CHASING THE MATHEMATICAL EXISTENCE

1.1 *Origins of tools*

Primordial challenges mankind had to face in order to survive, together with the need of availing oneself of the environmental resources, induced the necessity of ordering, counting, comparing objects and processes. Thus, basic concepts as quantity, comparison, disposition and shapes, held as innate by some[1], naturally arose very early in our evolutionary history.

Surely, stepping from physical entities to idealized concepts requires to clean up raw observational facts from contingent aspects and accidental qualities. This involves the processes of **abstraction** and **generalization** that lie at the core of mathematical modelling. Results were also required to be independent from individuals, to be shared and transmitted; this drove the evolution of human thinking towards conceptual tools built such as to be **universal** and **formalized,** sowing the seeds of the first mathematical representations.

Furthermore, observing, making use and wondering about the world, man very early caught *regularities* in natural phenomena, that appeared to follow fixed prescriptions. Even where such regularities were not manifest, a deeper search very often flushed out a number of hidden ones. In order to spot these regularities in Nature it is again crucial to discard unessential aspects and specificities of the target event as well as the impact of the observer. Thus, recognizing regularities and eliciting **universal statements** requires the same process of **abstraction** and **generalization** entailed in building up the conceptual tools for counting and measuring.

1.2 *Origins of effectiveness*

Therefore, on one hand, discerning order and correlations in Nature does require handling abstract concepts; on the other hand, idealized entities are the most natural and suitable tools to master Nature and its phenomena. These abilities carry evident values in evolutionary sense: being able to manage quantities and measurements, or foresee repeating events, allows making predictions, optimizing efforts and better exploiting available resources. For short, they dramatically enhance chances of survival. Thus, all cognitive faculties entailed in developing those mental tools have been genetically favoured by natural selection along human evolution because of their benefits in species competition. We may conclude that the abstract analysis of idealized concepts –we may now call it Mathematics– was born and developed to account for, describe and steer the natural environment, both in causal terms and in evolutionary biology perspective.

Such considerations should prevent us from being astonished by the effectiveness of Mathematics in describing the world: that is exactly what it is meant to do, and it ought to amaze us no more than a can opener effectiveness in opening cans, or a picture's resemblance to the subject!
Nonetheless, something else is baffling.

1.3 Mysterious fecundities

It often happens that when some of the formal tools developed to describe a specific circumstance are pushed forward, through further derivations and deductions, an entire universe of unexpected properties and relations discloses. Surveying it reminds the discovery of new, pre-existing land, waiting to be discovered. Abstract constructions show such richness and depth neither included nor predictable in the original scope, apparently autonomous and independent of whoever will. And in fact, one has not run into mere logical consequences or barren speculations: the conceptual tools derived anew extend their reach and validity in describing the world far beyond what they were designed for. Mental constructions stretch over new domains, allowing precise predictions of what will or would happen, under proper conditions. Where do such richness and descriptive power come from? Maybe, Math itself can help our understanding of this occurrence.

Let us consider the space of observed phenomena, Φ, and the space of conceived[a] mathematical structures, $\mathcal{M}$. The human effort of describing a phenomenon $\mathcal{I}\in\Phi$, where we can imagine $\mathcal{I}$ as a stream of events $\{\phi_i\}_{i\in I}$, can be understood in terms of finding elements $m\in\mathcal{M}$ such that all the connections between various events $\{\phi_i\}_{i\in I}$ are properly mapped into relations among the elements m; we'll label the elements of $\mathcal{M}$ useful to describe $\mathcal{I}$ as m_I. In this perspective, a **description** of reality is nothing but a **morphism** $\mathcal{D}$ between two spaces that preserves relations. This picture allows to understand why properties of abstract concepts derived logically may be fitting other phenomena: if elements of the mathematical structure m_I possess connections, previously unnoticed, to other elements of $\mathcal{M}$, say m_J, it may well be that they are the image of the map $\mathcal{D}$ of other phenomena ϕ_J, linked to ϕ_I in a similar, functional fashion as the mathematical properties m_I and m_J. In a more qualitative way, we can think of a situation where some kind of representation or *analogy* we made between two frameworks (the morphism $\mathcal{D}$) keeps holding while frames evolve.

