Two Principles of Quantum Gravity in the Condensed Matter Approach

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Abstract. In the absence of empirical testability, research on quantum gravity (QG) typically relies on appeals to guiding principles. This essay frames two such principles within the context of the condensed matter approach to QG. I first identify two distinct versions of this approach, and then consider the extent to which the principles of asymptotic safety and relative locality are supported by these versions. The general hope is that a focus on distinct versions of a single approach may provide insight into the conceptual and foundational significance of these principles.

1. Introduction.

In the absence of empirical testability, approaches to quantum gravity (QG) typically rely on appeals to guiding principles. This essay is concerned with the extent to which a particular approach to QG satisfies two such principles: asymptotic safety and relative locality. In senses to be made more precise in Sections 3 and 4 below, these principles state the following:

> Asymptotic Safety: *A theory of QG must scale towards an ultra-violet (UV) fixed point with a finite number of UV-irrelevant couplings.* (Weinberg 1979.)
>
> Relative Locality: *A theory of QG must entail that coincidence of events in spacetime ("locality") is relative to an observer's energy/momentum.* (Amelino-Camelia *et al*. 2011.)

The particular approach to QG I will consider is the condensed matter approach. I'll begin by identifying two distinct versions of this approach and then consider how these versions satisfy the above principles. The general hope is that a focus on different versions of a single approach to QG may provide insight into the nature of these principles. There are many other guiding principles in the literature on QG (duality, minimal length, holography, background independence, *etc*.). However the above two seem particularly relevant in the context of the condensed matter approach for the following reasons: First, being clear about the principle of asymptotic safety will require being clear about the notion of an effective field theory (EFT), which plays an essential role in the condensed matter approach. Moreover, in distinguishing between an EFT and an asymptotically safe theory, this principle raises the question, *What is a fundamental theory?* Second, being clear about the principle of relative locality will require being clear about the role that topological invariants play in both versions of the condensed matter approach, and how they relate to momentum space curvature. Moreover, this principle raises the question, *Is a state description of a physical system in terms of energy/momentum variables more fundamental than one in terms of spacetime variables?*, and so questions the fundamentality of spacetime.

2. The Condensed Matter Approach: Two Versions.

The goal of the condensed matter approach to QG is to construct a low-energy effective field theory (EFT) of a condensate that mimics general relativity (GR) and the Standard Model. This is an approach to QG insofar as the latter attempts to reconcile GR with quantum theory. The reconciliation here takes the form of a common origin in the condensate, the low-energy excitations of which take the form of the gauge, matter, and metric fields of GR and the Standard Model.

EFTs play a fundamental role in this approach. One way to understand the nature of an EFT is by means of the concept of a renormalization group transformation. This involves three steps: Given a "high-energy" theory encoded in an action,

$$S[\mathbf{g}] = \sum_a g_a \mathcal{O}_a \tag{1}$$

where the g_a are coupling constants and $\mathcal{O}_a$ are combinations of (derivatives of) field variables,

(i) Impose an energy cutoff $\Lambda(s) = s\Lambda_0$, where $s < 0$ and Λ_0 is a relevant energy scale.
(ii) Separate the field variables into low- and high-energy parts with respect to the cutoff, and integrate out the high-energy parts.
(iii) Absorb subsequent changes in the action into re-definitions of the couplings.

These steps define a map $R : \mathbf{g} \mapsto \mathbf{g}'$ in the abstract parameter space of the theory, and successive iterations of this map generate a flow. A fixed point $\mathbf{g}^*$ of a flow is a point that is invariant under further transformations: $R(\mathbf{g}^*) = \mathbf{g}^*$. Such a point encodes a theory with scale-invariant parameters.

