Reductionist Doubts

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Abstract

According to reductionism, every complex phenomenon can and should be explained in terms of the simplest possible entities and mechanisms. The parts determine the whole. This approach has been an outstanding success in science, but this essay will point out ways in which it could nevertheless be giving us wrong ideas and holding back progress. For example, it may be impossible to understand key features of the universe such as its pervasive arrow of time and remarkably high degree of isotropy and homogeneity unless we study it holistically – as a true whole. A satisfactory interpretation of quantum mechanics is also likely to be profoundly holistic, involving the entire universe. The phenomenon of entanglement already hints at such a possibility.

1 Reductionism's Strengths and Weaknesses

"Nature does not begin with elements as we are obliged to begin with them. It is certainly fortunate that we can, from time to time, turn aside our eyes from the overpowering unity of the All and allow them to rest on individual details. But we should not omit ultimately to complete and correct our views by a thorough consideration of the things which for the time being we left out of account" (Ernst Mach, 1883).

To get an idea where reductionism's strong and weak points lie, let's go to its source in Newton's worldview: utterly simple laws that govern the motions of bodies in space and time.

The law of inertia is the most basic: a force-free body moves uniformly in a straight line forever. Things are almost as simple if bodies interact. When far apart, they move with near perfect inertial motions, but when closer they can, through forces like gravitation, begin to change each other's motions. If body A has a greater mass than body B, then A affects B more than B does A. With allowance for the masses, action and reaction are equal.

The behaviour of a large system of many bodies, in principle the whole universe, can, according to Newton, be entirely explained by the inherent tendency of each body to move inertially modified merely by the forces exerted by all the other bodies in the universe. Just as reductionism proposes, the whole truly is the sum of its parts. Or is it?

The weak spot in Newton's scheme is the very thing that makes it reductionist. The position and motion of individual bodies such as atoms, the simple entities, are defined relative to invisible space and time, the framework in which simple laws can be formulated. Mach [1] and others argued that in reality the position of any one object is defined relative to every other object in the universe. That is obviously a far more complicated approach and it is clearly holistic. Mach argued that nevertheless it could still reproduce all of Newton's successes because force-free bodies are observed to move rectilinearly relative to the stars, which suggests that, in their totality, they exert a powerful causal effect on individual objects. Newton could have mistaken this empirical fact as evidence for what he called absolute space.

It is easy to see how this could have led to seriously wrong ideas about the universe. Barely over 100 years ago, most scientists thought we lived in an 'island universe' of a few million stars (our Galaxy) with nothing but space outside it. According to Newton's laws, such an island universe could exist and have angular momentum, L, about an axis, making it oblate like the rapidly rotating Jupiter. This was to be expected.

However, a simple implementation of Mach's ideas [2] rules it out. A Machian island universe must have angular momentum *exactly equal to zero*: $L = 0$, even though subsystems of bodies in small regions of the Galaxy can behave as Newton's laws predict and have $L \neq 0$. It is merely necessary for the values of L for all the subsystems to add up to zero. Seeing Newton's laws confirmed in the solar system, which does have $L \neq 0$, astronomers had no reason to question any of the predictions of Newtonian theory, which in no way is capable of forcing an island universe to have $L = 0$.

But suppose astronomers had, on Machian grounds, been convinced around 1900 that the universe must have $L = 0$ and had found this was not so for the Galaxy. Far from giving up the Machian theory, they would have *predicted* that our Galaxy cannot be the entire universe. They would have predicted a universe with more matter whose angular momentum exactly balances the Galaxy's.

How does the universe we can now observe meet this expectation? Extraordinarily well. It is full of galaxies and clusters of galaxies. Each and every one manifestly possesses some angular momentum, but there is no indication that the individual angular momenta add up to give a significant non-zero angular momentum of the universe as a whole.

In fact, my example is too simplistic, being based on Newtonian theory and not its superior successor in the form of Einstein's general relativity. But I think it makes my point. A reductionist standpoint may be very misleading.

