daerengD Time Paradigm

This essay is the result of six years of reflection on time. I believe the fundamental nature of time is one change which distinguishes a before and after. In general I believe time is fully derived from combinatorics. The arrow of time is the result of entropy that is derived from microstates, macrostates and multiplicities. For time in quantum mechanics, I believe fundamental objects with no internal structure must experience change through interactions. The best mathematics for this approach is the derangement. I have focused only on wavefunctions and the measurement problem. I show that derangements produce the linear time evolution of wavefunctions and provide a mechanism for collapse. Sections I-IV will be a review for experts but I consider the ideas essential for a complete understanding of time.

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Newton believed that time was continuous and absolute, later Einstein would prove that it was not absolute but he still assumed it was continuous. The continuous assumptions of relativity and the discreetness of quantum theory produce problematic paradoxes. In relativity the time dilation may take any value, but quantum mechanics predicts that no interval of time is shorter than the Planck time. As stars collapse into black holes, relativity predicts infinitely small singularities, but quantum mechanics predicts the Planck length as the smallest distance. I believe that time is a derangement and that real, conceptual, wavefunctions are consequences of deranged mathematics.

I – The Science of Counting

Combinatorics, the mathematics of arrangements, forms the bedrock foundation of probability theory. Combinatorics is deduced with logic; it has few assumptions and many simple concepts. Despite the importance of the subject it is often neglected and understudied. I believe four ideas in enumerative combinatorics fully explain time: arrangements, derangements, and both ordered and unordered lists. The possible seating arrangements of three people: Larry, Jerry and Harry--sitting in a row are as follows:

$$LJH, LHJ, JLH, HLJ, JHL, HJL$$

Each of these possibilities is an example of a permutation or arrangement. Larry, Jerry and Harry may be arranged in six separate ways. Initially, the men may choose one of three seats. After Larry selects his seat, Harry and Jerry may choose one of two seats. After Jerry has selected, Harry must sit in the only remaining seat. Three, two and one multiplied together equals six, and six is the total number of seating arrangements. n!, n factorial, is equivalent to the integers from one to n multiplied together. An n element set may be arranged n! ways:

$$n! = n(n-1)(n-2)(-3)\ldots(3)(2)(1) \tag{1}$$

A derangement is an arrangement with no element in its original position. For instance, LJH and JHL are derangements of each other. For a collection of n objects the number of derangements is n_i and [n!/e] is the nearest integer function:

$$n_i = [n!/e] \tag{2}$$

Five card poker hands are unordered lists. The poker hand (Q♠,J♣,A♥,K♦,Q♥) is identical to (J♣,Q♠,K♦,A♥,Q♥) or any similar rearrangement. C(n,k), "pronounced n choose k", is the binomial coefficient; it counts the number of possible unordered lists:

$$C(n,k) = n!/[k!(n-k)!] \tag{3}$$

The number of possible five card hands "chosen" from a fifty-two card deck is as follows:

$$C(52,5) = 52!/(5!*47!) = [(52)(51)(50)(49)(48)]/5! = 2,598,960 \text{ hands}$$

A poker dealer may distribute five cards to players in many different ways this exemplifies ordered lists. Although the hands (Q♠,J♣,A♥,K♦,Q♥) and (J♣,Q♠,K♦,A♥,Q♥) are identical, they are received from the dealer in a different order. A slightly modified form of (3) counts the possible number of ordered lists:

$$k!C(n,k) = n!/(n-k)! \tag{4}$$

Again the values of n and k are fifty-two and five:

P(52,5) = 52!/47! = 311,875,200 ways to deal all poker hands.

II – Random Chance

Physics renames permutations, unordered lists and ordered lists as microstates, macrostates and multiplicity. The physics nomenclature has similar sounding words, so they are easily confused. To avoid ambiguity, a useful analogy is to picture a macrostate as a destination, and a microstate as one path to a destination. The multiplicity is the number of different paths leading to the same destination.

A microstate is specific to the outcome of each element. The total number of microstates for a set is the number of arrangements n!. When three coins are tossed, each coin has a one in two chance of landing as either heads or tails:

TTT, TTH, THT, HTT, HHT, HTH, THH, HHH (microstates)

A macrostate describes the overall set. Macrostates are independent of the elements order. The above microstates are simplified by writing them as the macrostates: no heads, one head, two heads and three heads, (0H,1H,2H,3H).

0H, 1H, 1H, 1H, 2H, 2H, 2H, 3H (macrostates)

The multiplicity is the number of microstates or rearrangements possible for each macrostate. The 0H macrostate has a multiplicity of one because it has only the TTT microstate. The 2H macrostate has a multiplicity of three: HHT, HTH and THH.