These concepts allow one to distinguish three distinct notions of an EFT. The first is simply the scale-invariant theory of a fixed point:

$$S[\mathbf{g}^*] = \sum_a g^*_a \mathcal{O}_a. \tag{2}$$

Another notion of an EFT is a theory of a point $\mathbf{g}'$ on an RG flow that intersects a fixed point:

$$S[\mathbf{g}'] = \sum_a g'_a \mathcal{O}_a \tag{3}$$

where $R^n(\mathbf{g}') = \mathbf{g}^*$ (for some appropriate number n of iterations of the map R). A final notion of an EFT is a theory of a point $\mathbf{g}''$ in the neighborhood of a fixed point $\mathbf{g}^*$, but *not* on a flow that intersects $\mathbf{g}^*$. Such a theory can be formally approximated by small perturbations about the fixed point:

$$S[\mathbf{g}''] = S[\mathbf{g}^*] + \sum_a g''_a \mathcal{O}'_a \tag{4}$$

where the $\mathcal{O}'_a$ are in general distinct from the $\mathcal{O}_a$ that appear in (1), (2) and (3). Associated with the first two notions, (2) and (3), of an EFT is the concept of a universality class. This is an equivalence class of high-energy theories that all flow to the same fixed point (in other words, they all have the same low-energy/macroscopic behavior, but may have different high-energy/microscopic characteristics). In the condensed matter context, fixed points and universality classes are typically associated with spontaneously broken symmetries, and internal order is characterized by symmetry. Since the third notion (4) of an EFT is not directed related to a fixed point, these concepts do not (directly) apply to it.

These different notions of an EFT can be seen to inform two distinct versions of the condensed matter approach. The first version is associated with the first two notions of an EFT, and focuses on condensates characterized by spontaneously broken symmetries and universality. An example of this version is Volovik's (2003) EFT of superfluid helium 3-A, which belongs to the same universality class as the massless sector of the Standard Model above electroweak symmetry breaking. The essential features of this version are that the EFT is characterized by universality, and the internal order of the condensate is characterized by symmetry.

A second version of the condensed matter approach implicitly adopts the third notion of an EFT. It focuses on condensates for which universality (defined in terms of a renormalization group flow) does not apply, and internal order is not characterized by symmetry. Such condensates are rather characterized by topological order. An example of this version is Zhang and Hu's (2001) EFT of the edge of a 4-dim fractional quantum Hall liquid, which describes (3+1)-dim zero-rest-mass fields. (The hope is that GR and the Standard Model can be reconstructed from such fields.)

3. Asymptotic Safety

I'd now like to consider the principle of asymptotic safety. Recall that this requires that a theory of QG must scale towards an ultra-violet (UV) fixed point with a finite number of UV-irrelevant couplings. A *UV fixed point* is a fixed point associated with the renormalization group parameter s going to infinity (*i.e.*, high-energies). In contrast, an *infra-red (IR) fixed point* is a fixed point associated with s going to 0 (*i.e.*, low-energies). An *irrelevant coupling with respect to a fixed point* is a coupling that *decreases* towards the fixed point. Thus a UV-irrelevant coupling decreases as s goes to infinity, whereas an IR-irrelevant coupling decreases as s goes to zero. In contrast, a *relevant coupling with respect to a fixed point* is a coupling that *increases* towards the fixed point. So a UV-relevant coupling increases as s goes to infinity, whereas an IR-relevant coupling increases as s goes to zero.

These distinctions allow one to characterize theories in the following way (see Table 1). A *renormalized theory* is associated with an IR fixed point $\mathbf{g}_{IR}*$ and possesses no IR-irrelevant, and a finite number of IR-relevant couplings; a *renormalizable theory* is associated with an IR fixed point and possesses a finite number of IR-irrelevant and IR-relevant couplings; and a *non-renormalizable theory* is associated with an IR fixed point and possesses an infinite number of IR-irrelevant and a finite number of IR-relevant couplings. Finally, Weinberg (1979) defined an *asymptotically safe theory* (AST) as a theory associated with a *UV* fixed point $\mathbf{g}_{UV}*$ and possessing a finite number of UV-irrelevant couplings and a (potentially) infinite number of UV-relevant couplings. An AST is essentially the UV mirror-image of a non-renormalizable theory.

