Math Tells Us What Works, Not What Is Real Or Why: A Study In Comparative Dimensional Physics

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Abstract:

Just how "unreasonably effective" is mathematics, anyway? First, why does math accomplish so much for us in (apparently, in practice at least) understanding the "real world" around us? Then, what are the limits of what it can accomplish? Can it tell us why there is anything substantial at all, and why the world is the way it is? Here I argue related theses. First, mathematics is good at judging relative consistency (such as, whether a given model universe will be able to satisfy conservation of mass-energy.) This is demonstrated through a novel argument explaining why our universe must have three large spatial dimensions, in terms of the self-consistency of electromagnetic relations. Then, I argue that math has no ability to explain its own effectiveness in a "real world," or to explain the particular foundational properties of our world. Furthermore, math cannot explain why some possible worlds have "concrete" or "real existence" and others do not. Indeed, the very idea there is such a distinction is surprisingly problematic and questionable, as argued by Max Tegmark and others. Finally: any such existential distinction would be inaccessible to computational (AI-theoretical) intelligences.

"I don't really know what is meant by someone who says that mathematical objects are abstract whereas donkeys, even other-worldly ones, are concrete." [1]

- David Lewis (1986), p. 111.

Although arguments continue about many issues in physics, almost everyone agrees that mathematics is supremely useful in characterizing or at least predicting so much of what happens (or statistically) in our universe. It is more debatable whether we should be surprised or perplexed by this "unreasonable effectiveness of mathematics" [2]. The question of why this is so, has challenged foundational thinkers. Perhaps the orderliness is the expression of the mind of God, whether that is taken literally or as a Platonic metaphor for the structure of foundational and generative reality. Perhaps it is just a brute fact of *whatever* underlies or embodies reality. (In both cases, by "reality" we mean something more than math, that math describes.) Here, we look at the promise and limitations of mathematics applied to our universe, using both foundational arguments and an example of comparison of "model" worlds that introduces a novel argument (in its scope) for why space is three-dimensional.

The major thesis is indeed: math can tell us what works, but not what is real or why it is real (in the purported sense of *transcending* mathematics, of course.) That is, math can analyze questions of *relative* consistency, such as: given a requirement for conservation of mass-energy in any model world with laws analogous to our own, which class of models will accomplish this? We explore this capability through a novel argument that worlds with laws like ours must have three large dimensions. (I have previously presented the case, and similar, in various fora and in my blog [3].) However, mathematics has no resources to tell us that class of "possible worlds" must therefore be embodied as something more "concrete" than the mathematical structures used to describe them. Indeed, math and logic cannot even characterize or explain such a distinction. Furthermore, a "mind" based on computations or otherwise characterizable through AI protocols or even *any* abstraction, is not able to model or imagine such a distinction (even if it acts as though it did.) This is due to the self-contained construction and reliance of such a "mind" on relations between abstractions (as will be elaborated later.)

That incapacity lends credence or at least respectability to notions such as modal realism and the mathematical universe hypothesis: the idea that there really is not any more in existence than the "Platonic realm" of mathematical structures and their kin. That is, the idea and even experience of an existentially special "concrete world" is – fashionably – an illusion. However, one can argue from the other end and accept that our experiences of being more than abstractions are valid. Hence, the world and our minds as well, cannot be fully modeled by mathematics of any kind.

Why is Space Three-dimensional?

Before delving into ultimate metaphysics, let's explore a question that is foundational yet subject to specific mathematical analysis. This can be approached through relative consistency, aside from whether or why any model world can also "really exist." The question of why space has three dimensions (at least, tangible ones) and one of time, has perplexed for centuries. This uncertainty continues despite great overall progress, suggestive hints, and interest in foundational explanations such as string theory. However, is it possible that most investigators are neglecting avenues actually simpler and easier to explore with models? Here we sketch out a comparison of model worlds to see if physics reasonably like ours can be extrapolated to spaces of any number of spatial dimensions. (Here we take one-dimensional time for granted, although other arrangements are possible.) This theoretical exploration considers electromagnetic interactions. Perhaps surprisingly, it doesn't *need* to consider quantum mechanics, particle physics, gravitational issues, or anthropic considerations at all. The results: *fully* consistent physics analogous to ours is only possible in a three-dimensional space.

First, some background. We seek to extrapolate known physical laws to be as similar as possible in realms with other numbers of large spatial dimensions *D*. This requires some judgment calls, but certain principles are long accepted as reasonable. First, the equivalent Coulomb law for static electric fields should follow:

$$\mathbf{E} = qr^{(1-D)}\,\hat{\mathrm{r}}\,. \tag{1}$$

The wave equation surprisingly does not provide a unique value of c when D is even, and other distortions instead in odd dimensions. Still, analogous influences cannot propagate instantaneously. We assume that SRT applies *per* a *limit* velocity c.