But we can make a step further, moving one level up in a Category Theory view: let us consider Φ and $\mathcal{M}$ as categories, and let the elements of Φ be linked to each other through some functorial map $\mathcal{H}$, a kind of *causal functor*; think for simplicity to the time evolution operator in Quantum Mechanics (QM). So we write $\mathcal{H}(\mathcal{I}) = \mathcal{J}$ if happens that $\forall\, i\in\mathcal{I}\ \ \exists\, j\in\mathcal{J}:\ \mathcal{H}(\{\phi_i\})=\{\phi_j\}$ meaning that phenomena $\mathcal{I}$ and $\mathcal{J}$ are *physically dependent* in some way. Likewise, suppose the elements of the mathematical structure $\mathcal{M}$ are bounded together by some kind of *Linkage functor* $\mathcal{L}$, say logical derivation, conditional operator or something alike. Then, the map between Φ and $\mathcal{M}$, our description of reality, can be understood in terms of the high-level concept of *natural transformation* (NT): a family of morphism from Φ to $\mathcal{M}$ that associates to every phenomenon $\mathcal{I}$ a descriptive morphism $\mathcal{D}_I$ between functors, $\mathcal{D}: \mathcal{H}\Rightarrow\mathcal{L}$, such that:

$$\mathcal{D}_J\circ\mathcal{H}(\mathcal{I})=\mathcal{L}\circ\mathcal{D}_I(\mathcal{I})$$

that is, $\mathcal{D}$ enters in the vertical arrows of the following commutative diagram:

$$\begin{array}{ccc} m_I & \xrightarrow{L} & m_J \\ {\scriptstyle D_I}\uparrow & & \uparrow{\scriptstyle D_J} \\ \{\phi_i\}_{i\in I} & \xrightarrow{H} & \{\phi_j\}_{j\in J} \end{array}$$

[a] Considering instead '*all possible* mathematical structures' it would require scrutinizing what is meant by '*possible*'; one could e.g. restrict to computable structures, but at this qualitative level we deem more appropriate referring only to what is known at a given time.

meaning that the natural transformation $\mathcal{D}$ maps the empirical causal connections among phenomena to the logical derivation of mathematical concept needed to describe them. NTs preserve the relations of the objects that the functors are acting upon, either at the level of phenomena and of mathematical structures. Both these relations are functional ones, even though in different domains. In this perspective, the emerging of new effectiveness in 'old' mathematical tools appears very natural; actually, it would be more surprising that a mental set-up were suitable just for a specific phenomenon and losing its validity in a larger context.

In conclusion, looking to the relation between Reality and Mathematics with the eyes of Evolutionary Biology and hints from Category Theory, offers a satisfactory explanation about the interplay between them.

Nevertheless, the question of what really is the ontological status of mathematical concepts is definitely legitimate; indeed, it is so natural that it sprang already in the ancient times, when their effectiveness was still unexploited. Also, the great relevance of Math in the main (if not the only) reliable way of knowing the world – namely Science– does require an adequate epistemological collocation. We shall discuss these two points right away.

2.1 Math and Physics *au pair*?

Both issues are nicely framed in the Quine–Putnam[2] (Q-P) 'Indispensability Arguments':

P1. We ought to have ontological commitment to all* and only** the entities that are indispensable to our best scientific theories.

P2. Mathematical entities are indispensable to our best scientific theories.

C. then **we ought to have ontological commitment to mathematical entities**[3].

What '*indispensable'* really means, which mathematical structures are meant to be so, and to what extent they are supposed to be so, are questions largely debated. Someone[4] argues that the 'amount' of Mathematics really needed in Physics[b] is indeed very limited; others dare to claim that it would be possible for Science to get along without Math[5]. In any case, it seems that all the technical tools needed for scientific calculations are derivable in the framework of second-order arithmetic, or from the axiom system of the Zermelo-Fraenkel set theory. For our concerns, it is enough to acknowledge that the 'mathematical Universe' contains objects and models so peculiar that they could not be instantiated in any way, ***therefore it has contents distinct from the physical Universe***.
Premise P1 stands upon two implicit general assumptions:

1. the Scientific Method is the only** way to attain trustworthy knowledge of reality (namely, Naturalism);
2. if a theory gets confirmed, one is compelled to hold as confirmed any* of its elements and implications (the so called 'Confirmational Holism').