2 Einstein, Mach, and General Relativity

Mach's idea that Newton's invisible absolute space could be replaced by an effect of the entire universe had a deep influence on Einstein, who called the idea *Mach's principle* [3]. It was the single greatest stimulus to the creation of his wonderful theory of gravity, general relativity (GR).

If GR implements Mach's principle, reductionism will be challenged. Simple parts and their interactions will not determine the whole; the whole will determine the way the parts behave. In fact, we shall see that the whole to a considerable degree determines what we perceive as parts.

So does GR implement Mach's principle? This has been a matter of controversy ever since Einstein created GR, mainly because he set about implementing Mach's principle in an indirect manner that left its status in GR obscure [4]. In fact, his definition of Mach's principle [3] was inconsistent [4], p. 93. I have given what I believe is the correct definition of Mach's principle [5] and argued that if the universe is closed up in three dimensions like the earth's surface in two then GR does implement Mach's principle [5]. If the universe is spatially infinite, the answer is equivocal. It is Machian however far you can imagine, but infinity is unreachable, and one can never establish a complete sense in which the whole determines the part.

Section 1 showed how reductionism can mislead if we conceive the universe as a mere sum of its parts, but it did not suggest any need to change the conception of the parts, the individual stars, but only the way they interact. The Machian interpretation of general relativity is much more sophisticated and changes radically the way we conceive the parts and not just the way they interact. In this section I show how the standard representation of GR presents the universe as a sum of parts. In Sec. 3, I give the alternative Machian interpretation, in which GR appears much more holistic. In the final section, I discuss possible implications of a holistic quantum view of the universe.

By parts in GR I mean little (infinitesimal) regions of spacetime. Imagine a two-dimensional surface that, at each point, is smooth and curved in accordance with a definite local law: measurements of the curvature in the immediate vicinity of every point on the surface would confirm the law holds at that point. Any surface for which this is true may exist. This is a reductionist situation: the parts (infinitesimal elements of surface) and the local law they satisfy determine what wholes (extended two-dimensional surfaces) can exist.

In general relativity, in the simplest situation in which no matter is present, the infinitesimal elements have not two but four dimensions: one of time and three of space. Otherwise the situation is very similar to what I described. Infinitesimal regions of spacetime satisfy Einstein's famous field equation $R_{\mu\nu} = 0$, and his theory permits any spacetime in which this local law is satisfied everywhere. Presented in these terms, it is a great triumph of reductionism; the predictions of GR have been very well confirmed for subsystems of the universe (the solar system and binary pulsars) and are not in conflict with cosmological observations. But in fact these present numerous puzzles whose solution may well call for a truly holistic approach that restricts the solutions GR allows and puts them in a different perspective. I now turn to that.

3 Gravity, Angles, and Distances

In Sec. 2, I identified the 'parts' in GR as infinitesimal pieces of spacetime. Their structure is determined by the fundamental quantity in GR: the metric tensor $g_{\mu\nu}$. Being symmetric ($g_{\mu\nu} = g_{\nu\mu}$) it has ten independent components, corresponding to the four values the indices μ and ν can each take: 0 (for the time direction) and $1, 2, 3$ for the three spatial directions. Of the ten components, four merely reflect how the coordinate system has been chosen; only six count. One of them determines the four-dimensional volume, or *scale*, of the piece of spacetime, the others the *angles* between curves that meet in it. These are angles between directions in space and also between the time direction and a spatial direction.

Now Mach's attitude to physics was not only holistic but also that all theoretical concepts should stay as close as possible to directly observable quantities. Using grand philosophical terms, the gap between epistemology – what can be observed – and ontology – what is assumed to exist – should be as small as possible. Ideally, there should be no gap at all. From this perspective, Einstein's ontology as expressed through the metric tensor $g_{\mu\nu}$ can be questioned on two grounds.

First, time is taken to be just as real as space. But Mach argued that time cannot be a primary notion; it must be derived from motion. In the first of these fqxi essay competitions, I showed how Newton's notion of an independent absolute time can be derived from motion. In Einstein's theory, Newton's global absolute and nondynamical time is replaced by local proper time, which interacts with space. This is a major change, but time, like space, is still taken to be a primary ontological concept, not something to be derived from more basic concepts.

