to b, while differentiation is only attributed to b. Hence we need integration constants that get lost with differentiation. Belittling R+ as special case of R gave rise to speculative physics. Many related false friends like dimension, field, and universe deviate from their original physical meaning.

2 Reasons to reinstate old definitions more precisely

2.1 *Notion of number as not simply a point but a measure between zero and a virtual endpoint*

Euclid defined: "A point is something that does not have parts", and he defined a (positive integer) number as a measure that counts of how often the basic quantum one is repeated [2]. Rational numbers are splits, commensurable to the same basic quantum. If it stands for a length, then the third power means a volume. At the time of Gauss, Euclid's notion of number got replaced. Gauss preferred attributing a complex number to the coordinates of a point in complex plane. Dedekind [3] still mentioned: Restriction to just one endpoint of a vector tacitly refers to its origin at the point zero. Intentional or careless neglect of this trifle led to considering the line as a compact set of points and to using the notion point in the sense of a vector in expressions like "dimensions of a point" or "rotation of a point". In its original meaning, a point has no extension in any dimension.

Imagine the half-line R+ of positive real numbers, which extends from zero to infinity, and consider on it a continuous piece along which a function f(x) has the value one. The piece begins at $+0$ and ends “at” x=c where c does not as usual denote a unique position, but it stands for a limit point of x approaching the end of the piece from inside. The included piece may be considered as an integral over a down-step, which is an integral over a half-sided delta-impulse that only extends within the piece and whose width approaches zero. Accordingly, x=c is a measure of length c directed from zero to the virtual endpoint “at” x=c. An included piece cannot include its endpoints as elements. The unfortunately common identification of a number with just one point instead of the distance between zero and a virtual limit point may be considered a relapse to the time of abacus when numbers were imagined like pebbles or like the patterns on a dice.

Real numbers strictly speaking require reinstating the Euclidean notion of a number as a measure as suggested above. This will resolve a lot of either ignored or denied trouble, not just with Buridan’s ass. It will hopefully allow a less arbitrary foundation of mathematics. Present mathematics suffers from problems that relate to pebble-like imagined numbers: So far, Dirac impulses cannot be located “at” the boundary, and the unilateral Laplace transformation cannot begin “at” zero without contradiction [4], [5]. Integral tables give misleading solutions for a singular x, see Appendix A.

2.2 *Peirce’s perfect continuum of mere potentialities*:

Anaxagoras argued in favor of infinite divisibility: Splitting of an object does not make it disappear. The possibility of repetitious splitting is theoretically unlimited. Hence there is no lower limit to the size of a part. Charles S. Peirce still defined an ideal continuum as “something, every part of which has parts” [6]. Hausdorff’s notion of continuum is much more complicated. It requires understanding a nonempty, compact, and connected metric space. Even worse, it relates to a number as an embedded single pebble-like imagined point instead of a measure between two fictitious endpoints. Mathematics suffers twice. At first, modern topology is not even in position to perform a symmetrical cut that separates the entity of all real numbers R into positive ones R+ and negative ones R- without a remnant number zero in between. Secondly mathematics adheres to the illusion R\{y} is executable. While N\{x=1} is commonly understood as removing the empty piece between 0.5 and 1.5 from the line, there is no smallest measure to be removed between real numbers. Fractals belong to rational numbers. There is no Cantor’s dust in Peirce’s true continuum.

Peirce’s ideal continuum, number, point, line, area, and volume belong to beautiful mathematical ideals. Nobody worries about treating a street as if it was a line and a crossroad like a point. Engineers know that an electric line current and a point charge are likewise just simplifications. Nonetheless such ideal models must obey logics. Strictly speaking Peirce’s continuum and Euclid’s number are ideal concepts that mutually exclude and complement each other. For good reasons, mathematicians did and do not accept this calamity. They are happy with Cantor’s paradise. The next two paragraphs explain why.

