



# **Inverting the Phenomenology of Mathematical Lawfulness to Establish the Conditions of Intention-like Goal-oriented Behavior**

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## **Abstract**

To understand how physical and living systems wander towards a goal, we must first admit that mathematical laws describing such entities cannot be imposed in a top-down manner, but rather emerge from physical and energetic constraints. This can lead to goal-directed behavior in systems that have no brain. In fact, there is an entire class of systems that exhibit "brainless" or "aneural" cognition, which produce autonomous and even intentional behaviors. Yet even then, the intention-like behavior of a system does not emerge in a brute force fashion. Rather, the generation of quasi-intelligent behavior results from a set of processes involving function, structure, and emergent dynamics. To understand how statistical regularities become mathematical laws, we can turn to both a statistical mechanics-inspired discrete model of combinatorial representations and the Ashby's Law of Requisite Variety. Taken together, these systemic features provide a means to understand how mathematical laws become discoverable, and how systems generate features captured using a formal mathematical language.

## **Introduction**

This essay will explore the space between mathematical realism and anti-realism, while also providing a possible answer to the "mindlessness" of natural (not necessarily mathematical) laws [1]. We will also consider biologically-inspired models of physical systems, as well as consider intention-like goal-oriented behavior in the context of combinatorial representations. To set the mood, we must recognize that mathematical formulations are simply cultural constructs, and that lawlike behavior is something that is often beyond the scope of formal mathematical language. Statistical regularities, such as they are, serve only as a heuristic for building mathematical equations and lawful relationships. One consequence of this non-traditional view of mathematical laws is that I will sometimes present statistical regularities as laws, such as Fitts' Law and the Law of Requisite Variety. For lack of a better semantic framework, we can consider all such laws as the statistical regularities or invariant features of a system exhibiting intentionality and goal-directed behavior.

With a clearer philosophical view of the problem, our ultimate aim becomes a bit easier to define as well. Namely, how do the regularities and constraints of physical systems contribute to intention-like goal-oriented behavior. To answer this, we can turn to the literature on brainless/aneural cognition [2], which refer to systems exhibiting adaptive behaviors in the absence of a brain. In examples of brainless/aneural cognition, systems can produce intention-like goal-oriented behaviors without a direct cognitive mechanism. This may be critical to explaining seemingly intelligent behavior in many physical systems, and stands in contrast to physical



scenarios that require the generation of Boltzmann Brains [3] or other spontaneous hypothetical mechanisms.

### **Brainless/Aneural Cognition**

For a brainless/aneural system (or distributed collective of brains with no centralized control) to exhibit something resembling cognition, that system must meet a number of conditions. The first of these is autonomous behavior, which means that the entity in question must be able to generate and utilize energetic inputs in their own. A second attribute is that of interactivity, or the ability of the autonomous agents to interact with the environment [4]. Yet a third attribute is complexity, or the ability to generate a large number of possible behaviors. In some cases, the function of unicellular organisms (e.g. Amoeba) can solve well-established problems such as the two-armed bandit [5].

We will now focus on two model systems for demonstrating brainless/aneural cognitive systems, although there are many more forms such systems can take. The following examples demonstrate the key features of brainless/aneural cognition, in particular how the emergence, structure, and function of such systems result in the production of intention-like goal-oriented behavior. The first type of brainless/aneural goal-orientation is that of swarm-like collective behavior. Examples include insect swarms, bacterial colonies, bird flocks, and slime molds [6]. Collective behaviors are emergent, and do not necessarily operate according to formal mathematical laws. Yet they exhibit intention-like goal-oriented behaviors in a coordinated fashion. There are, however, central tendencies and other typical behaviors of these structures that are not present when the collective is not coherent. While autonomy combined with coordination is necessary, intentionality is not. In fact, in self-organized systems, agency can occur through simple feedback mechanisms [7].

We find a second type of brainless/aneural goal-orientation in passive dynamic walking (PDW) systems [8]. PDWs approximate the complexity of lower-limb movement of a bipedal organism in a passive fashion. In this case, passive refers to both limiting the amount of input energy, and removing neural control of limb movement. PDW principles demonstrate that intention-like goal-oriented and mathematically lawful behavior (in this case, stable bipedal walking) while generating life-like movements simply through physical constraints and inertia. Whereas collective behavior does not lend itself easily to formalized mathematical laws, the laws of motion govern PDW systems. Yet PDWs demonstrate intention-like goal-oriented behavior and coordination similar to that of collective behavior. In a sense, a mechanical system that approximate the lower limbs of a mammal are a collective system. The coordinated movement of both limbs occurs according to a finite set of rules and can interact in a coordinated fashion. Yet unlike the collective system example, the PDW demonstrates the role of physical constraint in determining the number of possible configurations the system can exhibit.