Renormalized theory	$S[\mathbf{g}_{IR}*] = \sum_a g*_a \mathcal{O}_a$	no IR-irrelevant couplings finite # IR-relevant couplings
Renormalizable theory	$S[\mathbf{g}'] = \sum_a g'_a \mathcal{O}_a\,,\quad R^n(\mathbf{g}') = \mathbf{g}_{IR}*$	finite # IR-irrelevant couplings finite # IR-relevant couplings
Non-renormal-izable theory	$S[\mathbf{g}''] = S[\mathbf{g}_{IR}*] + \sum_a g''_a \mathcal{O}'_a$	infinite # IR-irrelevant couplings finite # IR-relevant couplings
Asymptotically safe theory	$S[\mathbf{g}'''] = S[\mathbf{g}_{UV}*] + \sum_a g'''_a \mathcal{O}''_a$	finite # UV-irrelevant couplings infinite # UV-relevant couplings

Table 1. Theory types.

An example of a non-renormalizable theory is GR formulated as a quantum field theory: it has an infinite number of IR-irrelevant couplings that supposedly blow up at high-energies. An example of an AST is quantum chromodynamics (QCD). The UV fixed point of QCD is the free theory: the strong force goes to zero at high energies (thus, not only is QCD asymptotically safe, it is also asymptotically *free*). Weinberg (1979) originally suggested that the formulation of GR as a quantum field theory might be another example of an AST. If it has a UV fixed point (not necessarily a free-theory fixed point), its IR-irrelevant couplings would be tamed, and the theory would be well-behaved at all scales. This suggestion has spawned a research programme that attempts to identify UV fixed points of GR, hence the associated guiding principle of asymptotic safety (see, *e.g.*, Percacci 2009).

An initial assessment of this principle in the context of the condensed matter approach might begin with the following claim:

The EFTs in both versions of the condensed matter approach should aspire to be ASTs.

This claim seems reasonable to the extent that both versions attempt to reproduce the QCD sector of the Standard Model (and, potentially, the GR sector of QG). On the other hand, this would seem to mean that the EFTs in both versions should aspire to be associated with two fixed points: An IR fixed point defined with respect to the "high-energy" theory of the condensate, and a UV fixed point associated with the QCD and GR sectors of QG.

One potential worry here is whether it's consistent to consider an EFT as an AST. Under Weinberg's interpretation, an AST is a fundamental theory to all orders, insofar as it is supposed to get the fundamental degrees of freedom right: If GR is an AST, then "...the appropriate degrees of freedom at all energies are the metric and matter fields..." (Weinberg 2009, pg. 17). An EFT, on the other hand, is typically not taken to be fundamental in this sense. It's typically interpreted as restricted to a given energy range, beyond which new physics is supposed to arise (or, minimally, beyond which one should remain agnostic). Indeed, under a literal interpretation of the condensed matter approach, the fundamental degrees of freedom are those of the condensate, and the degrees of freedom associated with GR and the Standard Model are simply low-energy approximations of the former.

On the other hand, the relation between an EFT and a high-energy theory need not be interpreted as one of approximation. If one can argue that an EFT is autonomous, in an appropriate sense, from its high-energy theory, one need not view the latter as fundamental and the former as less so. For instance, if one can describe the relation between an EFT and a high-energy theory as one of emergence (in some sense), then, at least conceptually, it may be consistent to claim that an AST can emerge in the form of an EFT of a fundamental condensate. Thus whether or not it's consistent to consider an EFT as an AST will depend, in particular, on how the relation between an EFT and a high-energy theory is cashed out.

As an example, suppose the relation between the third notion (4) of an EFT and a high-energy theory (1) can be characterized by the following properties:

(a) *Failure of law-like deducibility*. The phenomena described by an EFT are not deducible consequences of the laws of a high-energy theory.

