Is No Drama Quantum theory Possible?

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I. INTRODUCTION

Dear Reader,

Do you think it is possible to offer a "no drama" quantum theory? Something really simple? Something that does not need any new principles, does not turn philosophy upside down? You may tell me: "No way", go away from this essay and forget about it. Or, if you are a bit more lenient, you may ask me: "How simple is "really simple""? No new principles beyond what set of principles?

I have in mind something as simple (in principle) as classical electrodynamics - a local realistic theory described by a system of partial differential equations. Again, you may tell me: "No way", go away from this essay and forget about it. If, however, you are even more lenient, you may ask me: "How about the Bell theorem"?

Well, I have little new to say about the Bell theorem, and this essay is essentially not about the Bell theorem. However, I cannot "sweep it under the carpet" and will have to discuss it in more details. In doing so, I use other people's arguments. Most of them are outlined in the numerous posts by nightlight in physicsforums (see, e.g., Ref. [1]) and can be summarized as follows.

First, I agree with opponents of local realistic theories that the Bell inequalities cannot be violated in such theories. However, I don't believe these inequalities can be violated either in experiments or in quantum theory. First, in spite of anything that was written about experimental demonstration of such violations, there is a consensus among experts that all experiments so far were not free of some loopholes (detection loophole, locality loophole, etc.)Let me quote Abner Shimony and Anton Zeilinger, who are no fans of local realistic theories:

"The incompatibility of Local Realistic Theories with Quantum Mechanics permits adjudication by experiments, some of which are described here. Most of the dozens of experiments performed so far have favored Quantum Mechanics, but not decisively because of the detection loophole or the communication loophole. The latter has been nearly decisively blocked by a recent experiment and there is a good prospect for blocking the former. [2]"

"All recent experiments confirm the predictions of quantum mechanics. Yet, from a strictly logical point of view, they dont succeed in ruling out a local realistic explanation completely, because of two essential loopholes. The first loophole builds on the fact that all experiments so far detect only a small subset of all pairs created ... It is therefore necessary to assume that the pairs registered are a fair sample of all pairs emitted. In principle this could be wrong and once the apparatus is sufficiently refined the experimental observations will contradict quantum mechanics. [3]"

Second, to prove theoretically that the inequalities can be violated in quantum theory, one needs to use the projection postulate (loosely speaking, the postulate states that if some value of an observable is measured, the resulting state is an eigenstate of the relevant operator with the relevant eigenvalue). However, such postulate, strictly speaking, is in contradiction with the standard unitary evolution of the larger quantum system that includes the measured system and the measurement device (and the observer, if you wish), as such postulate introduces irreversibility, whereas there is no reversibility for the larger system, and, according to the quantum recurrence theorem, the larger system will return to a state that can be arbitrarily close to its initial, pre-measurement state. Therefore, mutually contradictory assumptions are required to prove the Bell theorem, so it is on shaky grounds both theoretically and experimentally.

In Dirac's "new electrodynamics" (Ref. [4]), the gauge condition of classical electrodynamics is chosen in such a way that the resulting equations of motion for the potentials of the electromagnetic field (the Maxwell equations) also describe motion of electrons in accordance with the Lorentz equations. Schrödinger (Ref. [5]) applied Dirac's approach to a quantum theory – (non-second-quantized) Klein-Gordon-Maxwell electrodynamics. He demonstrated that the equations obtained in the unitary gauge (Ref. [6],[7]), where the charged matter field is real, closely resemble those of "new electrodynamics". However, to the best of my knowledge, it was not noticed then or later that the equations of Schrödinger's work share another unique and important feature of "new electrodynamics": after natural and possibly obvious elimination of the matter field wave function, they also describe independent dynamics of electromagnetic field in the following sense: if components of the 4-potential of the electromagnetic field and their first derivatives with respect to time are known in the entire space at some time point, the values of their second derivatives with re-