3 Atomism, finitism, and the two aspects of infinity

Atomism denies infinite divisibility. A first elaborate version of atomism has been ascribed to Demokrit who speculated that everything tangible is entirely composed of various smallest while imperishable and atomos, i.e., indivisible parts. Demokrit’s atomist philosophy was not a lucky guess but a result of experience and reasoning. While the existence-monists Zeno agreed with him in that there is a reality, Zeno meant all true being evades measurement, and he rejected the idea that a continuum is composed of atoms. Zeno argued: Rest and beginning motion exclude each other. Indeed, any finite amount of immobile points cannot constitute a continuum, and continuous motion is strictly speaking at odds with atoms of time. Aristotle clarified: While any finite amount of atoms cannot constitute a strictly continuous object, any real object or property has nonetheless lower and upper limitations of size.

Atomism was developed further by Epikur (341-270). Gassendi (1592-1655) revived it. Then Boyle, Dalton, Meyer, Planck, and others made it foundational to chemistry and physics of particles.

There are however no mathematical atoms alias Urelemente. Already the theorem of Pythagoras led to irrational measures that are incommensurable to the unit one. Originally they were dubbed alogos. Galilei, Kepler, and others used an idea of Eudoxus for treating the continuum as if it was exhaustibly composed of many indivisibles. On this pragmatism arose DEQs.

When Galilei got aware of the possibility of bijection between the natural numbers 1, 2, 3, … , and their squares 1, 4, 9, … , he concluded “The quantitative relations smaller than, equal to, and larger than do not apply for infinite quantities” [7]. Galilei still understood being infinite as the ideal property of a measure that can neither be increased nor be exhausted, cf. Appendix B.

While forbidden in finitist mathematics, the division by zero fits well to that ideal meaning of infinity. There is no justification for different degrees of ∞ if the distinction between finite and infinite is independent from the distinction between discrete and continuous. There are however two mutually excluding and complementing aspects of infinity: The fictitious boundaries of R or R+ are $-\infty$ and $+\infty$, or 0 and $+\infty$, respectively. They act like ideal mirrors, which constitute the basis for an expansion into repetitious periods of sinusoidal components. Here infinity is seen as something perfect from outside, i.e., from an ideal continuum R.

Ebbinghaus [8] mentioned that it is hard for mathematicians to understand in what real numbers are different from the rational ones. Extended real numbers are a pleonasm to those who understand in what property R logically differs from Q: The realistic “inner” view sees an unreachable, so called potential infinity form inside Q. Sommerfeld’s radiation condition states: No wave is reflected from infinity in finite time. Standing waves are strictly speaking approximations.

Artifacts of apparent symmetries belong to real numbers, not to rational ones. The fictitious outer view can easily be unmasked if compared with the physical point of view: While apparent symmetries are always perfect, symmetries in reality tend to be more or less imperfect.

So far, it seems to be most reasonable to consider the world neither finite nor actually infinite but potentially infinite towards smaller as well as larger values of spatial and temporal distance. While Planck length and Planck time are too small and Planck energy is too large as to be of practical use, Planck mass amounts 22 microgram, which would be easily measurable.

4 Why foundational pragmatism outperforms pseudo-foundational formalism

Using the expression “incomparably small”, Leibniz deviated from the original meaning of infinity. Nonetheless in his understanding, every number is finite while the infinitesimals as well as the infinite large are well-founded fictions to be used as if they did really exist [9]. Berkely objected to infinitesimals: “They are neither finite quantities, nor quantities infinitely small, nor yet nothing”

[10]. Today there is no doubt: Together with the introduction of in principle once redundant values between $-\infty$ and $+\infty$ they proved foundational for physics.

Mathematicians of the 19th century strived for a rigorous basis. Weierstrass applauded G. Cantor who ignored Galilei when he demonstrated that R must be larger than Q. Cantor's naïve ambitions arrived at transfinite cardinal numbers in excess of infinity. Then paradoxes and missing proofs gave rise to speak of a foundational crisis of mathematics. In 1900, Hilbert put the proof of CH on top of the ten foremost important mathematical problems. In 1904, Cantor failed to prove well-ordering the real numbers. Zermelo supported him by inventing the axiom of choice. Zermelo's evidence was based on the exhaustion of infinity which was traditionally understood as inexhaustible. As way out of the crisis, Hilbert and others declared Cantor's set theory naïve and replaced it with sets of axioms, e.g. ZFC. Bourbaki even called set theory the basis of mathematics. This might be at least exaggerated. Cantor's transfinite cardinalities were pseudo-foundational. CH is not even wrong.