Second, there is a big difference between, on one hand, quantities with scale, like lengths and volumes, and, on the other, angles: the former cannot be specified without the human choice of some unit – say metres or yards. That is arbitrary. In contrast, an angle can be specified in dimensionless terms as a fraction of a radian, which itself is a dimensionless quantity.

Let me spell this out to underline the Machian aspect. Suppose yourself at the centre of a circle of radius r looking at two points on the circumference separated by r. The angle that you see between them is by definition one radian. Key here is that the angle is the same whatever the radius r. This brings out the difference between angles and lengths.

Now imagine yourself on a clear night at high altitude in a desert. What do you see? Thousands of stars studding the black sky, all with definite angles between them. It's beautiful. The scientific point is that you directly see the angles between the stars. But you cannot tell their distances.[1]

There are two possible reactions to the difference between dimensionful and inherently dimensionless quantities like angles. The standard one in science is to say that it is not a big deal; one must simply express everything in terms of ratios. Astronomers, for example, express interplanetary distances as ratios, taking the semi-major axis of the Earth's orbit as unit. Moreover, these distances have *dynamical* effect: Newton's gravitational force is inversely proportional to the square of the distance.

The alternative attitude to dimensionful quantities is to deny them any fundamental role. Is this feasible? I think so. Note first that, despite being defined through ratios of distances, angles can be truly said to belong to a point. The radius of the circle taken to define a radian can always be taken as small as one likes. In contrast, a distance is necessarily associated with two points, the ends of the considered interval. In the standard formulation of GR, angles and distances both have ontological – dynamical – status. Distances do because the curvature of spacetime plays a central role in GR, and curvature is defined by comparisons over distances even though they can be taken arbitrarily small.[2]

Curvature, and with it distance, is so deeply embedded in the conceptual and technical foundations of GR it would seem only a madman would try to deny it a deep dynamical significance. General relativity without curvature seems like Hamlet without the Prince of Denmark. Remarkably, a new formulation of GR, called *shape dynamics* [6, 7, 8], suggests that, in a well defined sense, it is only angles that count. Because it shows how reductionism could be misleading us, I will sketch how the significance of angles emerged.

Dirac [9] and Arnowitt, Deser, and Misner (ADM) [10] took the first step over 50 years ago by replacing Einstein's vision of spacetime as a four-dimensional 'block' universe by a picture in which a three-dimensional entity evolves, building up spacetime in the process. This is known as the ADM formalism. The hope, still held by many, is that the Dirac–ADM approach will one day lead, as its creators intended, to a quantum theory of gravity.

I need to say what the Dirac–ADM three-dimensional 'entity' is. It is a Riemannian three-geometry (named after the great mathematician Riemann, who introduced the concept). A two-geometry is easy to visualize: it is like the two-dimensional curved surface of the Earth, which closes up on itself. A closed geometry is needed to model a universe as a whole. A closed three-geometry is much harder to imagine, but is mathematically possible. In the ADM formalism angles and infinitesimal distances are, dynamically, on an equal footing.

However, a hint that angles might have priority emerged from subsequent work related to the technically important ADM initial-value problem. There is no need for me to give details except to say that without its solution it is simply impossible to make any practical use of GR to establish, for example, what happens when two black holes coalesce. Among other things, this work, by James York [11] with input from his student Niall Ó Murchadha [12], established the amount of information that is needed to determine a physically sensible solution of GR.

The required information can be expressed in terms of a 3-geometry and the rate at which it is changing. As with spacetime, at each point of a 3-geometry there is encoded angle and scale information, 2 and 1 numbers respectively (compared with $5+1$ for four-dimensional spacetime). The rates at which these are changing are also characterized by $2+1$ numbers. York showed that the purely angle part of this information, i.e., $2+2=4$ numbers at each point, is sufficient to solve the initial value problem and hence determine a complete spacetime that satisfies Einstein's equations.