The number of microstates for n coins is 2^n not n!; because the coins are identical elements and may be in two separate states. n+1 is the number of macrostates and the multiplicity of a macrostate is C(n+1, m) (m=0H, 1H,…). Summing the multiplicities of every macrostate equals the total number of microstates:

$$\sum C(n+1,m) = n! \tag{5}$$

The total number of microstates for three coins is $2^3=8$, and three coins have 3+1=4 macrostates. $\Omega(m)$, the multiplicity of the mth macrostate, for each coin: $\Omega(0H)=1$, $\Omega(1H)=3$, $\Omega(2H)=3$, $\Omega(3H)=1$. The sum of the multiplicities is $1+3+3+1=2^3=8$, the total number of microstates.

The probability of obtaining a macrostate is the multiplicity of the macrostate divided by the total number of microstates. The larger the multiplicity of a macrostate, the more likely random chance favors its outcome. The probability of obtaining the 0H macrostate is 1/8 and the probability of the 2H macrostate is 3/8.

The multiplicity of macrostates varies significantly with a small change in the number of elements. The change in multiplicity is important because not all the macrostates receive an equal number of microstates. For a large number of elements, the likely outcome becomes certain due to changes in the relative probabilities of the macrostates. For coins, the number of microstates doubles for each coin added, but the number of macrostates increases only by one. For three coins or three-thousand coins, the number of microstates for the 0H and all heads macrostate remains one. However, the difference between the number of microstates for three-thousand coins and three coins is enormous. A few macrostates have a large multiplicity increase, and others do not change as dramatically. This is the reason a large number of coins has an overwhelming probability for the half heads macrostate, nH/2.

Both time and an increase in entropy are irreversible. Entropy provides a much deeper understanding of time and the measurement problem. The entropy equation is:

$$S = k\ln(\Omega) \tag{6}$$

Ω is the multiplicity of a system. The energy distribution of all physical systems is related to the macrostate with the greatest multiplicity. The multiplicity of a cup of coffee is a very large number written with multiple exponents, such as $10^{20^{15}}$. The natural logarithm in the entropy equation (6) converts large numbers to manageable sizes by the property, $\ln(a^b) = b\ln(a)$. k, is Boltzmann's constant and it changes multiplicity into units of energy and temperature.

Everything in the universe is in constant interaction with everything else, and these interactions increase the overall multiplicity of the universe. If a hot cup of coffee and a cold one are put into contact, given enough time, both cups will reach the same temperature. Each cup has an amount of energy spread randomly among the coffee's atoms. When two cups are placed together, the total energy is the energy of the first cup added with the energy of the second cup. It is unlikely every coin will land tails if a million coins are tossed. It is even more unlikely to find all the energy in the cups of coffee in only a handful of atoms. As the entropy of the universe increases, some macrostates that were possible become impossible. The second law of thermodynamics says entropy must always increase and never decrease. A decrease in entropy would result in a possible past arrangement of the universe.

III - The Wavefunction

Wavefunctions in quantum physics provide chance predictions for what classical physics predicted with certainty. The Copenhagen interpretation postulates the unsquared wavefunction as a mathematical object with no physical significance. The wavefunction is a superposition of imaginary bases states.

$$\Psi(x,t) = C[a_1\psi(1) + a_2\psi(2) + \ldots + a_n\psi(n)] \tag{7}$$

All physical predictions in quantum mechanics must be real probabilities. Imaginary numbers become real numbers by squaring them. Squaring is often referred to as the inner product of the wavefunction; inner products are always real numbers. If the basis states are orthonormal, then the squared wavefunction will not contain cross terms. For example, if ψ_i and ψ_j are orthonormal, squaring $(\psi_i+\psi_j)$ removes the $\psi_i\psi_j$ cross term requiring it to equal zero. Squaring a wavefunction with these properties may only be a math-e-magical way to eliminate imaginary numbers.

$$\psi_i+\psi_j)^2 = \psi_i^2 + \psi_i\psi_j + \psi_j^2 = \psi_i^2 + \psi_j^2 \quad (\langle\psi_i|\psi_j\rangle = \delta_{ij}) \tag{8}$$

The wavefunction must also be normalized to provide probabilistic predictions. Normalizing a wavefunction allows the physicist to solve for the C coefficient in (7). C is independent of time. Knowing C turns the squared bases coefficients into the probabilities of obtaining that basis state in a measurement. The bases states' coefficients are meaningless unless the wavefunction is squared and normalized. Obtaining real probabilistic wavefunctions in quantum physics requires a tremendous amount of theoretical work. Much of the mathematics provides little in conceptual understanding. A physical explanation of the wavefunction may answer many of the important foundational questions and greatly simplify the theory.