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spect to time can be calculated for the same time point, so the Cauchy problem can be posed, and integration yields the 4-potential in the entire space-time. Thus, the broad range of quantum phenomena described by Klein-Gordon-Maxwell electrodynamics can be described in terms of electromagnetic field only. This unexpected result not only permits mathematical simplification, as the number of fields is reduced, but can also be useful for interpretation of quantum theory. For example, in the Bohm (de Broglie-Bohm) interpretation (Refs. [8–10]), the electromagnetic field can replace the wave function as the guiding field. This may make the interpretation more attractive, as "If one believes that the particles are real one must also believe the wavefunction is real because it determines the actual trajectories of the particles. This allows us to have a realist interpretation which solves the measurement problem, but the cost is to believe in a double ontology. [11]" Independent of the interpretation, quantum phenomena can be described in terms of electromagnetic field only.

Similar, but less general results are derived for the Dirac-Maxwell electrodynamics.

Then I show (using other people results) how the "one-particle" theories can be turned into "many-particle" theories, which look very much like quantum field theory, with little or no extra complications.

II. DIRAC'S "NEW ELECTRODYNAMICS"

This work heavily uses the results of Refs. [4, 5], so let us summarize and reformulate some of them here (a system of units where $c = \hbar = 1$ is used). In Ref. [4], Dirac considers the following conditions of stationary action for the free electromagnetic field Lagrangian subject to the constraint $A_\mu A^\mu = k^2$ (k is a constant):

$$\Box A_\mu - A^\nu_{,\nu\mu} = \lambda A_\mu, \tag{1}$$

where A^μ is the potential of the electromagnetic field, and $\lambda = \lambda(x)$ is a Lagrange multiplier. The constraint represents a nonlinear gauge condition. One can assume that the conserved current in the right-hand side of Eq. (1) is created by a distribution of particles of mass m, charge e, and 4-velocity

$$u^\mu = \frac{dx^\mu}{d\tau} = \frac{1}{\sqrt{1-\mathbf{v}^2}}(1, \mathbf{v}) = \zeta A^\mu, \tag{2}$$

where τ is the proper time of the particle ($(d\tau)^2 = dx^\mu dx_\mu$), $\mathbf{v} = (v^1, v^2, v^3)$ is 3-velocity, and ζ is a constant. If these particles move in accordance with the Lorentz equations (Ref. [12], §23)

$$\frac{du^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} u_\nu, \tag{3}$$

where $F^{\mu\nu} = A^{\nu,\mu} - A^{\mu,\nu}$ is the electromagnetic field, then

$$\frac{du^\mu}{d\tau} = u^{\mu,\nu}\frac{dx_\nu}{d\tau} = u_\nu u^{\mu,\nu} = \zeta^2 A_\nu A^{\mu,\nu}. \tag{4}$$

Due to the constraint, $A_\nu A^{\nu,\mu} = 0$, so

$$A_\nu A^{\mu,\nu} = -A_\nu F^{\mu\nu} = -\frac{1}{\zeta} F^{\mu\nu} u_\nu. \tag{5}$$

Therefore, Eqs. (3,4,5) are consistent if $\zeta = -\frac{e}{m}$, and then $u_\mu u^\mu = 1$ implies $k^2 = \frac{m^2}{e^2}$ (so far the discussion is limited to the case $-\frac{e}{m} A^0 = u^0 > 0$).

Thus, Eq. (1) with the gauge condition

$$A_\mu A^\mu = \frac{m^2}{e^2} \tag{6}$$

describes both independent dynamics of electromagnetic field and consistent motion of charged particles in accordance with the Lorentz equations. The words "independent dynamics" mean that the Cauchy problem can be posed: if values of the spatial components A^i of the potential ($i = 1, 2, 3$) and their first derivatives with respect to x^0, $\dot{A}^i$, are known in the entire space at some time point ($x^0 = \text{const}$), then A^0, $\dot{A}^0$ can be eliminated using Eq. (6), λ can be eliminated using Eq. (1) for $\mu = 0$ (the equation does not contain second derivatives with respect to x^0 for $\mu = 0$), and the second derivatives with respect to x^0, $\ddot{A}^i$, can be determined from Eq. (1) for $\mu = 1, 2, 3$.