5 Twice redundant while equivalent

While some details of physical reality look pretty countable and finite, a superior simplification can often be achieved by pragmatically modeling something that is finite in reality as if it was infinitely repetitious or at least mirror-symmetrical. So called analytic continuation is in particular used in order to prepare measured data for Fourier transformation (FT) with integration from $-\infty$ to $+\infty$.

A preparation is necessary for measured data because these always belong to a finite window of time. Even if Peirce was perhaps correct in that there is no limitation to measures like temporal and spatial distances in reality, tangible object of physics are always finite. The beginning of any process can be ascribed to an emergence of relevant influences. Consideration of any process ends at the last considered moment; in case of considered reality this is the very moment. Therefore it is always possible and most natural to avoid the arbitrary distinction between positive and negative values of time and choose a unilateral f(t) with t=0 at either begin or end of the process under consideration. For the frequency analysis of a snapshot, it does not matter that the end is subsequently sliding with respect to the ordinary time scale. Heaviside suggested the commonly used trick how to deal with the unilateral f(t): Set the missing f(-t) zero and split f(t) into an even and an odd component. Continuation and splitting adds three more identical copies to the single original unilateral f(t). After FT this double redundancy utters itself as Hermitian symmetry of the complex f(ω): mirror symmetry of its real part and anti-symmetry of its imaginary part. When written as matrix, the original is a real-valued half-matrix. Its Hermitian correlate is a symmetrical matrix whose elements are conjugate complex to those mirrored at the main diagonal.

With the alternative continuation f(-t)=f(t), FT degenerates to the real-valued cosine transformation (CT). Restriction to R+ from the very beginning instead assuming R also leads to CT. A comparison shows that the simpler CT of an originally one-sided function is equivalent to the allegedly more general complex FT. The latter carries additional information merely on the chosen at will point of reference and the assigned direction. Having nothing to do with the reality of the original unilateral f(t), these additions are needless with CT in R+. Nonetheless they are essential with FT in R.

At least signal processing benefits a lot from use of the twice redundant complex representation. Harmonic analysis allows solving otherwise difficult or not at all solvable linear DEQs like algebraic equations. Differentiation and its inverse simply shift sinusoidal functions by a quarter of period.

Already the once redundant DEQ were favorable. While twice redundant, complex representations offer further benefits. Complex calculus is so excitingly superior that one is tempted to interpret its results immediately in complex plane. Sometimes, this does actually work. Engineers do not bother much about stunning apparent symmetries including negative frequencies, perplexing non-causality of ideal transfer functions, notorious problems with spectrograms, and the like. They even used to ascribe physical properties to imaginary power and evanescent modes. However in order to strictly avoid misinterpretation, one has to perform careful all four steps of transformation: at first continuation from R+ into R for once redundant DEQs, then FT into the twice redundant complex domain, inverse FT as third step, and finally back to R+.

Unfortunately the one-sidedness of a finite and merely potentially infinite view on reality is at variance with the deeply rooted belief in an a priori given block universe. Most physicists are proponents of the idea that the world can be reduced to eternal laws given as DEQs and at least one primordial element. Accordingly they arrive at seemingly natural symmetries of virtually everything.

As an example of an originally always positive quantity we may consider the pressure of air. Acoustics is based on a linearization. DEQs of acoustics are dealing with the alternating component of pressure, which has of course positive as well as negative values. Original physical quantities do not have opposites: The distance from A to B is the same as from B to A. No temperature is lower than zero Kelvin. No atom is less heavy than nothing, etc. Any increase is naturally directed from zero to larger values. Basic quantities that describe phenomena of possibly opposite directions, for instance acceleration, force, and velocity of motion consist of an arbitrarily defined arrow of size one multiplied by an absolute value whose square also refers to the neutral element of addition.