(b) *Ontological distinctness*. The degrees of freedom of an EFT characterize physical systems that are ontologically distinct from physical systems characterized by the degrees of freedom of a high-energy theory.

(c) *Ontological dependence*. Physical systems described by an EFT are ontologically dependent on physical systems described by a high-energy theory.

Property (a) understands the laws of a theory encoded in an action to be its Euler-Lagrange equations of motion, and is thus suggested by the formal distinctions between the EFT (4) and the high-energy theory (1), and their corresponding Euler-Lagrange equations of motion. In the case of property (b), this suggests that the degrees of freedom of an EFT are dynamically distinct from those of a high-energy theory (in the sense of satisfying different dynamical laws); moreover, the former are typically encoded in field variables that are formally distinct from those that encode the latter; *i.e.*, different field variables, $\mathcal{O}'_a$, $\mathcal{O}_a$, appear respectively in the actions of an EFT (4) and a high-energy theory (1). On the other hand, the fact that the degrees of freedom of the former can be identified as the low-energy degrees of freedom of the latter suggests property (c): the physical systems described by an EFT do not completely "float free" of the physical systems described by a high-energy theory.

One way to connect these properties of the relation between an EFT (of type (4)) and a high-energy theory to a notion of emergence is to conceive of the latter as embodying both a notion of *novelty* (in the sense that emergent properties should not be deducible from fundamental properties), and a notion of *microphysicalism* (in the sense that the emergent system should ultimately be composed of microphysical systems that comprise the fundamental system). One might then attempt to argue that properties (a) and (b) underwrite novelty, whereas property (c) underwrites microphysicalism (see, *e.g.*, Bain 2012).

Of course this attempt to flesh out a concept of emergence for EFTs is based on viewing the latter in terms of the third notion (4). More work needs to be done in assessing the feasibility of this view of emergence for (4), as well as the extent to which it applies to the notions of EFTs embodied in (2) and (3).

4. Relative Locality

I'd like to move on to the principle of relative locality. This requires that a theory of QG must entail that coincidence of events in spacetime is relative to an observer's energy/momentum (Amelino-Camelia *et al*. 2011a). The idea is that this is due to momentum space curvature. To understand this, consider the phase spaces of special and general relativity (see Table 2, after Amelino-Camelia *et al*. 2011b). The phase space of special relativity is given by the Cartesian product $\mathcal{M} \times \mathcal{P}$ of configuration space $\mathcal{M}$ and momentum space $\mathcal{P}$, where $\mathcal{M}$ is taken to be flat Minkowski spacetime. The phase space of GR is given by the cotangent bundle $T^*\mathcal{M}$ over $\mathcal{M}$, which is allowed to be a Lorentzian manifold with nontrivial curvature. In both cases, $\mathcal{P}$ is assumed to be flat. In contrast, the momentum space of a theory that satisfies relative locality is allowed to be curved, and its phase space is given by the cotangent bundle $T^*\mathcal{P}$ over $\mathcal{P}$. Thus in such a theory, there's a separate spacetime $\mathcal{M}_p$ for each point $p \in \mathcal{P}$. And if $\mathcal{P}$ is curved, then the $\mathcal{M}_p$'s will differ from point to point.

Γ = phase space (x^μ, p_μ)	$\mathcal{M}$ = configuration space (x^μ)	$\mathcal{P}$ = momentum space (p_μ)
$\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$	flat	flat
$\Gamma^{GR} = T^*\mathcal{M}$	curved	flat
$\Gamma^{RL} = T^*\mathcal{P}$	flat	curved

Table 2. Theories and their phase spaces.