This suggested that GR is a theory of the way angle data change and that distance data play no dynamical role. However, this possibility was never taken too seriously because York's technique seemed to violate a foundational principle of GR – that there is no distinguished definition of simultaneity in the universe. For this reason, York's work, which does single out a notion of simultaneity, has usually been regarded as a merely technical device, albeit very valuable, for solving problems but not settling issues of principle.

4 Shape Dynamics

To what extent does shape dynamics (SD), which I mentioned earlier and for which the above has been a preparation, change this?

Imagine a 'toy' model universe of just three particles in two dimensions. Picture them as dots on an infinite sheet of grid paper. Then two coordinates determine the position of each. The six coordinates together define a *Newtonian configuration*, the sheet playing the role of absolute space. But one can ignore the positions of the dots on the sheet and regard as real only the three distances between each particle pair. That defines a Machian *relative configuration*. Defined in its own intrinsic terms, it does not exist anywhere in space. The final step is to say only the shape of the triangle, defined by two of its angles, is real.

The progressive elimination of non-Machian data takes us from Newtonian configurations through relative configurations to *shapes*. Now comes a key fact.

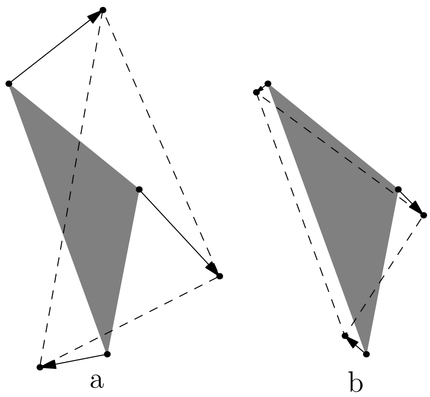

Figure 1. a) An arbitrary relative positioning of two triangles determines an initial Newtonian configuration (vertices of the grey triangle) and its rate of change (length of the arrows to the vertices of the dashed triangle). In the best-matched positioning b), shifting, rotating and scaling has brought the dashed triangle as close as possible to perfect overlap (congruence) with the grey one. An appropriate mathematical expression [5] measures the residual incongruence, the determination of which depends only on the shapes, not on their sizes or positions in space. The procedure is holistic, since the shapes alone determine the outcome, which is moreover expressed tractably through the coordinates of the vertices of the dashed triangle.

Mathematically it is vastly easier to describe change in terms of coordinates, at the level of Newtonian configurations. That is why Newton introduced absolute space – and with it reductionism. Because shapes are irreducibly holistic, it is much harder to work with them and achieve the gold standard of dynamical theory: the ability to determine the future from an initial state of the system. In Newton's dynamics, a configuration and its rate of change define an initial state. In shape dynamics, we want a shape and its rate of change to determine an initial state.[3] That's where the problem, the passage from reductionism to holism, lies. It is solved by *best matching*, which is explained in Fig. 1.

Once the shape-dynamic problem of defining initial states has been solved, it is easy to determine how shapes change according to a law that in no way relies on absolute duration, on position in an frame like grid paper, or on an external scale that measures size. The idea can be applied very generally, in particular to Riemannian geometries from which all information except that relating to angles between curves has been abstracted. Then we are dealing with *shapes of closed geometries*, just as York did, but now with some differences.

First, SD has led to an improved understanding of York's technique. This is a technical matter that I will relegate to an end note[4] for experts.

Second, and relevant for us, York did not solve the initial-value problem following any clearly formulated first principles but by exploring different mathematical techniques until he found one that worked, supremely well in fact. In contrast, SD derives not only Einstein's equations but also York's method for solving the initial-value problem on the basis of clearly formulated Machian first principles, of which there is a 'holy trinity': duration, position, and size must all be relative. The first two of these were foundational principles of Einstein's original derivation, but the third was not. In the shape-dynamic reformulation of GR, Einstein's relativity of simultaneity is traded for relativity of size.