Are there anti-quantities in reality? While hypothetical anti-particles were expected to exist as frequently as usual ones, this proved wrong. A positron always occurs together with its negative counterpart as do the two halves of a dipole field.

Dirac, one of the fathers of quantum mechanics, was misled by common sense when he uttered what they were convinced altogether: Wave length as well as frequency must not be negative [11]. Accordingly, they considered the Hamiltonian as a positive quantity. This would have required to identify complex functions of time in a somewhat unusual manner as a so called analytic signal, i.e., as fictitious symmetrical functions of positive as well as negative time with symmetrical real part and anti-symmetrical imaginary part in complex domain. Correct inverse FT would have led to an unilateral f(t), with non-zero support restricted to the time before the moment of consideration.

6 Equivalence as a touchstone for theories

In order to remove restrictions to mathematical operations, positive real numbers R+ were extended by adding negative real numbers, which were further extended by adding imaginary numbers. This must not be mistaken: Complex numbers C do not provide a more general description of reality. While it is indeed advantageous to calculate with them, evanescent modes, alternating components, negative dB, negative °C, or the like, do not directly relate to reality. Objects and measures of reality have natural zeros: distance, absolute temperature, passed time, mass, pressure, probability, entropy, etc. Any attribution of negative or imaginary numbers to them is arbitrary and redundant.

Pauli and also Stueckelberg [12], [13], [14] stated that quantum mechanics is the first and only discipline that cannot be formulated without the complex unit i.

Heisenberg's quantization condition is known to demand an imaginary difference between the products of two non-commuting quantities:

pq – qp = ih/2π

When Minkowski's introduced ict as fourth dimension, he confessed not to understand why it is imaginary [15]. With v=velocity, $x = \int v dt$ would correspond to $x = -ivt$. The imaginary unit means a rotation in complex plane. Not even the minus sign would be a problem because it is arbitrarily chosen anyway. If there is a problem in this case then it relates to the twin paradox due to Lorentz transformation, cf. Appendix C. However, how to justify ih?

The following analysis will make the issue of continuous vs. discrete more transparent. The quantities p and q are so called canonically conjugated quantities as also are time t and frequency ω, or radius r and wave number k. Products like pq/h, ωt, and kr are zero-dimensional and therefore possible arguments of nonlinear functions, for instance cos(ωt). Planck's constant h is just required as to get a dimensionless argument. It may be called quantum of action, but it has the meaning of the smallest quantum of energy E=hω only on condition there is a lower limit to the circular frequency ω. Wave guides have such a cut-off frequency for transversal waves. Heisenberg's uncertainty relation likewise applies for a conjugated pair time and frequency. Corresponding hyperbolas are shown in Fig. 1. They resemble cochlear frequency analysis.