One motivation for taking $\mathcal{P}$-space curvature seriously is that it entails non-commutativity of spacetime coordinates, and various approaches to QG employ non-commutative geometry. Moreover, some advocates of relative locality have suggested that $\mathcal{P}$-space curvature has observable effects that are measurable by current technology (Amelino-Camelia *et al*. 2011b).

How does relative locality relate to the condensed matter approach? According to its advocates,

> ...just as some condensed matter or fluid systems provide analogues for relativity and gravity, it may be that condensed matter systems with curved momentum spaces may give us analogues to the physics of relative locality. (Amelino-Camelia *et al.*, 2011, 084010-12.)

I'd now like to consider how this might be made a bit more precise in the context of the two versions of the condensed matter approach. It turns out that both versions encode aspects of their EFTs in aspects of $\mathcal{P}$-space *topology*, and these topological aspects can then be related to $\mathcal{P}$-space curvature. An example of such a relation is the Gauss-Bonnet-Chern theorem, which relates an aspect of the topology of a given parameter space to an aspect of its geometry:

$$2(1 - g) = 1/(2\pi) \int_S K\, dA \tag{5}$$

where the integral is over a surface S without boundary, K is the local curvature of S, and the integer g is the number of handles characterizing the topology of S (Avoron *et al*. 2003, 40). Intuitively, one can identify analogues of (5) in both versions of the condensed matter approach.

The first version fleshes this out in the following three steps (after Volovik 2003):

1. One first encodes low-energy dynamics in the form of a (single-particle, retarded or advanced) Green's function on $\mathcal{P}$-space:

   $$\mathcal{G}(p_0, \mathbf{p}) = [ip_0 - \mathcal{H}(\mathbf{p})]^{-1} \tag{6}$$

   where $\mathcal{H}(\mathbf{p})$ is the condensate Hamiltonian. For superfluid Helium 3-A, low-energy excitations correspond to poles in the Green's function, which are represented by points in $\mathcal{P}$-space (referred to as "Fermi points").

2. One then demonstrates that the low-energy dynamics is stable under perturbations. Mathematically, one can construct a topological invariant,

$$\mathcal{N} = (1/24\pi^2)\epsilon_{\mu\nu\lambda\gamma}\,\mathrm{Tr}\int_\Sigma dS^\gamma\, \mathcal{G}\partial_{p_\mu}\mathcal{G}^{-1}\mathcal{G}\partial_{p_\nu}\mathcal{G}^{-1}\mathcal{G}\partial_{p_\lambda}\mathcal{G}^{-1} \quad (7)$$

given by the integral in $\mathcal{P}$-space over a surface Σ surrounding the Fermi points, where the integrand depends on the Green's function and derivatives of its inverse (Volovik 2003, 97). This defines a nontrivial winding number of the map from Σ to the space of Green's function matrices. This means that $\mathcal{N}$ is invariant under continuous deformations of the Green's function. And this means that it's invariant under low-energy perturbations of the Hamiltonian, which means that $\mathcal{N}$ defines a fixed point/universality class.

3. Finally, one can relate the $\mathcal{P}$-space topological invariant $\mathcal{N}$ to $\mathcal{P}$-space curvature. Intuitively, $\mathcal{N}$ encodes topology, while the integral on the RHS of (7) encodes $\mathcal{P}$-space geometry (as in the Gauss-Bonnet-Chern theorem). More specifically, one can show that, in the case of the integer quantum Hall effect, the quantized Hall conductance is given by a topological invariant that can be obtained from $\mathcal{N}$ *via* dimensional reduction (Volovik 2003, pp. 136, 269). And it's been shown that the Hall conductance can be encoded in the adiabatic curvature of the relevant parameter space (Thouless *et al*. 1982). This suggests that the integral expression that defines $\mathcal{N}$ in (7) also encodes parameter space (*i.e.*, $\mathcal{P}$-space) curvature.