### **Role of Constraints**

Given these two examples, perhaps the common factor is not mathematical law but rather simple behaviors that work against physical constraints. In the process, this produces outcomes that approximate behavior consistent with statistical regularities of cognitive systems. As just one



example, let us take a special case of the PDW we will refer to as a passive dynamic reacher (PDR) and see if we can fit its operation to Fitts' Law [9]. Fitts' Law is a well-established psychomotor statistical regularity that predicts that a physical reaching mechanism will conform to a speed-accuracy tradeoff. While Fitts' Law usually applies to empirical characterizations of cognitive performance, performance in a brainless/animal system can also be described using Fitts' Law.

According to Fitts' Law, the size of a given goal (target location) determines the accuracy or speed of performance in a placement task. In the PDR, reaching to a goal depends on the limb geometry, the size of each limb segment, and the input energy. In this case, the constraints refer to places where the PDR cannot reach. This could be due to physically impossible configurations, or configurations that require implausible amounts of energetic input for a passive mover. The consequence of constraints on such a system are two-fold: they limit the variety of configurations in the system, and limit the number of different types of movements to a target that are energetically equivalent. This provides us with a functional tradeoff based on optimizing the variety of movements generated by the constrained system. According to Ashby's Law of Requisite Variety [10], the variety must be great enough to avoid catastrophic failure in the face of perturbation, but not so much as to prevent optimal states from being determined. While the relationship between Fitts' Law and the Law of Requisite Variety remains unexplored, they both demonstrate how regularities emerge from coordinated, intention-like goal-oriented systems not organized through a mind.

Are there similar laws one might assign to a group of agents exhibiting collective cognition? Lawful relationships exist in the crowd dynamics [11] and "wisdom of crowds" [12] literature. Where there are not formally proposed laws, however, there are regularities. These regularities serve as an organizing principle for higher-order goal directedness. One of the mathematical features that distinguishes PDR from collective dynamics is the ability to predict intention-like goal-oriented behavior as an inverse problem. In the PDR case, behaviors generation is reversible: we can determine behavior by variational diversity but can also be predicted from configurational diversity. By contrast, collective dynamics are harder if not impossible to reconstruct in a reversible manner. Nevertheless, collective dynamics can produce coordinated, goal-directed behaviors that might allow us to connect the phenomena of emergence and the Law of Requisite Variety. In this case, the Law of Requisite Variety allows for a number of equivalent configurational states, but restricts those states to a set of precursors that favor self-organization at multiple scales.

### **Discrete Model of Combinatorial Representations**

At this point, there are some points of clarification regarding non-standard terminology. Specifically, I present two terms not used in the standard way. However, their use in this essay allows us to address in a conceptually manner the emergence of spontaneous goal-driven behavior. The first of these is configurational diversity, and differs from the concept of configurational entropy in statistical mechanics. In this usage, configurational diversity represents all possible configurations of the system under analysis. We can further define configurational diversity as a measure of all unconstrained degrees of freedom in the system. This distinction is important, since depending on the system, constraints can play a significant role in shaping diversity. The second



non-standard term is variational, which also differs from its usage in statistical mechanics. In this case, variational refers to many equivalent solutions to the same problem. This draws from the idea of variation amongst the solution set, no matter what the criterion for optimization or selection.

This leads us to a quasi-Evolutionary discrete model of combinatorial representations. Briefly, there are three steps to this process. The first is to characterize all discrete pathways in the system under analysis. This could be via computational representation, or through directly characterizing the discrete degrees-of-freedom (e.g. the PDR example mentioned earlier). Secondly, we must characterize the configurational diversity of the system. This results in a possibility space for the system's operation. This also provides us with a bridge to second-order concepts such as the adjacent possible [13]. This leads us to variational diversity, which result from either functional or structural selection on configurational diversity. One example of this are the interactions of agents in a system of collective action. For example, a mathematical model of bird flocks and fish shoals shows that homogeneity of behaviors due to sensory constraints can yield coherent large-scale behaviors with respect to the individual's flock or shoal [14]. Yet, particularly in large-scale emergent systems, there might be a number of possible macro-behaviors even given homogeneity at the individual level. Interestingly, variational diversity is often missing from many mathematical laws, and part of the reason why such mathematical descriptions are often incomplete.