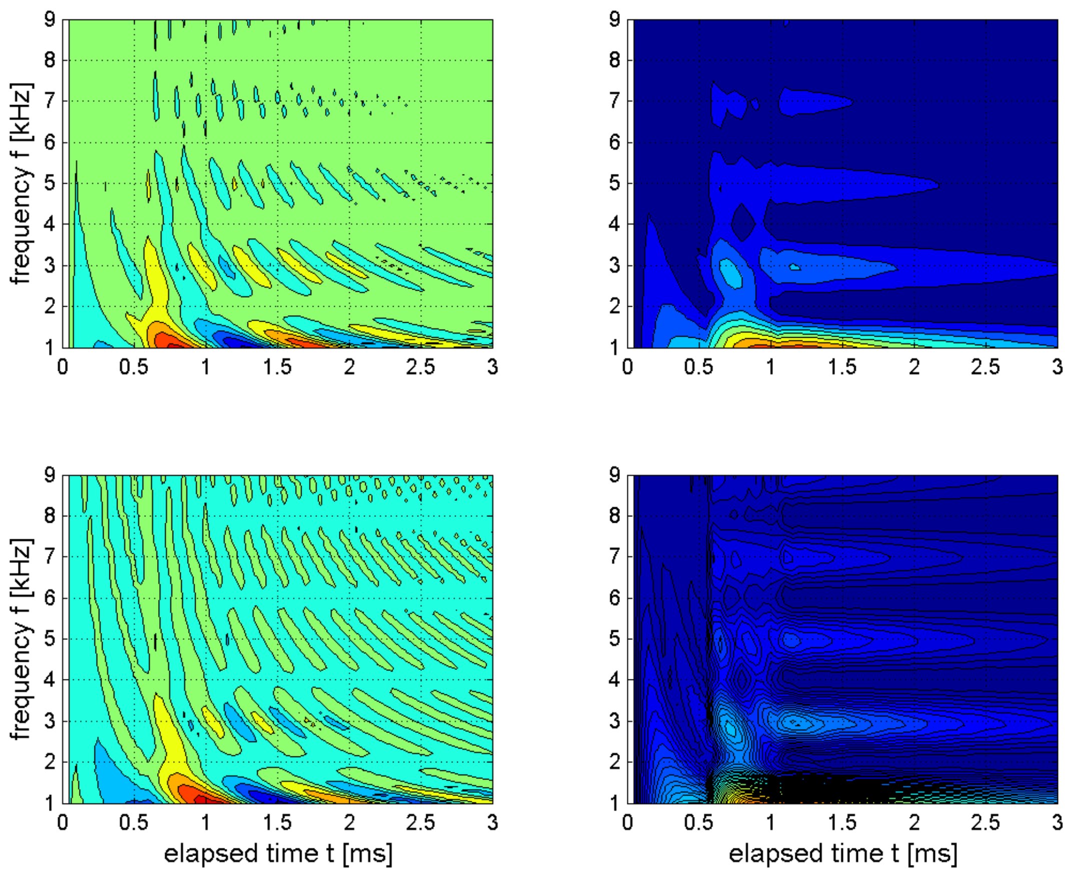

Fig. 1 Frequency vs. time calculated by means of cosine transformation. Stimulus: Haar wavelet, stepping at 0.5 and 1ms. Real result (top left), imaginary part (bottom left), and magnitude (right)

Superimposed CTs of an originally discrete unilateral function of time provide a continuous plot, showing hyperbolical ripples of alternating amplitudes [16]. This real-valued result was complemented by means of Hilbert transformation with an imaginary part Y_{imag}. Then the envelope to both parts was calculated as the magnitude abs(Y_{real} – i Y_{imag}) in order to compare it with the usual representation. As expected, strictly horizontal spectral lines of constant magnitudes are to be seen for lower (top right) as well as for higher (bottom right) resolution.

A rather rapid decay (3.5 ms) of cochlear response was chosen for Fig.1. With 500 ms, the decay is hardly to be seen in http://home.arcor.de/eckard.blumschein/3nat3reim500.pdf .

In principle, R+ is sufficient for description of potentially infinite one-sided processes. Compelling arguments were already found in physiology of audition [16]: The ear is bound to causality. It does not know any agreed reference of time and cannot perform complex calculus. It rather performs a CT that refers to the very moment.

This confirms a surprising fact: The imaginary condition of quantization is only necessary if the complex FT is preferred. Otherwise, physics can be less complicated. CT is tailor-made to reality. Therefore it allows to easily calculate and to plot absolutely causal reliefs, which illustrate how continuous and discrete representations are corresponding to each other.

Incidentally, doesn't Pauli's exclusion principle also simply resemble the real-valued relationship between the property of sin(x) to be anti-symmetrical as are fermions and of $1-\cos(x) = 2\sin^2(x/2)$ being mirror-symmetrical as are bosons?