Similar steps can also be identified in the second version of the condensed matter approach:

1. One first encodes the *internal order* of the condensate in its ground state degeneracy (GSD). The condensate in this case is a fractional quantum Hall (FQH) liquid, and one can show that two distinctly ordered FQH states (given by distinct filling factors) can have the same symmetries but different GSD (Wenn 2004, 342). Thus the internal order of FQH states cannot be characterized by symmetry, but can be (partially) characterized by GSD.

2. One can then demonstrate that the GSD of FQH states depends on topology, and is robust under arbitrary perturbations, which indicates it's encoded in a topological invariant (Wen & Niu 1990, pg. 9378).

3. Finally, one can relate GSD to $\mathcal{P}$-space curvature in the following way (Wen 1990): FQH states can be classified by matrices K and described by an effective topological quantum field theory, where the determinant of the K matrix encodes the GSD of a given FQH liquid. Wen then showed that K can be encoded in the Berry phase characterizing adiabatic deformations of the FQH Hamiltonian, and the Berry phase is an ingredient in the definition of the adiabatic (*i.e.*, parameter space) curvature.

Thus, charitably, both versions of the condensed matter approach to QG may be said to satisfy the principle of relative locality, to the extent that both can be associated with curved momentum spaces. I'd now like to consider what, if any, questions of fundamentality this raises.

Advocates of relative locality suggest that it entails that descriptions of physical systems in terms of their energies/momenta are more fundamental than descriptions in terms of their spatiotemporal properties:

> Our most fundamental measurements are the energies and angles of the quanta we emit or absorb, and the times of those events. Judging by what we observe, we live in energy-momentum space, not in spacetime. (Amelino-Camelia *et al*. 2011, 1.)

> We do not live in spacetime. We live in Hilbert space, and the classical approximation to that is that we live in phase space. (Amelino-Camelia *et al*. 2011, 12.)

(where, charitably, perhaps we should allow that the terms "Hilbert space" and "phase space" in the second quote are intended to refer to momentum space.) Moreover, they also claim spacetime emerges from the dynamical interactions of particles in momentum space:

> We take the point of view that spacetime is an auxiliary concept which emerges when we seek to define dynamics in momentum space. (Amelino-Camelia *et al*. 2011, 5.)

These remarks suggest the following interpretation of the condensed matter approach: Reality consists of a fundamental condensate whose low-energy excitations constitute the phenomena described by GR and the Standard Model. Essential aspects of these phenomena are encoded in an appropriate curved momentum space. This entails that the momentum space ($\mathcal{P}$-space) state descriptions of these phenomena are more fundamental than their configuration space ($\mathcal{M}$-space) state descriptions; and, moreover, that the relativistic spacetime associated with these phenomena emerges from the dynamics of their $\mathcal{P}$-space state descriptions. Here are a few concerns one might wish to address to further elaborate this interpretation:

(a) First, the sense in which spacetime emerges from the phenomena associated with relative locality should be fleshed out in a bit more detail. The condensed matter approach suggests it might be cashed out in terms of the relation between an EFT and its high-energy theory, insofar as, in the condensed matter approach, the phenomena associated with relative locality are described by EFTs. Section 3 above offers one suggestion on how this might proceed.

(b) Second, the presence of the condensate may complicate the claim of advocates of relative locality that $\mathcal{P}$-space state descriptions are fundamental. Under a literal interpretation, it is the condensate that is fundamental, and not the phenomena associated with GR and the Standard Model. The latter are merely low-energy approximations of the former. The worry then is that it is only aspects of the "less fundamental" low-energy approximations that are associated with curved $\mathcal{P}$-space geometry; the fundamental "high-energy" theory of the condensate is not. This seems to suggest that the principle of relative locality does not apply to the condensate itself, but only to aspects of its low-energy excitations. Thus if the condensate represents fundamental reality, then perhaps $\mathcal{P}$-space state descriptions of reality are not fundamental. One way to address this potential problem would be to again take seriously the notion that an EFT emerges in a sense that makes it sufficiently autonomous from a high-energy theory to underwrite talk of the fundamentality of the EFT's $\mathcal{P}$-space state descriptions.