About point 2, one may object[6] that theories are meant 'just to work', without necessary 'be true' in any deeper sense. Plus, one should also stick to the 'pessimistic meta-induction': all theories happened to be provisional, so we may expect to see each of them overturned sooner or later. This is definitely not a good credential of 'reality' for the corresponding mathematical objects.
These considerations seem ***drawing apart reality and Math.***
Penelope Maddy nicely recasts the Q-P arguments, emphasizing the parity of the groundings of physical and mathematical entities:

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[b] Since Physics is the discipline making the heaviest use of Math, it is enough to consider just it.

A. We have good reason to believe our best scientific theories, and mathematical entities are indispensable to those theories, so we have good reason to believe in mathematical entities. **Mathematics is thus on an ontological par with natural science**.
B. Furthermore, the evidence that confirms scientific theories also confirms the required mathematics, so **Mathematics and science are on an epistemological par as well.**[7]

This formulation appears weaker, since being "on an ontological and epistemological par" seems not implying the Q-P "ontological commitment': having common *legitimations* does not make natural science and Math converging to the same *essence*.

Also, the distinction between *ontological* and *epistemic* confirmations (addressing *existence of something,* either logically or physically, versus *knowing* something) is possible only if one accepts a priori a ways of knowing *different from the empirical one.* But it is not clear how a sound ontological legitimation could ever stand without valid empirical findings at all[c]. In short, **the difference between a '*good reason to believe*' and an '*evidence that confirms*' appears fading**.
But let us see what each of these pairing implies.

2.2 What's on the market

Generally speaking, the 'epistemological par' is endorsed by those philosophical positions, such as e.g. *conventionalism* and *quasi-empiricism,* that see Mathematics as no more than an empirical instrument to intend the world. 'Ontological par', on the other hand is a bit harder to swallow. It can be brought for in two ways: the first one, basically the ***mathematical realism***, consists essentially in promoting the existence of mathematical entities, up to the level of real objects. Early Platonism and its developments constitute the core of realistic approaches[d]. The second way, on the contrary, explores the possibility of voiding the physical reality of any subsistence *different from the one possessed by abstract concepts*. This corresponds to the other main school of though emerged through time, i.e. ***nominalism***.

Over-simplifying, the tension is between admitting that e.g. the 'number 2' exists as an electron does or, otherwise, claiming that an electron, ultimately, is just an abstract entity. Put in these terms, both alternatives seem untenable. However, after having dug into each one, we shall see how they can converge into a compelling framework, at one with different experimental and philosophical indications.

2.3 Heavens and Earth

Nominalist approaches[8] in general tend to deny an independent ontology to the mathematical objects. Math is conceived as a mere human, mental construct: a formal instrument, a conventional language functional to what is devoted to describe. Questions concerning its correspondence with reality are easily dismissed, referring them to the mental, linguistic and even social sphere. This vision can also take advantage of the findings of neurophysiology and linguistics in addressing the effectiveness of Mathematics, talking out its relation with functional abilities of the human brain.

[c] In fact, admitting the possibility of *having believes with no evidences of any kind* means stepping out from Reason, so it would be hard to make any progress.
[d] Even though Plato was putting archetypal ideas beyond the heaven, in the Hyperuranion, i.e. not an ordinary, physical 'place'. Thus Plato, claiming the existence of abstract concepts, had to postulate a distinct kind of existence for them, in an exotic 'reality'.

Alternatively, one could be oriented towards realistic positions, especially considering the apparent unavoidability of Mathematical truths, which seems to *make them real*. In fact, *they seem more **necessary** that the physical reality itself*, which on the contrary, appears deeply ***contingent***. A Universe completely different is easier to imagine than a mathematical landscape totally disconnected from the one we know; we can easily conceive even having no Universe at all, but barely finger out how 2+2 could differ from 4. The hard task for the mathematical realism though is to explain how mathematical entities could 'exist', and having causal impact on reality, without being physical. A possible way out could be forcing their stance of 'being real', *equating the physical reality to a mathematical structure*. That is the core of the '**Mathematical Universe Hypothesis**'[9] (MUH) of Max Tegmark; a paradigmatic framework so general than it can be thought as a Theory of Everything (ToE).