Also, it is necessary to have electromagnetic inertia, so to conserve mass-energy. The work either generated or needed by changing the separations between electric charges must be conserved. That energy is considered to be stored in the fields around the charges, but how does that affect the charges? In the case of bound charges, EMI manifests as a change in their *collective* inertia, equal to the mass equivalent of the work done. So, push like-sign charges closer and then bind them together. If you accelerate this collection, its effective total mass m will be greater *per* $\mathbf{f} = m\mathbf{a}$, than the summed effective masses of the charges by themselves. Likewise for a compressed spring. Of course, any individual charge would already have its own EMI, but its "mass" is effectively a given. (There has been much argument over how this applies to elementary particles, not fully resolved.) And so forth, for other configurations of charges. The EMI is also called the electromagnetic mass (or mass component) of the body, although the energy is considered to reside in the fields. Theories designed to show that *all* mass has an electromagnetic origin failed, but Higgs theory explains base mass in the same vein (*i.e.*, as ultimately interactive, not just a "given" of each particle.)

For the case of two tiny charges q_1 and q_2 that are at a given separation r from each other in a space of dimension D, integration of (1) from infinity to separation r yields the "collective" EMI equivalent mass m_{EM}:

$$m_{EM} = \text{EMI} = q_1 q_2\, r^{(2-D)}/(D-2)\, c^2 \qquad (2)$$

Again, other mass and the starting EMI of the c*onstituent* charges is treated as given. The integral for $D < 3$ diverges to infinity, which (obscurely!) rules out such spaces as viable.

What *directly causes* the change in inertia? Richard Feynman's discussion ([4], now available online) is interesting and readable. There are two effects involved. They are unrelated in kind yet somehow "conspire" to produce the correct EMI in a 3-D space. (There is still some controvery over these issues and the related radiative self-force, somewhat reminiscent of quantum interpretations yet obscure.) First, there is the effect of causal delay on the reception of fields "projected" to parts of the charge distribution, from other portions. For comparison: the mutual forces between co-moving inertial charges cancel out. When a charge system is accelerating, the field propagated from any part of the system won't reach other parts until they have accelerated an extra distance. That displacement increases the travel distance to portions that are forward relative to the acceleration. It decreases the distance to portions to the "rear" of other portions. In combination with radiative effects (acceleration inherently effects the fields emitted by a charge), this causes the net forces between charges to no longer cancel. This interaction thus leaves a net "self force" $\mathbf{f}_{\text{self}}$, which *seems promising* as the causal basis for EMI. If $\mathbf{f}_{\text{self}}$ opposes acceleration, it acts like increased mass m_{SF}, and *per* the converse, such that $\mathbf{f}_{\text{self}} = -m_{\text{SF}}\, \mathbf{a}$ in the rest frame. However, we are not done. What if $m_{\text{SF}} \neq m_{\text{EM}}$?

It has been known for decades that the direct change in net forces caused by altered **E** fields does *not* give the correct value for EMI! This discrepancy is sometimes known as "the 4/3 problem" (based on the case of a hollow spherical charge distribution), and has been a bone of controversy in physics since discovered. Usually, it is considered solved by an effect of stress in the material constraining the accelerating charges, famously referred to by Feynman as "rubber bands." Einstein introduced [5], and later he and others developed the idea of stress-altered values of momentum and energy for a moving body. He had a dramatic insight about the implications of forces applied to *extended* bodies in relativity theory, a subject now wrongly in neglect. Due to relativity of simultaneity, forces applied "at the same time" at separated points of an object in its rest frame, are not applied simultaneously in a frame in which the application points are separated along the line of motion. The significant consequences are missing from popular treatments of relativity, and inadequately developed in academic treatments. These stress-corrections are *crucial* to achieving correct EMI for accelerating configurations of charge.

Consider a rod of any sturdy elastic material of proper length L_0 in its rest frame. By that standard, we simultaneously initiate and maintain weak ("test") compressive forces f_e at the ends of the rod. In a frame in which the rod moves length-wise at v along y, the "rear" force was applied at a time $\Delta t = -\gamma L_0 v/c^2$, earlier than the opposite force at the rod's "front." Hence, from consistency and conservation rules, the rod must acquire an additional momentum $\mathbf{p}_{corr}$ and additional energy U_{corr}, compared to its unstressed state. (NB: These quantities are *in addition* to those constituted from simple velocity-transformation of the mass-energy of elastic compression itself!) For the stress-corrected values, we find $p_{corr} = f_e \Delta t = -\gamma f_e L_0 v/c^2$ along y, and $U_{corr} = f_e v \Delta t = -\gamma f_e L_0 v^2/c^2$.