IV - The Measurement Problem

Time is in trouble. An ontological enigma, the measurement problem, exists at the foundation of quantum theory. The problem is the two separate time-dependencies of the wavefunction. One is from the irreversible collapse of the wavefunction, and the other from the reversible time evolution of the wavefunction:

$$\Psi(t) = \exp(-i\alpha t) \tag{9}$$

Measurement forces a wavefunction to collapse or reduce into one of the possible outcomes called eigenvalues. The eigenvalue a wavefunction collapses to can not be predicted with certainty but can only be assigned a probability. Once the wavefunction has collapsed, any additional measurements will find the same eigenvalue with certainty. The wavefunction collapse increases the entropy of the universe and time moves forward.

Some believe a measurement and the wavefunction collapse are from human or metaphysical consciousness. The many worlds interpretation hypothesizes that multiple universes exist. The many worlds interpretation believes that a wavefunction collapses into every eigenvalue, a universe for each eigenvalue. These imaginative ideas lack empirical evidence yet dominate the headlines and colloquiums. I believe that the collapse of the wavefunction and the coin-tossing example are similar probabilistic phenomena. In the coin-tossing example if two different sets of coins are brought together, then some macrostate is found with certainty. This is similar to the description of a measurement as the measured wavefunction interfering with the wavefunction of the measuring device.

V - Deranged Time

A change distinguishing before and after is the simplest definition of time. A discrete fundamental object cannot be subdivided. Therefore, such an object is not capable of internal change. To experience change it would need to interact with other objects in the surrounding environment. Assume W, X, Y and Z are discrete fundamental objects capable of interacting with each other. X and Y will experience a change if X and Y interact. Similarly, W and Z will experience a change through interaction. To experience another change, every object must interact with something new and different: W with X and Y with Z. This is a derangement. From the object's perspective the change was instantaneous, moment-to-moment.

The probability any new permutation of objects is a derangement is $\exp(-1)$. Derangements are mutually exclusive outcomes. If there are two permutations, the probability both are derangements is $\exp(-2)$. For n derangements the probability is $\exp(-n)$ ($n = 0,1,2\ldots$). This is identical to the time-dependence of a real wavefunction.

If the properties of multiplicity and derangements solve the measurement problem, then a small number of elements must have an exp(-n) time-dependence. The derangement converges to exp(-1) as the limit of n, the number of elements, approaches infinity. If derangements do not converge rapidly to exp(-1), then derangements are incompatible with multiplicity. The n=4 set [1,2,3,4] has nine derangements, $4_¡=9$:

$$[4321], [4132], [4312], [3421], [3412], [3142], [2413], [2341], [2143]$$

The total number of arrangements is 4!=4*3*2*1=24, and $4_¡/4! = 9/24 = 0.375$. The probability that a permutation is a derangement in a four element set is within two percent of exp(-1):

$$[0.375 - \exp(-1)]/\exp(-1) = 2\%.$$

Any new arrangement is either a complete derangement or partial derangement. Either every element is in a new position, or some elements are and some are not in their original positions. If an instant is a complete derangement, then every element in the set would experience the same instant. Time would be absolute, but this assumption is removed with partial derangements. Deducing the partial derangement equation is straightforward. In a set of n elements choose k of them to remain in their original position and derange the remaining n-k elements.

$$P(n,k) = C(n,k)((n–k)_¡) \tag{10}$$

$$P(n,k) = C(n,k)[((n–k)!)\exp(-1)] \quad \text{for } (n-k \geq 4) \tag{11}$$

Every arrangement is a complete or partial derangement. Therefore the sum of all partial derangements and the complete derangement is n!, the number of possible arrangements.

$$n! = \sum[P(n,k)] = \sum C(n,k)[(n-k)!\exp(-1)] \tag{12}$$

$$1 = (1/n!)\sum C(n,k)[(n-k)!\exp(-1)] \tag{13}$$

Recall, the number of microstates, macrostates and multiplicities are n!, n+1 and C(n+1,m) respectively. Summing all the multiplicities is equivalent to the total number of microstates (5). The probability of obtaining a macrostate is the macrostate's multiplicity divided by the total number of microstates. A small change in the number n increases entropy and creates overwhelmingly likely macrostates.

For derangements, n! is also the total number of microstates. A macrostate is the number of elements remaining in their original position. A set has n+1 partial and complete derangements. The multiplicity with a exp(-1) dependency of derangements and wavefunctions becomes (11) making (5) and (12) identical.

If (12) is divided by n!, then the coefficients for each partial derangement term become probabilities, (13). This will also simultaneously satisfy the completeness relation. Mathematically 1/n! is a constant; and physically 1/n! is not a derangement and therefore independent of time.

A deranged wavefunction is identical to the wavefunctions used in quantum physics today. However, the deranged wavefunction is independent of imaginary numbers, inner products, orthonormal bases, normalization and the measurement problem. Derangements provide a derivation for the time evolution of the wavefunction and the mechanism of collapse. Fundamentally, time is a derangement, and the temporal arrow is a phenomenon of derangements and entropy.

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