Ambitious goals require strong supports, i.e. verifiable predictions. One would be, in the Author's opinion, that humans will keep discerning mathematical structures into the physical world. This claim seems fallacious for two reasons. Firstly, it is not falsifiable: even if at some point humans will stop to describe the world through mathematical models, there would be no ways to be sure that this lingers on forever. Secondly, the argument seems methodologically misguided: the fact that natural phenomena are effectively described by abstract structures is exactly the kind of facts that MUH is meant to explain, rather than an evidence for it.

If the Universe is a purely mathematical structures, the incompleteness theorems ensure that there will be undecidable statements around; so we may wonder what they would represent in physical terms; perhaps, aspects of reality not addressable in mathematical terms.

In our opinion, some open issues about MUH are:

- What would be the role of the Tarski's undefinability theorem for a purely mathematical ToE? If no sufficiently powerful language is strongly-semantically-self-representational[e], how could the purely mathematical ToE be complete in physical sense (i.e. not needing external tools to explain 'everything')?
- What about the fact that sentient beings, supposed to be framed in a computable mathematical ToE, are able to conceive and handle abstract concepts far beyond the computability? If computability is meant to be the bound of instantiated mathematical structures, that should be logically impossible.
- What if we consider Leibniz's profound idea[10] that *if a theory is not substantially simpler than what it's meant to explain, it does not explain anything*, and we combine it with the evidence[11] of relatively simple **computationally irreducible** systems [12]? In terms of 'Leibniz's comprehension', the role of the ToE in describing those aspect of reality would be nothing but an abstract rephrasing of their behaviour.
- A purely mathematical ToE would need an 'external' validation anyway, unless we are ready to throw away the entire Scientific Method. Hence, a distinction between what a theory *is* and what *it refers to* is always implicitly assumed; thus, an ontological difference between the two domains is in place as well. That implies we will never be able to scientifically *verify* that reality *is* a mathematical structure.

Nonetheless, looking at reality as a mathematical structure has a number of interesting features.

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[e] meaning that the interpreted language contains predicates and function symbols defining *all* the semantic concepts inherent to itself.

First of all, it is a smooth way out to the dichotomy of either the need of a first cause, or an infinite chain of causes ('*regressus ad infinitum*'), because it grants the *necessity* possessed by mathematical structures to the physical world. Secondly, MUH offers a perfect understanding of the basic physical quantities, ascribing to them the **relational** nature of mathematical properties**.** That means ***making physical and ideal existence converge***.

In fact, we will see that carefully pondered, the concept of a genuinely intrinsic quality appears untenable, both theoretically and epistemologically.

3.1 Properties meant to be intrinsic

Let us consider the main ingredients of our best physical theories, such as particles and fields, and have a look to some fundamental properties that could be naively held as intrinsic: mass, spin, charge.

The problem of defining mass in General Relativity (GR) is complex; one has to deal with the gravitational field energy, and in general it is not always possible to single out its contribution to the total energy for all observers, meaning that only isolated systems can have a coordinate-independent mass. This actually led to different definitions of mass[13]. But the main point here is that is hard to build a quantity describing the mass of a system, specified only in terms of physical properties defined within the finite region containing it. Thus, mass appears to be hardly tractable as a local property. This reflects also what is held to be an attractive feature of GR and Grand Unified Theories in general, namely the Background Independence, another characteristic deeply *relational*[14]. In Quantum Field Theory we currently understand the mass of particles as generated by their *interaction* with another object, namely the Higgs field, via the spontaneous symmetry breaking mechanism. So again, mass is not intrinsically defined, but arises relationally.

The particles spin then is related to the collective behaviour of the system through the spin-statistics theorem. In QM, spin is best understood in terms of the exchange symmetries possessed by a multi–body system, i.e. possible symmetries of the overall wave function (something in turn, stated in terms of probability and hence informational status of external observers). Again, something intimately relational.

Finally, charge also does not appear anywhere near to an intrinsic property in the intuitive sense: each of the charges we currently know in Physics are associated with symmetries that systems hold *as a whole*: local actions performed on them may exhibit differential symmetries such that a conserved current can be built.