III. ELIMINATION OF MATTER FIELD FROM KLEIN-GORDON-MAXWELL ELECTRODYNAMICS

In his comment on the Dirac's work, Schrödinger (Ref. [5]) considered interacting scalar charged field ψ and electromagnetic field $F^{\mu\nu}$ with the Lagrangian

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\psi^*_{,\mu} - ieA_\mu\psi^*)(\psi^{,\mu} + ieA^\mu\psi) - \\ -\frac{1}{2}m^2\psi^*\psi \tag{7}$$

and the Klein-Gordon-Maxwell equations of motion

$$(\partial^\mu + ieA^\mu)(\partial_\mu + ieA_\mu)\psi + m^2\psi = 0, \tag{8}$$

$$\Box A_\mu - A^\nu_{,\nu\mu} = j_\mu, \tag{9}$$

$$j_\mu = ie(\psi^*\psi_{,\mu} - \psi^*_{,\mu}\psi) - 2e^2 A_\mu \psi^*\psi. \tag{10}$$

For each solution A^μ, ψ of these equations there is a physically equivalent (i.e. coinciding with it up to a gauge transform) solution B^μ, φ, where φ is real. For real scalar field the equations of motions can be written in the following form (see also Ref. [6]):

$$\Box\varphi - (e^2 B^\mu B_\mu - m^2)\varphi = 0, \tag{11}$$

$$\Box B_\mu - B^\nu_{,\nu\mu} = j_\mu, \tag{12}$$

$$j_\mu = -2e^2 B_\mu \varphi^2. \quad (13)$$

Schrödinger emphasized two circumstances. Firstly, except for the missing constraint, the equations for the electromagnetic potentials coincide with Eq. (1) (if we replace B_μ with A_μ and $-2e^2\varphi^2$ with λ). Secondly, the fact that the scalar field can be made real by a change of gauge, although easy to understand, contradicts the widespread belief about charged fields requiring complex representation.

Obviously, the equations for B_μ and φ are not gauge invariant, as the gauge has already been fixed by the condition that φ is real – unitary gauge (Ref. [6],[7]). It should be noted that these equations can be obtained from the following Lagrangian (Ref. [13]):

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}e^2 B_\mu B^\mu \phi^2 + \frac{1}{2}(\varphi_{,\mu}\varphi^{,\mu} - m^2\varphi^2). \quad (14)$$

Actually, it coincides with the Lagrangian of Eq. (7) up to the replacement of the complex scalar field by a real one.

Rather surprisingly, Eqs. (11,12,13) also describe independent dynamics of electromagnetic field in the following sense (assuming φ and B^0 do not vanish identically): if components B^μ of the potential and their first derivatives with respect to x^0, $\dot{B}^\mu$, are known in the entire space at some time point ($x^0 = \text{const}$), Eqs. (11,12,13) yield the values of their second derivatives, $\ddot{B}^\mu$, for the same value of x^0, so integration yields B^μ for any value of x^0. Indeed, φ can be eliminated using Eq. (12) for $\mu = 0$, as this equation does not contain $\ddot{B}^\mu$ for this value of μ:

$$\varphi = \sqrt{(-2e^2 B_0)^{-1}(\Box B_0 - B^\nu_{,\nu 0})}. \quad (15)$$

Then $\ddot{B}^i$ ($i = 1, 2, 3$) can be determined by substitution of Eqs. (13,15) into Eq. (12) for $\mu = 1, 2, 3$. Conservation of current implies

$$0 = \partial_\mu(B^\mu\varphi^2) = (\partial_\mu B^\mu)\varphi^2 + 2B^\mu\varphi\partial_\mu\varphi, \quad (16)$$

or

$$0 = (\partial_\mu B^\mu)\varphi + 2B^\mu\partial_\mu\varphi = (\dot{B}^0 + B^i_{,i})\varphi + 2B^0\dot{\varphi} + 2B^i\varphi_{,i}$$