The generalization of best matching to 'shapes of geometries' is direct but so far reaching it is worth outlining. One starts from $2 + 2$ given pieces of angle information at each point and considers how one could add to them in all possible ways $1 + 1$ pieces of 'trial' information that specify distances and their rates of change. The extra $1+1$ pieces of information are then varied in all possible ways until best matching is achieved. The mathematics is sophisticated but conceptually fully analogous to adjusting the dashed triangle to the best-matched positioning in Fig. 1. As in that case, there is always just one unique way, for given $2+2$ angle information, in which the best-matching condition can be satisfied. One then has an initial state that determines the future uniquely.[5]

Prior to the best matching, there is *nothing* in the specification of the initial $2 + 2$ pieces of angle information at each point of space that determines the distance between nearby points, the rate at which time flows at each space point, or the path of an inertially moving body. What I find really remarkable is that best matching applied to the distance-free $2 + 2$ pure angle information leads to complete determination of distances, rates of time flow, and inertial motion everywhere.[6] It is a triumph of Mach's intuition, going far beyond his original tentative suggestions.

Best matching is profoundly holistic. The procedure based on it that determines how the extra $1+1$ pieces of distance information are added at each point has to take into account the angle information *everywhere* – at all points. What is fixed *here* is determined by everything that is *there*.[7] There is a remarkable delicate interdependence of everything.

You may well ask: has anything been achieved, or is SD merely a different way of interpreting GR? The real test will be whether SD helps in the creation of a quantum theory of gravity. I shall consider that in Sec. 5. It is however noteworthy that GR allows a great number of solutions that seem physically most implausible; this has long been a concern. The ADM formalism, if regarded as the true form of the theory, already considerably restricted the allowed solutions. Shape dynamics goes significantly further. It requires the universe to be compatible with generation from York initial data and to be spatially closed. This what the truly holistic approach requires. In this respect, it runs counter to much current thinking in cosmology. It is always good for a theory to make bold predictions and 'live dangerously'.

The main aim of this section has been to show that a reductionist approach can give a seriously misleading intuition about 'what the world is made of' and how it 'works'. In certain key respects, the standard representation of GR remains close to Newton's world view. Time and space are fused together, but locally their ontological nature and inertial motion in them are barely changed. Distance in space and duration in time are both real and foundational. Shape dynamics suggests that only angles are real.[8]

5 Quantum Implications

Consider the archetypal quantum two-slit experiment in which, in Dirac's famous words, "each photon interferes with itself". This implies a particle that somehow splits in two and manages to 'go through' both slits at once. This obviously stretches the imagination.

Could a reductionist mindset be misleading us in trying to interpret quantum experiments in terms of particles moving in space and time? It's a very Newtonian picture. We see the Moon but never a photon. All we see is equipment and an event that we attribute to a photon. We see a photon-generating laser, a filter to ensure that only one photon at a time reaches the slits, and then the screen on which spots appear one after another.

The observed events are certainly discrete, but are we right to try to interpret them in terms of point-like particles? We do so because the experiment results in those spots on the screen or some other localized responses in apparatus. Of course, Bohr above all emphasized the complementary nature of quantum phenomena: particle-like or wavelike features are manifested depending on the macroscopic arrangement of the experiment. Change the arrangement and complementary aspects of reality are manifested. One can measure either momentum or position of a particle but never both at once.

This led Bohr to insist on the impossibility of formulating any meaningful statements about quantum phenomena without the framework provided by the classical, non-quantum world of macroscopic instruments. In this view quantum theory is not about 'what is' but about 'what we can say'.

But perhaps quantum theory *is* about 'what is' and the problem is that reductionism has led us to the wrong picture of 'what is'. Instead of thinking of particles in space and time, we should perhaps be thinking in terms of complete shapes of the universe. Then a photon would not be a particle that moves from one place to another but a law telling us how one shape of the universe becomes another. After all, that is what happens in the two-slit experiment. A spot appears on a screen that was not there before. In the first place, this tells us the shape of the universe has changed. Of course, it is only one change among an immense multitude, but it is a part of the totality of change. We may be doing violence to the universe – Mach's overpowering unity of the All – by supposing individual events are brought about by individual things, by parts. Hard as it may be, I think we need to conceptualize in terms of wholes.