### **Limiting Variational Diversity**

The goal of the combinatorial representational process is to generate a phenomenological space or network that is continually constrained and expanded depending on the operational context. As configurational diversity refer to the number of possible and/or likely paths a system will take during dynamic behavior, variational diversity is but a subset of configurational diversity. It becomes easier for systems such as the PDW and collective dynamics examples to produce intention-like goal-oriented behaviors when constraints act in a systematic manner. One way this can occur is by limiting the degrees of freedom so that the number of possible configurations in a system is also limited over time. In the PDW example, limiting configurational diversity results in a deterministic manner, as the limb lengths, number of joints, and input energy provide strict limits on what is possible. For systems exhibiting collective dynamics, simple behaviors amongst individuals have a collective effect that can in part be determined through stochastic influences. In such cases, both spatial and temporal coordination and various constraints of the system (e.g. group size, individual interconnectedness) limit configurational diversity. In short, limiting variational diversity without destroying viability of the system itself is the key to extracting an intention-like goal-oriented outcome.

### **Games Against Nature**

Another way to state variational dynamics is to observe them in the context of a discrete system such as a game against nature. Games Against Nature [15] feature a single player that makes moves against a stochastic agent which shares the same strategy suite as the single player. In this scenario, the single player makes intentional, goal-directed moves against unknown countermoves. This is analogous to predictions a farmer might make against the weather, or a



gambler might make against a slot machine. On first take, one might assume that meeting one's goals in such a system is like hitting the lottery. However, by limiting variational diversity on the part of the player's strategy suite, particularly against the wide range of strategies exhibited in a stochastic signal, a game of nature has a much more predictable outcome. This favorable outcome for the autonomous player results from the emergent properties of the system in question.

In the game against nature, playing against a stochastic agent requires a set of strategies that maximize the advantage of being constrained to a series of regularities. This is much like the role of configurational diversity and variational diversity in the PDW and collective behavior examples. While this might seem to limit what the non-stochastic player can accomplish against an opponent with access to all possible configurations, it actually provides a means for taking advantage of randomness in a directed manner. In the PDW and collective behavior examples, constraints act at lower levels of organization in a deterministic manner. Joints can only move in a finite number of directions just as individual organisms can only exhibit a finite number of behaviors. These constraints also act to select for strategies that favor goal-directed behavior at the macro-level. As with the PDW and collective behavior examples, systems that play a game against nature are subject to the Law of Requisite Variety. There will be enough advantageous strategies to provide variety in dealing with the potentially disruptive effects of stochastic noise, but this set is limited due to the structural constraints of the system itself.

### **Variational Homogeneity and Brainless Cognition**

How do we get from training variational diversity to intention-like goal-oriented behavior? On its surface, this statement is somewhat of a paradox: to mimic the richness of cognition, we must restrain the size of the possibility space. This actually suggests that cognition is also a process of transforming many degrees of freedom into a constrained number of configurations to an even smaller number of intelligent responses. We do know how cognition takes advantage of heuristics such as chunking and affordances to support cognition, yet it is not clear how those attributes of cognition can be distilled into natural laws and ultimately be describable as mathematical statements. In the case of brainless cognition, we see this same secondary attribute of cognition, but on a limited scale.

### **Conclusion**

In conclusion, the main argument in this essay is one of emergent phenomena. When a system stumbles towards a goal, it does so without the direct confines of mathematical laws. Rather, mathematical laws emerge from the interactions of the systems itself. In the cybernetics literature, there is a notion of control as a mathematical abstraction of real-world processes [16]. While the locus of control is the Cybernetician's view is a bit unclear and abstract, we can begin to sketch out a process of obtaining mathematical laws from real-world systems.

Using the PDR as an example, we begin by defining both the available degrees of freedom and constraints. While this constrains the total universe of dynamical responses, the existence of adaptive behavior and feedback leads to the formation of small-scale control motifs. These might be as simple as a series of first-order feedbacks, but might also include nonlinear motifs such as cross-inhibition. This leads to a process of emergence from many small-scale control motifs to



large-scale control strategies. Once macro-scale adaptive behaviors become established, mathematical regularities become established. These (as well as repeated and well-coordinated small-scale motifs) become apparent as mathematical regularities and discoverable as invariant features of the system.

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