(Greek indices in the Einstein sum convention run from 0 to 3, and Latin indices run from 1 to 3). This equation determines $\dot{\varphi}$, as spatial derivatives of φ can be found from Eq. (15). Differentiation of this equation yields

$$0 = (\ddot{B}^0 + \dot{B}^i_{,i})\varphi + (\dot{B}^0 + B^i_{,i})\dot{\varphi} + \\ +2(\dot{B}^0\dot{\varphi} + B^0\ddot{\varphi} + \dot{B}^i\varphi_{,i} + B^i\dot{\varphi}_{,i}). \quad (17)$$

After substitution of φ from Eq. (15), $\dot{\varphi}$ from the previous equation, and $\ddot{\varphi}$ from Eq. (11) into Eq. (17), the latter equation determines $\ddot{B}^0$ as a function of B^μ, $\dot{B}^\mu$ and their spatial derivatives (again, spatial derivatives of φ and $\dot{\varphi}$ can be found from the expressions for φ and $\dot{\varphi}$ as functions of B^μ and $\dot{B}^\mu$). Thus, if B^μ and $\dot{B}^\mu$ are known in the entire space at a certain value of x^0, then $\ddot{B}^\mu$ can be calculated for the same x^0, so integration yields B^μ in the entire space-time. Therefore, we do have independent dynamics of electromagnetic field, although one cannot choose arbitrary values of B^μ and $\dot{B}^\mu$ at a certain time point as, for example, the argument of the square root in Eq. (15) must not be negative. However, there is no need to prove that the set of solutions of the relevant equations is broad enough, as it includes all solutions of the Klein-Gordon-Maxwell equations Eqs. (8,9,10) (up to a gauge transform). If φ or B^0 vanish identically, the dynamics of electromagnetic field is also independent but different. This indicates that the dynamics can be sensitive to arbitrarily small fields.

Apparently, it is possible to introduce a Lorentz-invariant Lagrangian with higher derivatives that does not include the matter field, but is largely equivalent to the Lagrangian of Eq. (14) (the significance of some special cases, e.g., $\varphi = 0$ and $B^\mu B_\mu = 0$ (see below)) is unclear. To this end, the latter Lagrangian can be expressed in terms of φ^2, rather than φ, using, e.g., the following:

$$\varphi_{,\mu}\varphi^{,\mu} = \frac{1}{4}\frac{(\varphi^2)_{,\mu}(\varphi^2)^{,\mu}}{\varphi^2}, \quad (18)$$

and then φ^2 can be replaced by the following expression obtained from the equations of motion Eqs. (12,13):

$$\varphi^2 = -\frac{1}{2e^2}\frac{B^\mu(\Box B_\mu - B^\nu_{,\nu\mu})}{B^\mu B_\mu}. \quad (19)$$

It should be noted that this article only deals with local, rather than global properties of the relevant equations, although global properties can also be physically important.

The result remains valid if conserved external currents are added to the right-hand side of the Maxwell equations (Eqs. (9). This is important for description of the hydrogen atom, the Aharonov-Bohm effect, and other quantum phenomena in terms of electromagnetic field only.

This result can also be relevant to interpretation of quantum theory. For example, it allows an interesting modification of the Bohm interpretation (Refs. [8–10]). Loosely speaking, in the Bohm interpretation (for one particle) the charged field represents an ensemble of point-like particles guided by the field and moving along the lines of current. For example, for the Klein-Gordon field, the current is defined by Eq. (10) or Eq. (13). The results of this work suggest that electromagnetic field, rather than the matter field wave function, can be regarded as the guiding field, and in each point the particles move along the potential B^μ.