The Machian interpretation of GR gives us hints of how this could change things. It suggests that what we observe locally has a part that is 'really here' – in the first place angle information – and a framework and basic laws that 'seem to be here' – local distances, times and inertial motion – but are in reality the effect of the complete universe.

In many ways, Bohr had a Machian mindset: the interpretation of quantum events depends crucially on the relationships that define the experimental layout. But where does that end and what determines it? Quantum entanglement measurements are now performed between the Canary Islands and the west coast of Africa and would be impossible without clocks that measure photon travel times. But if the Machian interpretation of GR is correct, the very distances over which travel times can be measured and even the times themselves are in a real sense created by the universe.

Thus, I think Bohr was on the right track but that one needs to go much further along it. Where will it take us? I don't know but will hazard a conjecture or two.

First, the quantum mechanics of the universe, like the Machian interpretation of GR, is holistic: it is about shapes. However, in contrast to shape dynamics, in which one shape follows another, in the quantum universe *all* shapes are present with different probabilities. But then whence comes our tremendously strong impression that time passes and we have a unique history?

In *The End of Time* [13], I attempted to answer this question with the notion of a *time capsule*: a very special configuration of the universe whose structure suggests that it is the outcome of a process that has taken place in time in accordance with definite laws. In [13] I still took distance to be real. The ideas that later led to shape dynamics were only briefly mentioned (in Box 3). I now distinguish carefully between relative configurations, in which both distances and angles are fundamental, and shapes defined by angles alone.

In [13] I conjectured a 'timeless quantum competition' between all possible configurations of the universe in which time capsules get the greatest quantum probabilities. We experience them as instants of time bearing every appearance of the transition from a definite past to an indefinite future. I justified this conjecture by the pronounced asymmetry of what I called *Platonia*, the collection of all possible relative configurations of the universe. But with distance removed from the foundations, we are left with shapes. Platonia is *shape space*.

In this view, the quantum universe is *doubly holistic*: the possible shapes of the universe are wholes, each a unity in itself, but there is also shape space, the higher whole formed by the totality of possible wholes. Shape space is the ultimate quantum arena.[9] That is my second conjecture.

Seen from a 'god's eye' perspective, the striking asymmetry of Platonia that I noted in [13] becomes even more pronounced and suggestive: in any space of shapes there is always one unique shape, appropriately called Alpha, that is *more uniform* than any other possible shape.

It interesting to note in this connection that the theory of cosmological inflation has some significant achievements to its credit but nevertheless must assume, as a hitherto unexplained initial condition, that the presently observed universe emerged from a state that was highly isotropic and homogeneous: in a word, very uniform. This suggests to me, as my third conjecture, that we should not be looking to explain the isotropy and homogeneity by a special initial condition but through the dominant status of Alpha in the universe's shape space. This is reflected in the fact that every shape has a degree of complexity that ranges from the most uniform possible to shapes of ever increasing complexity.[10] One never reaches Omega.

If there is no Omega in shape space, there must still be an end to this essay. Let it be my final conjecture, which sharpens the one in [13]. Our experiential life is dominated by time and a great asymmetry: the transition from past to future, from birth to death. Science has so far failed to explain the asymmetry and has been forced to attribute it to a special – *highly uniform* – initial state of the universe. I suspect we need look no further than the structure of shape space. Its highly uniform Alpha, which holistic shape dynamics tells us must be deeply significant, seems to match perfectly what we take to be the uniform initial state of the universe.

Mathematics, the tool of theoretical physics, can only describe becoming through differences of structures that simply are – they exist in a timeless Platonic realm. Becoming is the ever mysterious moving reflection of being. We cannot explain the indubitable asymmetry of becoming that we experience unless being is asymmetric. It is.

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Notes

[1]Even when distances are determined by parallax, it is changes of observed angles that determine the distances, which are themselves ratios of the trigonometric base length.

[2]Technically, curvature involves second derivatives of the metric tensor, whereas angles are defined by the undifferentiated metric tensor.