Eventually, if one can safely agree upon the relational nature of properties we deem fundamental, and all other physical features are meant to be emergent, what else is left to be genuinely intrinsic?

These considerations indicate that the concept of 'intrinsic' is misleading, and suggest that ***the idea of independent properties should to be totally dismissed***.

3.2 Gauging the world

The act of *attributing properties* obviously requires observations, measurements, or equivalent operations in order to address them; thus all non-trivial issues entailed in such processes step in. When properties to be addressed are the fundamental ones, QM is in play; when dealing with ordinary 'macroscopic' qualities, subjective and/or epistemological concerns may arise. But in both cases, the issues come from the process meant to assess them.

The first, basic tool would 'direct observation'[15]. If this process is meant to be of sensorial nature, nothing could be more subjective and floating. If, otherwise, one has in mind the founding process of the scientific endeavour, namely measurements, one is necessarily dealing with Sentient Agents (SAs) performing it. Their interplay with the observed system is notoriously a critical aspect of QM; whatever interpretation of QM is endorsed, one cannot rule out the presence of other systems with *relational interactions* with the target. Finally, one may then consider a large sample of 'measure-like' actions, collectively performed and processed, conveying independent assessments and merged into verified statements. Clearly, it is what Science is all about[16]: a world-wide effort devoted to gather data, perform analyses and build models, aiming to reach a suitable level of agreement and understanding, in order to work out sure statements. But what does really drive any assessment? Evidently, not some kind of universal protocol, nor the output of formal quantifications, nor any procedural algorithms. Instead, it is always the *evidence* of results, the *consensus* built about them, and the *expert elicitation* that ultimately ground any scientific achievement. All of that entails subjective instances, conveniently selected and filtered by collective experiences, by elicitation and counselling[f]. Ironically, it is through a plethora of *subjective* practices that humans cast what they deem 'objective'.

Ultimately, the process of attributing specifications to whatever entity ends up to be unavoidably related to the Agents observing and processing it. In fact, its very meaning[g] carries the remainder of an *awareness* that is built in someone's mind.

Let us stress here that 'subjective' does mean in any way 'arbitrary', 'moot' or even less 'representationally unreliable'; rather, it simply refers to processes necessarily entailing interactions with one or more subjects. 'Subjective' hence refers to assessments rooted into features and dynamics of other systems or SAs and, therefore, it is linked to the concept of 'relative' and 'relational'.

3.3 Please someone watch over the Moon!

At this point we should be ready to address the question of what it means for something *to exist* from a new perspective. If existence cannot be a barely intrinsic property too, in which sense can it be relational?

The obvious answer is that existence is a quality relationally attributed to entities by Sentient Agents, and it pertains *only* to their current status or informational content. We are forced to say that something exists *if an only if* a SA is aware of it. We are not denying physical reality of course, but simply stressing that there cannot be any other meaningful way to make sense of the basic ontological quality of 'existence'. Its very meaning is the universal property SAs attribute to entities when they establish a substantial relation of **acquaintance** with them. Without SAs around at all, committed to sanction existence, there is no way to ascertain any entity. Pleading existence does pertain to the cognitive domain.

It is not under consideration here if the ways through which one attains awareness of entities is adequate or effective, but only talking out the very meaning of 'to exist', showing that it is necessarily related to knowledge and awareness.

Are we endorsing idealism or, rather, some kinds of internal/indirect realism?[17] Not really; instead, we do bash *physical realism.* This concept is a bit overloaded, and no wonder that a universal and unambiguous formulation is missing[18]. However, in many

[f] constituting an epistemological basis *de facto* much wider than any individual, codified procedure of truth finding.

[g] etymologically, from the Latin verb *tribuere*, 'to assign', i.e. the act of giving something to someone else.

physicists' mind is it often framed as the claim that "the Moon *is there* even if nobody is watching it"[19]. But if '*is there*' is rephrased as '*it exists*', in the light of the previous discussion we shouldn't be so sure anymore. If not *any* SAs can validate existence, the Moon, as any other entity, would be out of reach of anyone's knowledge, as e.g. an astronomical object beyond the cosmological horizon, or *as a Mathematical structure still to be discovered*. In these situations there is no existence at all that can be dealt with, tautologically.