It should be mentioned that there is some controversy about the Bohm interpretation of the Klein-Gordon field, in particular, because current may be spacelike for this field. For example, in Refs. [14, 15] it is contended that these difficulties do not lead to inconsistencies; a different definition of particle trajectories is given in Refs. [16, 17]

(see, however, Ref. [18]); there is also an opinion that bosons are fields, and they have no particle trajectories (Ref. [9]). This author focuses, however, on electrodynamics and does not consider any massive bosons, so the Klein-Gordon equation is regarded just as a reasonably decent approximation for electrons. Therefore, the inevitable next step would be to replace the Klein-Gordon field by the Dirac field. However, the latter has more components than the former, so the approach of this article yields less general results for Dirac-Maxwell electrodynamics (Ref. [19], pp. 4-5, Ref. [20]). The results for the Dirac field are presented in the next section. Other gauge theories (such as the Standard Model) have not been considered yet. As for second-quantized theories, nightlight indicated that such theories can be obtained from nonlinear partial differential equations by a generalization of the Caleman linearization procedure (Ref. [21]). This procedure generates for a system of nonlinear partial differential equations a system of linear equations in the Hilbert space, which looks like a second-quantized theory and is equivalent to the original nonlinear system on the set of solutions of the latter.

Let us consider a nonlinear differential equation in an (s+1)-dimensional space-time $\partial_t\boldsymbol{\xi}(x,t)=\boldsymbol{F}(\boldsymbol{\xi},D^\alpha\boldsymbol{\xi};x,t)$, $\boldsymbol{\xi}(x,0)=\boldsymbol{\xi}_0(x)$, where $\boldsymbol{\xi}:\mathbf{R}^s\times\mathbf{R}\rightarrow\mathbf{C}^k$, $D^\alpha\boldsymbol{\xi}=(D^{\alpha_1}\xi_1,\ldots,D^{\alpha_k}\xi_k)$, α_i are multiindices, $D^\beta=\partial^{|\beta|}/\partial x_1^{\beta_1}\ldots\partial x_s^{\beta_s}$, with $|\beta|=\sum_{i=1}^{s}\beta_i$, is a generalized derivative, $\boldsymbol{F}$ is analytic in $\boldsymbol{\xi}$, $D^\alpha\boldsymbol{\xi}$. It is also assumed that $\boldsymbol{\xi_0}$ and $\boldsymbol{\xi}$ are square integrable. Then Bose operators $\boldsymbol{a^\dagger(x)}=\left(a_1^\dagger(x),\ldots,a_k^\dagger(x)\right)$ and $\boldsymbol{a(x)}=(a_1(x),\ldots,a_k(x))$ are introduced with the canonical commutation relations:

$$\left[a_i(x),a_j^\dagger(x')\right]=\delta_{ij}\delta(x-x')I,$$
$$[a_i(x),a_j(x')]=\left[a_i^\dagger(x),a_j^\dagger(x')\right]=0, \tag{20}$$

where $x,x'\in\mathbf{R}^s$, $i,j=1,\ldots,k$. Normalized functional coherent states in the Fock space are defined as $|\boldsymbol{\xi}\rangle=\exp\left(-\frac{1}{2}\int d^sx|\boldsymbol{\xi}|^2\right)\exp\left(\int d^sx\boldsymbol{\xi}(x)\cdot\boldsymbol{a}^\dagger(x)\right)|\mathbf{0}\rangle$. They have the following property:

$$\boldsymbol{a}(x)|\boldsymbol{\xi}\rangle=\boldsymbol{\xi}(x)|\boldsymbol{\xi}\rangle, \tag{21}$$

. Then the following vectors in the Fock space can be introduced:

$$|\xi,t\rangle=\exp\left[\frac{1}{2}\left(\int d^sx|\boldsymbol{\xi}|^2-\int d^sx|\boldsymbol{\xi}_0|^2\right)\right]|\boldsymbol{\xi}\rangle$$
$$=\exp\left(-\frac{1}{2}\int d^sx|\boldsymbol{\xi}_0|^2\right)$$
$$\times\exp\left(\int d^sx\boldsymbol{\xi}(x)\cdot\boldsymbol{a}^\dagger(x)\right)|\mathbf{0}\rangle. \tag{22}$$