This simplified treatment is for the rod as a whole, with sign of f_e negative for compression by convention. We can explore such issues in generality, like the peculiar lateral momentum vector (key to legitimate resolution of the "right-angle lever paradox") from shear stresses, through the stress-energy-momentum tensor (variously named and applied.) Some critics find the corrections to be distasteful, yet they can't be avoided or fairly replaced by contrivances such as "the asynchronous formulation" *etc*. There is excellent discussion of these issues in Reference [6].

There is a simple yet curiously obscure way to get clear insight into how all this works. The time derivative of $\mathbf{p}_{corr}$ under proper acceleration $\mathbf{a}_0$ creates *per* conservation requirements another kind of effective mass m_{corr}. In the simple case in the rest frame with acceleration parallel to $\mathbf{v}$:

$$m_{corr} = |\dot{\mathbf{p}}_{corr}|/a = -f_e L_0/c^2. \qquad (3)$$

Adding $m_{SF} + m_{corr}$ provides the correct final value of "apparent EMI" for our specific $D = 3$ physics. We will see this through a simple restricted case, applicable to any value of D. It *only* yields correct EMI when $D = 3$. First, we must find what the change in inertia should be in such spaces. Next, predict the effect of acceleration on the fields exchanged between charges.

Finally, we must apply the stress correction to the previous, to see whether the net effective mass matches adding the predicted EM inertia. Finding EMI in various spaces is straightforward. Radiation, however, is very tricky when $D \neq 3$. Yet we can bypass its complexities by considering only interactions along the line of acceleration. Finally, the stress correction is usefully a universal value independent of D.

Note: m_{corr} is consistent, as we expect from the equivalance principle, with a shift in force transmitted across changes in gravitational potential. This wrongly-obscure gravitational Doppler shift of effective mass during *mechanical* interaction (not to be confused with the "Nordtvedt effect") was explicitly described by K. Nordtvedt [7] and partially generalized by myself [8].

We will analyze the first stage of the problem in the manner of Griffiths and Szeto [9], who considered the case $D = 3$, and did not extrapolate or try to derive the correction from first principles as done here. (Note that in this context, "classical" means non-quantum but still using SRT.) So, consider a rod of length L with small charges q_1 and q_2 affixed respectively to each end, like a dumbbell. (For simplicity we won't *specify* proper values.) We are applying a force **f** such that the rod undergoes constant length-wise acceleration along coordinate y, at a low rate $a << c^2/L$ to ensure negligible proportional advance from acceleration during signal travel-time. So, during the time Δt it takes a signal at c to travel from q_1 at the "rear" of the rod to q_2 at the front, the rod accelerates forward from its emitting rest position by an additional, relatively small distance $s = a(\Delta t)^2/2$. This increases the separation of q_2 from q_1 by an amount $s = aL^2/2c^2$, since $\Delta t = L/c$. Since the projected (*aka* "retarded") field is parallel to the acceleration, there should be no radiative component in any space. Hence, we find the change in incident **E** at q_2 straightforwardly by multiplying s by the derivative of the static field value along y. This gives a change of the f_y on q_2, compared to the rest value, of $\Delta f = (1 - D)\, q_1 q_2\, r^{2-D} a / 2c^2$. The differential effect of q_2 on q_1 is the same vector, since q_1 accelerates *closer* to q_2 during the signal delay, but the applicable vector is of opposite sign.

The combination of these adjustments to mutually received **E**, leads to a total net self-force $\mathbf{f}_{\text{self}}$ on the rod, directed along y. We find $m_{\text{EM}} = -f_{self}/a$, sign reversed as noted, to yield:

$$m_{\text{EM}} = (D - 1) q_1 q_2 r^{2-D} / c^2. \qquad (4)$$

This is the infamous incorrect result. It is too much in $D = 3$ by a factor of two (not 4/3, because we have a different charge distribution.) So, we must add in the m_{corr}. The elastic forces are caused by basic Coulomb forces between q_1 and q_2, now neglecting higher-order corrections thereof. Like-sign charges cause tension, hence we change the sign after substituting $q_1 q_2$ etc. into (3). That yields the final result, showing the corrected, *effective* m_{EM} compared to the neutral equivalent:

$$m_{\text{EM}} = (D - 2) q_1 q_2 r^{2-D} / c^2 \qquad (5)$$

This is not a general result, it only applies to the simple case at hand. (Perhaps the difficulty of generalization to all configurations in all D, explains the reluctance to seek that.) However, it was not previously noted, that this result does allow us to at least tentatively rule out spaces of $D \neq 3$ from being truly analogous to ours. We do this by comparing this relation to (2) and seeing that there is only agreement when $D = 3$. Simply put: solve $D - 2 = 1/(D - 2)$. Elaboration is not needed to show that analogous other spaces *cannot* consistently express the correct EMI in their context. (I have found other ways to show inconsistencies in $D \neq 3$, not discussed here.)

The implications of this result are surely debatable. One point stands out: since the sort of relations that manifest EMI presumably require an actionable scale to work through, this constraint could explain why additional spatial dimensions – if there are any – need to be compactified. It also removes three-dimensionality from the list of features conditioned by anthropic requirements, since it is a necessity of basic consistency with the form of our laws. However, nothing in this prevents say, a classical-type 23-dimensional world from manifesting.

What Then of "Reality"?

This result should get attention for what it shows about self-consistency and the scope of possible worlds of a certain type, but our critical question here is: what does it tell us about "reality"? By reality, I mean what the person in the street means: "stuff" that actually "exists," not just mathematical models like the ones used to prove this point. Well? Actually, nothing about the preceding exercise tells us at all whether anything should exist at all, or whether "illegal" worlds can "concretely" exist but just break rules that apply to our world, etc. All we have accomplished, is to make the case that "a universe" cannot exist *if* it would be characterized by conjunction of the following traits: 1.) it has laws analogous to ours in the manner usually accepted and as utilized here, 2.) it will follow conservation of mass-energy in all cases, and 3.) it would have other than three large dimensions of space. Yet that is just a logical set-intersection, a tautology in effect. It's much like asking, "which numbers are divisible by seven, and between 34 and 71?" But all the other numbers are just as "real" (or none are genuinely real, as you prefer ...)

Indeed, "existence" beyond the ideals of mathematics has long been a mystery in philosophy. In what sense are material objects "more real" than a mathematical model that fully (?) describes them? If we can discuss and analyze the former in terms of the latter, what is extra? (Note: time mixes with space, they can be modeled together, and time's "flow" may be illusory – time does not force a distinction.) Yet we can't *describe* the presumptive existential difference as features of structure or behavior. It's not like a "real lion" has a different pattern of particles or hair, is more complicated *per se*, or acts differently; than an abstract lion (in principle, not to be confused with the quality of our models.) We rely on circular definitions. Sure, we can reference just about anything in statements – but our experiences inform the semantics of the referent. "Materially real" needs to be appreciated up front, like consciousness, then we can talk "about" it. But what does it mean? Are the mere forms of lions, however perfect, the kind of lions that "don't really exist"? What is this?

Some bold and able thinkers argue the astonishing hypothesis there actually is no meaningful distinction between "real things" and the forms of things. The mathematical universe hypothesis, is being promoted on behalf of physics by Max Tegmark [10]. It does not mean simulation by something concrete, nor relative mathematical transformations like the holographic principle. Often wrongly disconflated as "all mathematical forms must be physically manifested" (a principle of plenitude, not of identity), this refinement at its purest is mathematical monism: the idea that structures of math are all that exist. Tegmark postulates our universe and everything else literally consisting of the kind of mathematical model that physicists use to represent and simulate it. We could call our space-time, its content and its rules a "logical space." All this transcends even artful concepts of physicality such as bundle theory, which still existentially *privileges* the select properties. This is structuralism as ontology, not strategy. If "existence" is a precondition for having properties, then the Platonic solids *do* exist because we can describe them (more clearly, in fact, than material objects.) So: the ultimate map, better than any we devise, really *is* the territory.

MUH may seem absurd or intuitively wrong to most of us, but the problem is that math and logic ultimately constitute a self-referential universe, even when presumptively representing other things. They work only on their own ideals: structure not essence. Math and logic don't *have* the tools to reach beyond their realms and characterize the *status* of another existential level. Such systems can neither describe a more "substantial" realness *per se*, nor distinguish nor explain the *high*-level "accidents" of phenomenal existence – the apparent discriminatory actualizations of one (or more) mathematically-delineated possible worlds. Therefore, math cannot either describe what "concreteness" is, nor which if any model worlds should be manifested as "actual worlds." It cannot explain why any such transcendently "more real" world should be mathematically elegant or "simple," instead of messy and not effectively accessible *through* math.