It may seem we are falling into the metaphysical realm, with nothing but speculation[20]. But as many times in history, especially in the XX century, Physics wears away room for pure speculation, jumping in with empirical facts. Indeed, even though our claim may appear radical, there are already strong experimental findings supporting them. The Kochen-Specker theorem[21] has permanently questioned the possibility of unambiguously assigning "properties" to a quantum system, and various works on Einstein-Podolsky-Rosen experiments[22] brought to similar conclusions; as Bernard d'Espagnat put it, "the doctrine that the world is made up of objects *whose existence is independent of human consciousness* turns out to be in conflict with quantum mechanics and with facts established by experiment"[23]. Also, Rovelli's Relational QM[24] (RQM) put forwards pretty strong evidences that an objective description of reality, independent from observers, is an untenable mindset. Giving up the pretension of an objective Universe made up by entities with intrinsic well-defined properties does not rule out the possibility of a complete description of reality; but every description is possible only through interactions, and inevitably pertains *only* to a specific system; it does change from one another, and eventually, also incompatible descriptions may stand up.

This perfectly fits Bohr's view that "it is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what *we can say* about nature."[25]

4. Some answers

Both physics and philosophy provide strong indication that *there is no way to make sense of 'existing in absolute sense'.* The quality of existence comes out to be, firstly, deeply *relational*, and secondly, *inseparably related to the awareness and knowledge one has about entities*. These features are shared by both physical and mathematical entities; that is where they fit together, at the point that one can ultimately say they exist *exactly in the same terms*!

This perspective requires a paradigm shift, but indeed it solves many quandaries.

For instance, in this light the question if **Mathematics is 'invented' or 'discovered'** really dispels: the difference thins down because the two expressions become synonymous. Both 'to invent' and 'to discover' now means 'to become aware', and it makes no sense to wonder if a previously unknown 'thing', either physical or conceptual, were 'existing'; in that sense, the unknown doesn't exist yet by definition.

It also sheds light on the conundrum of the **mind** itself: 'knowing the existence' of our own mind translates into a cognitive loop, since it means to be acquainted with it, and it seems logically impossible that the entity seizing knowledge were at the same time the entity that knowledge is about.

From the dynamics of quantum systems down to ordinary dice throws, computing **probabilities** means quantifying our ignorance about systems, i.e. the knowledge we already have about them. They depend on the information gathered and conveyed by experimenters: one can state in advance the probability for each outcome of a fair die throw *only if one already knows the die is fair*. Without that knowledge or assumption, one would have no other ways than arrange long trials and observe the distribution of the outcomes, and then set limits on its fairness.

Now, most attempts made in order to quantify **information** are built with probabilities as main ingredient[26]. Thus, if probability entails awareness and relational knowledge, evidently information also will[27]. The same holds also for **complexity** and **randomness,** since both are usually defined in term of information, or related to it. If we consider the implications of their relational nature, it follows that all these properties cannot be barely intrinsic. Information and complexity of a given system then are meaningful only relatively to other systems, and their amount must be related to the knowledge possessed by systems interacting or aware of it. ***This sorts out negatively the question if there can be a universal, absolute measure for them***. At most, we envisage that there can be maximal and minimal values of all the combinatorial, relational knowledge among a set of interacting systems, setting in turn the possible range for the complexity of a target system.

Eventually, our position provides suggestions also to the issue of **phenomena potentially not addressable in mathematical terms**. In fact, rephrasing it as 'if there could ever *exist* aspects of reality non describable abstractly', and sticking to the conscious, relational meaning of existence, we may conclude that it cannot be the case. In fact, in order to assert existence, awareness must be built, that essentially is a mental projection in abstract term. This is, in turn, a description made through modular elements (neuron action potentials) with semantic content, which is, ultimately, ontologically equivalent to a formal mathematical description.

But on this issue, our opinion remains uncertain for two reasons: Tarski's undefinability theorem prevents Math from granting the truth on its own content; this provides hence an instance of something that will always be no manageble in strict mathematical terms, namely the Math foundations themselves. Moreover, we must keep in mind the role of qualitative, intuituitively rooted arguments[28] put forth in asserting the validity of the grounding axioms or in choosing them, such as self evidence, richness, teleological desire to have specific results… all these options seem to hide something unavoidably trascending the explanatory power that the axioms enclose.

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