Differentiation of Eq. (22) with respect to time t yields, together with Eq. (21), a linear Schrödinger-likeevolution equation in the Fock space:

$$\frac{d}{dt}|\xi,t\rangle=M(t)|\xi,t\rangle,$$
$$|\xi,0\rangle=|\boldsymbol{\xi}_0\rangle, \tag{23}$$

where the boson "Hamiltonian" $M(t)=\int d^sx\boldsymbol{a}^\dagger(x)\cdot F(\boldsymbol{a}(x),D^\alpha\boldsymbol{a}(x))$

IV. APPLICATION TO DIRAC-MAXWELL ELECTRODYNAMICS

So far we have only considered solutions of well-established theories – the Klein-Gordon-Maxwell and Dirac-Maxwell electrodynamics, so in this respect we have been on firm ground, no matter how controversial their interpretation may be. Now let us consider the standard Lagrangian of the Dirac-Maxwell electrodynamics and impose the constraint $\bar{\Psi}\gamma^\mu\gamma^5\Psi=0$ (the axial current vanishes). A similar approach to imposition of the Majorana condition was used in Ref. [22], but the specific procedure there raises some doubts as the constraints of that work make no contribution to the equations of motion. In our case the equations of motion are as follows:

$$(i\partial\!\!\!/-eA\!\!\!/+D\!\!\!\!/\,\gamma^5-m)\Psi=0, \tag{24}$$

$$\Box A_\mu-A^\nu_{,\nu\mu}=j_\mu, \tag{25}$$

$$j_\mu=e\bar{\Psi}\gamma_\mu\Psi, \tag{26}$$

$$\bar{\Psi}\gamma^\mu\gamma^5\Psi=0, \tag{27}$$

where, e.g., $D\!\!\!\!/=D_\mu\gamma^\mu$ (the Feynman slash notation), and D_μ are the Lagrangian multipliers. Every solution of this system is physically equivalent to a Majorana solution related to it via a gauge transform: Eq. (27) implies that the spinor Ψ may be represented in the form $\Psi=\exp(i\theta)\Phi$, where $\theta=\theta(x)$ is real, and Φ is a spinor satisfying the Majorana condition. Substituting this in Eqs. (24,25,26), we obtain equations for Majorana spinors:

$$(i\partial\!\!\!/-eB\!\!\!\!/+D\!\!\!\!/\,\gamma^5-m)\Phi=0, \tag{28}$$

$$\Box B_\mu-B^\nu_{,\nu\mu}=j_\mu, \tag{29}$$

$$j_\mu=e\bar{\Phi}\gamma_\mu\Phi, \tag{30}$$

where $eB_\mu=eA_\mu+\theta_{,\mu}$. Treating Eq. (28) in the same way as the Dirac equation (Eq. (**??**)), we obtain:

$$(i\partial\!\!\!/+D\!\!\!\!/\,\gamma^5-m)\Phi=0, \tag{31}$$

$$B\!\!\!\!/\,\Phi=0. \tag{32}$$

Again, Eq. (32) implies $B_\mu B^\mu = 0$, if $\Phi \neq 0$; if the vector B^μ is not zero, the equation also implies that there exists such λ that $j^\mu = \lambda B^\mu$. Therefore, we obtain the following system of equations with Majorana spinors:

$$(i\partial\!\!\!/ + D\!\!\!\!/\,\gamma^5 - m)\Phi = 0, \tag{33}$$

$$B_\mu B^\mu = 0, \tag{34}$$

$$\lambda B^\mu = j^\mu = e\bar{\Phi}\gamma^\mu\Phi, \tag{35}$$

$$\Box B_\mu - B^\nu_{,\nu\mu} = \lambda B_\mu. \tag{36}$$

Eq. (33) is linear in D_μ, so it is easy to eliminate D_μ from it. Again, Eqs. (34,36) describe independent evolution of the electromagnetic field, and Eq. (35) allows one to determine the trajectories in the Bohm interpretation from the potential of the electromagnetic field. It remains to be seen whether Eqs. (33,34,35,36) are compatible with experimental data or they may only be used as an interesting toy model for interpretation of quantum mechanics.

V. CONCLUSION

So is a no drama quantum theory possible? Using a popular phrase, "I'll give you a definite maybe".

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