Abbreviations

C	all complex numbers	N	all positive integer numbers
CH	Cantor's continuum hypothesis	Q	all rational numbers
CMB	Cosmic microwave background	R+	all positive real numbers
CT	cosine transformation	Z	all integer numbers
DEQ	differential equation	ZFC	Zermelo Fraenkel axiom system with axiom of choice
FT	Fourier transformation		

References (last accessed Jan. 21, 2011)

[1] F. Dyson (2001) http://www.edge.org/3rd_culture/dyson_ad/dyson_ad_index.html

[2] Euclid: Elements. http://aleph0.clarku.edu/~djoyce/java/elements/bookVII/bookVII.html

[3] R. Dedekind (1872) Stetigkeit und Irrationale Zahlen. Braunschweig: Vieweg

[4] J. Aseltine (1958) Transform Method in Linear System Analysis. New York: McGraw Hill

[5] E. Terhardt (2006) Laplace Transformation Updated. http://arxiv.org/PS_cache/math/pdf/0507/0507132v3.pdf

[6] Collected Papers of Charles Saunders Peirce. Vol. 5, p. 355, Cambridge: Harvard Univ. Press see also http://www.angelfire.com/super/magicrobin/peirce.htm

[7] G. Galilei (1639) Discorsi e dimostrazioni matematiche, intorno à due nuove scienze. Leiden: Elsevier

[8] H. D. Ebbinghaus et al. (1991) Numbers. New York: Springer

[9] D. M. Jesseph (1998) Leibniz on the Foundations of Calculus: The Question of the Reality of Infinitesimal Magnitudes. *Perspectives on Science* (MIT) **6**. 1&2, 6-40 http://www.webcitation.org/5nZWht6FE

[10] Berkely (1734) The Analyst. http://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/

[11] P. Dirac (1930) Principles of Quantum Mechanics. Oxford

[12] H. Atmanspacher et al. (2005) Der Pauli-Jung-Dialog. Berlin: Springer

[13] F. D. Peat: http://www.fdavidpeat.com/bibliography/essays/pauli.htm

[14] E. C. G. Stueckelberg (1951) *Helvetica Physica Acta*, **24** , p. 153

[15] H. Minkowski (1908) Raum und Zeit. *Phys. Zeitschr.* **10**, 75

[16] E. Blumschein (2006) Adaptation of Spectral Analysis to Reality. ISPI Workshop Berlin: IEEE, corrected by Fritzius: http://home.arcor.de/eckard.blumschein/M283.html

Appendix A

Dedekind equated a number x with the location of a point that cuts the line of real numbers into smaller, exactly equal, or larger values. Current mathematics treats x like a tiny piece of the line surrounding it. In case of a function f(x) that steps from 0 for negative x to 1 for positive x, f(x=0) is by definition equal to 1/2. Therefore, integral tables give for instance for |a|=1 the misleading solution

$$\int_0^\infty \frac{\sin x \cos ax}{x} \mathrm{dx} = \frac{\pi}{4}$$

Instead, the solution $\pi/2$ is required as to get correctly back the original function after performing CT and its inverse, cf. [16].

Let's instead reinstate Euclid's notion of number as a measure of distance: It does not belong to a point but to a directed piece of the real line between two endpoints. More precisely, these endpoints are fictitious limits, one of them toward zero, the other one toward the other end of the piece. They are not included in the piece and cannot be excluded from it. In case of a step function, the limits of adjacent pieces of the line are different at the same location, hence |sign(x)|=1 with no exceptional singular point for x=0.

Appendix B

The property of being infinite refers to an unchanged totality of all not excluded elements. For instance the possibility to count is endless within the fictitious totality N of all non-negative integers. It cannot be enlarged by addition of negative integers. The union Z of both non-negative and negative integers is a different but also endlessly countable totality. Z does not include more elements than does N. Cantor's second diagonal argument first assumes all real numbers fixed and then it modifies this totality. Therefore it is, cautiously formulated, not a convincing argument for more numbers in R as compared with Q, while R is nonetheless different from Q in that it is uncountable. Accordingly, there is no known use of any transfinite cardinality except $aleph_0$ and $aleph_1$, which correspond to Q and R, respectively.

Appendix C

While there is much work that claims to confirm the special relativity, fluctuations in CMB radiation were recently interpreted as indicating a flat universe. Among other serious objections, the twin paradox might indicate that already what Poincare dubbed Lorentz transformation could be inappropriate. Poincare's synchronization actually destroys synchrony because the seen by A travel from A to B and return to A is not symmetrical.