(c) Finally, a deeper concern is the following: We may grant that $\mathcal{P}$-space curvature entails $\mathcal{M}$-space state descriptions are relative. But it doesn't necessarily follow that $\mathcal{P}$-space state descriptions are more fundamental than $\mathcal{M}$-space state descriptions. That temporal intervals and spatial intervals are relative to an inertial reference frame in special relativity, whereas spatiotemporal intervals are not, does not, by itself, entail that time and space are less fundamental than spacetime in special relativity. In general, the distinction between an absolute property and a relative property doesn't necessarily map onto the distinction between a fundamental property and a derived property. To argue otherwise requires articulating metaphysical assumptions about the nature of these types of properties, assumptions that aren't necessarily worn on the sleeves of the theories in which they appear.

5. Conclusion

This essay has briefly looked at two principles of QG in the context of the condensed matter approach to quantum gravity. I've suggested that both versions of this approach should aspire to be asymptotically safe, but I've questioned whether an asymptotically safe theory can also be considered an EFT. A comprehensive answer will have to involve fleshing out interpretative options surrounding the relation between an EFT and a high-energy theory. I've also suggested that both versions satisfy relative locality, to the extent that they encode relevant quantities in momentum space topological invariants, and these invariants generate nontrivial momentum space curvature. But I've questioned the extent to which the ontological morals that advocates draw from relative locality apply in the condensed matter context. Again, further work needs to done in answering these questions on the relation between an EFT and a high-energy theory, and on the distinction between fundamental and derived properties on the one hand, and absolute and relative properties on the other.

References

Amelino-Camelia, G., L. Freidel, J. Kowalski-Gllikman, L. Smolin (2011a) 'Principle of Relative Locality', *Physical Review D, 84*, 084010.

Amelino-Camelia, G., L. Freidel, J. Kowalski-Glikman, L. Smolin (2011b) 'Relative Locality: A Deepening of the Relativity Principle', *General Relativity and Gravitation 43*, 2547.

Avron, J. D. Osadchy, and R. Seller (2003) 'A Topological Look at the Quantum Hall Effect', *Physics Today 56*, 38-42.

Bain, J. (2012) 'The Emergence of Spacetime in Condensed Matter Approaches to Quantum Gravity', *Studies in History and Philosophy of Modern Physics*, http://dx.doi.org/10.1016/j.shpsb.2012.05.001.

Percacci, R. (2009) 'Asymptotic Safety', in D. Oriti (ed.) *Approaches to Quantum Gravity*, Cambridge University Press, 111-128.

Thouless, D., M. Kohmoto, M. NIghtingale, M. den Nijs (1982) 'Quantized Hall conductance in a 2-dim periodic potential', *Phys Rev Lett 49*, 405.

Volovik, G. (2003) *The Universe in a Helium Droplet*, Oxford Univ. Press.

Weinberg, S. (2009) 'Effective Field Theory, Past and Future', arXiv:0908.1964v3 [hep-th].

Weinberg, S. (1979) 'Ultraviolet divergences in quantum theories of gravitation', in S. Hawking & W. Israel (eds.) *General Relativity: An Einstein Centenary Survey*, Cambridge Univ. Press, 790.

Wen, X.-G. (2004) *Quantum Field Theory of Many-body Systems*, Oxford Univ. Press.

Wen, X.-G. (1990) 'Topological orders in rigid states', *Int J Mod Phys B4*, 239.

Wen, X.-G. & Q. Niu (1990) 'Ground-state degeneracy of the fractional quantum Hall states in the presence of a random potential and on high-genus Riemann surfaces', *Phys Rev B41*, 9377.

Zhang, S. -C., and J. Hu (2001) 'A Four-Dimensional Generalization of the Quantum Hall Effect, *Science 294*, 823-828.