[3]I need to say something about what rate of change means in shape dynamics. It is not with respect to any external time but relates to the expansion of the universe, which cosmology indicates is a reality. In the context of shape dynamics, this introduces a single overall scale ratio: one can say the universe is twice as large now as it was in an earlier epoch. This scale ratio, which is why shape dynamics is based on volume-preserving conformal transformations, has nothing to do with angles but provides a single global parameter that defines the rate of change of the angles. Thus, the angles do not depend on time but on the scale ratio. I do find the need for a scale ratio mysterious. Perhaps it has to be there to provide a substitute for time.

[4]York based his technique, for a spatially closed universe, on full conformal transformations (which change local sizes but leave the angle information unchanged), whereas SD is based on volume-preserving conformal transformations. This is a tiny restriction, but it explains a bizarre feature of York's method that seemed completely *ad hoc*. I am referring to the scaling law that York adopted for the trace part of the momentum in the Lichnerowicz–York equation, which seemed incompatible with the law adopted for the trace-free part. SD provides a simple explanation for the law [6].

[5]My characterization of shape dynamics emphasizes its conceptual aspects. My collaborators Henrique Gomes, Sean Gryb, and Tim Koslowski also use the name shape dynamics for a specific gauge-theoretical implementation of the underlying ideas in a form that is likely to be best suited for technical developments.

[6]Soon after its completion, York's work suggested to Isenberg and Wheeler [14] a formulation of Mach's principle in somewhat similar terms but without a clear shape-dynamic underpinning.

[7]Best matching in the case of 'shapes of geometries' involves the solution of elliptic partial differential equations. As York and Ó Murchadha showed, these have very good existence and uniqueness properties. In the case of spatially closed universes, there are no boundary conditions to be specified; everything is determined intrinsically and holistically.

[8]I should add two caveats here. First, the discussion so far has ignored matter. That can be treated by York's method and hence added to shape dynamics without difficulty. Indeed it adds conceptual clarity to the picture. This is because matter fields define curves in space. The angles between any two curves are real (ontological), but distances along the curves are not; they are gauge. Second, all relativists would agree that a spacetime is determined by the specification of $2 + 2$ pieces of information at each point in a three-geometry. However, a majority would probably deny that these pieces of information must be exclusively and exhaustively angle data; they could be a mixture of angle and distance data. If relativity of simultaneity is taken to be sacrosanct, that is undeniable but it leaves one with a very indeterminate situation. What, if any, mixture of angle and distance data is correct? Three arguments speak for pure angle data: angles are conceptually more fundamental, the choice is clean (no indeterminate mixture), no other general and robust method apart from York's pure-angle method has been found to solve the initial-value problem.

[9]In the ADM approach to quantum gravity, the wave function of the universe for vacuum gravity is defined on superspace, the space of Riemannian three-geometries. The problem with this arena is the supreme difficulty, unresolved despite over 50 years of attempts, of implementing the ADM Hamiltonian constraint. I suspect that the root of the problem is the indeterminate mixture of angle and distance information encoded in superspace, which, as noted in the main text, is unavoidable as long as relativity of simultaneity (refoliation invariance) is held to be sacrosanct. In shape dynamics, the wave function of the universe is defined on conformal superspace. This brings undoubted conceptual clarity, as York already pointed out, but severe problems still remain. The ADM Hamiltonian may be conceptually hybrid but it is at least local; the shape-dynamic Hamiltonian is non-local.

[10]Complexity is a surprisingly difficult notion to capture in a definition. However, in the case of the Newtonian N-body problem, there is a uniquely appropriate measure of the complexity of an N-body system that I call the scale-invariant gravitational potential V_{si}. It is the product of the ordinary Newtonian potential and the square root of the centre-of-mass moment of inertia $I_{cm} := (1/M)\sum_{i<j} m_i m_j r_{ij}^2$, where r_{ij} is the distance between particles i and j and M is the total mass. It is a function on shape space and takes its maximum on the

most uniform shape the system can have. For large N, the corresponding distributions of the particles are remarkably uniform [15]. As the distributions become more clustered, V_{si} takes ever larger negative values. Given the close analogy between the Newtonian N-body problem when recast in Machian terms and GR, I anticipate the existence of a function on conformal superspace (the shape space for GR) that is analogous to V_{si}.