All that mathematics can do for us, is make relative judgments about model worlds in terms of various internal criteria. It cannot answer existential questions, such as "why does anything exist (in the sense of being more than math)" or even if there is such a thing as "more." All that math knows and can tell us in effect, is about math. When we think it is telling us about "the world," we are just finding out about the model that we are using. If we are lucky, they match, and if not, they do not. Why "this model," is a mystery. Our world does not and cannot follow "from first principles" of any *logical* or mathematical sort. Also, misleading attempts at explanation such as "from nothing" are not helpful. Space-time continua are effectively "things." They have dimensions, laws, effective time for probabilities to manifest, and so on. True "nothing" is not like a stuff or realm at all, it is the timeless logical negation of there being anything at all (other than math, if we take math to be "real.")

One way to put it: processes fully representable by models deal with predicates, and in the model context. They identify and represent whether a computationally realizable or otherwise abstract lion-form is large or small or crouching or running etc. Yet features, rules etc. logically exhaust the model. They do not relate to possible grades of existential status pointing beyond the features as abstractly definable. Nor do various arguable "levels" of realness in math form an analogy or path from form to concreteness. A distinction that is not a predicate, like "existing" as presumed in the material sense, cannot itself be modeled. It has no features, it is structurally void; it is "essential." We must transcend math and logic to grasp this.

But wait, don't we intuitively "know" that Max Tegmark is surely wrong, and this around us is "really real" in a way beyond mathematics? Not so fast. That depends on how our minds work. If AI theory is true, our minds are "information processing systems" and no more than that. If so, then the answer is: no we don't. As I wrote in my previous Essay [11] for FQXi:

So how does that relate to how our minds work? Well, suppose that a mind really works like a computer. That means, all it does is describable in terms of math. If that is so, then its operations don't depend on whether MUH is true or not. All your brain could know is the abstractions represented by its computational activity. The same computations happen anyway in either a purely abstract sense, or in a world that is "made of math", as would happen in a special material world. Mathematics is defined and operates only on its own ideals, even though it seems to us to represent "realness" and not just pure form. Hence there would be no way for your brain to detect its substantive existence in a material world that was not just an abstract model world. Descartes was wrong because a mathematical brain in any of the countless structure worlds of the UE, would be able to "think" in the sense of having analytical processes. But I don't think a mere "math brain" could have real feelings: love, nausea, itches and pains, delight, experiences of pretty color sensations, and above all: the basic "sense" of being alive and real.

Hence, an abstractable, model-like type of "mind" cannot know whether MUH is true or false, or even legitimately understand the question. *If* you do truly feel and know that the world is more than math, then it is, indeed – and so is your mind. I do not *know* what that "more" is, if it is so. We are free to decide what intuitions to trust in our quest for the horizon of what we and the universe *are*, and why. Mathematics cannot tell us about anything more than itself.

References:

[1] David Lewis, O*n the Plurality of Worlds* (Oxford: Blackwell Publishers, 1986.)

[2] E. P. Wigner, "The unreasonable effectiveness of mathematics in the natural sciences," C*ommunications on Pure and Applied Mathematics*, **13**, 1-14 (1960.)

[3] Neil Bates, "Paradoxer," http://tyrannogenius.blogspot.com.

[4] California Institute of Technology, Michael A. Gottlieb, and Rudolf Pfeiffer; http://www.feynmanlectures.caltech.edu: http://www.feynmanlectures.caltech.edu/II_28.html.

[5] A. Einstein, *Annalen der Physik (ser. 4)*, **23**, 371–384, (1907).

[6] *The Attraction of Gravitation: New Studies in the History of General Relativity (Einstein Studies, Book 5),* John Earman, Michel Janssen, John D. Norton (Editors); (Birkhäuser 1993.)

[7] Kenneth Nordtvedt, "The equivalence principle and the question of weight," *Am. J. Phys*. **43**, 256, (1975).

[8] Neil Bates, "The paradox of the force accelerating an extended body in relativity theory," P*hysics Essays* **1**, 127 (1988).

[9] David J. Griffiths and Ellen W. Szeto, "Dumbbell model for the classical radiation reaction," *Am. J. Phys*. **46**, 244 (1978).

[10] Max Tegmark, *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality* (Knopf, New York, 2014.)

[11] Neil Bates, "Flashlights, mirrors, real brains and willpower: steering ourselves to steer our future" (http://fqxi.org/community/forum/topic/2119, posted 4/24/2014.)