ABSOLUTE OR RELATIVE MOTION... OR SOMETHING ELSE?

Daniel Wagner Fonteles Alves

Universidade Federal do Ceará

dw-alves@hotmail.com

Abstract. Historically, motion has been conceived at the classical level either in a relationist or absolutist framework. It is argued that there may be other ways to conceive motion and that a systematic investigation of these different conceptions may produce new physics.

1. INTRODUCTION

The most striking challenge in contemporary physics is agreed by many as to be that of the theory of quantum gravity (QG), the unification of quantum mechanics (QM) and general relativity (GR). It is also the most longstanding problem ever: never in the history of physics it took so much to unify two incompatible theories [1] (and there is no irrefutable success yet). In a few words, there is at present no understanding of how the universe works: while general relativity tells us that space and time are modeled together as a 4-dimensional manifold with enough mathematical structure to conceive a generalized notion of curvature (and which explains gravitation), quantum physics was formulated using a classical background space and time. The successes of both theories are tremendous, particularly because we only have access to experiments in which one of them may be ignored. When we try to unify both theories in a single framework, the technical and conceptual problems are outstanding, and their origin has a lot to do with the different conceptions of space, time and motion in GR and QM. Finally, the scale at which quantum gravity effects are expected to be relevant, the Planck scale, is unreachable with current technology, and this is the main difficulty for the formulation of the unified theory. The absence of empirical evidence forces us to rely on *first principles* of what the theory should look like. Since some of the main conceptual divergences of GR and QM arise from the distinct conceptions of space, time and motion in both theories, it is reasonable that any questioning about those fundamental aspects is relevant for the debate on quantum gravity.

Throughout the centuries, motion was either considered from an *absolute or relational point of view*. The original dialectic goes like this: the relationists claim that since space and time are unobservable and all we can see are objects (which may be rulers and clocks), the description of motion should be built solely upon relative distances, that is, time and space should be derived concepts upon that of a physical object. The absolutists, on the other hand, consider time and space to exist, and see their invisibility as incidental. For if there is a relative distance between objects, where are they embedded? Even though modern physics has made much of this naïve discussion obsolete for a number of reasons, the debate is still important because *quantum mechanics uses a mathematical and conceptual formalism suitable for an absolutist picture while general relativity is almost perfect relational.* In this essay I will show, however, that there may be ways to conceive motion other than the absolute or relative pictures, and offer a possible qualitative alternative, although hints on how to make it rigorous will be discussed. That could have importance for the debate on QG.

2. THE ABSOLUTE X RELATIVE DEBATE

The mathematical and conceptual formalism of classical mechanics goes like this: the positions of objects that constitute a physical system and their evolution in time are expressed as a set of functions $q_i(t)$ which satisfy second order differential equations.

$$-(\nabla V)_i = m_i \ddot{q}_i(t)$$

Where V is a function that could in principle depend on (x,y,z,t).

But positions, by their own *definition*, are always expressed in relation to something and the $q_i(t)$ are expressed in relation to absolute space. But since absolute space cannot be observed, how could this theory be tested?

We could search and find a particular observable object which, when building the functions $q_i(t)$ in relation to it, the second order differential equations mentioned before would hold. This particular object chosen in this way is said to constitute an inertial frame of reference. It was shown in the 19^{th} century how to define an inertial frame of reference operationally. According to the differential equation itself applied to the inertial frame object, it must either be at rest or moving with constant velocity in relation to absolute space. So we could ''almost'' see absolute space, except that there would still be no operational way to discover absolute velocity and absolute position. It is a strange coincidence the fact that it turns out that no physical process actually depends on absolute position and absolute velocity. This is stated as the fact that classical mechanics is Galilean Invariant.

But there was a group of scientists who thought Newtonian absolute space and time were metaphysical excess and among them was the notorious and influent Ernst Mach. According to him, physics should deal with observable elements from the start, or there would be senseless terms at the heart of our theories. For example, if you look at the distant stars in the sky at night and start spinning in relation to the stars, you would feel the centrifugal force acting through your body. According to Newton, this is because you are accelerating through absolute space (as you can see by looking at the distant stars that constitute an inertial frame of reference). Now what would happen if the heavens started to rotate and you remained static in relation to the ground? Would you feel the centrifugal force? By following Newton´s rules that would not happen because you are no longer accelerating through absolute space and the distant stars have ceased to be a reliable inertial frame of reference. But for Mach, since any differences between the two situations are *unobservable,* they should result in the same physical situation: the centrifugal force should exist even for relative rotational motion. It does not matter whether the heavens or the Earth rotates: both situations are actually the same, described differently due to the excessive structure of classical mechanics: absolute space.

So the concept of inertia for Mach should be something built upon relative motion of all objects in the universe. There should be a law that causally relates inertial forces with relative motion. This kind of thought has inspired a lot of people, including Einstein when he formulated GR [2]. His laws are expressed with the usage of tensors which are mathematical objects which have an existence explicitly independent of the coordinate system used. Furthermore, Einstein´s equation relates the metric tensor (which is used to find the paths of inertial motion) with the stress-energy tensor. Thus, in the words of Wheeler, matter-energy here influences inertia there. In fact, when formulating GR, Einstein tried to give life to Mach´s thoughts and his theory has a lot in common with Machian ideas about the fundamentals of dynamics. However, instead of creating a theory where only observable data (relative distances) play any role, Einstein still uses invisible coordinate systems by presupposing a space-time manifold. GR is an indirect solution to the Machian problem of the origin of inertia and history tells that the reason why Einstein chose this path was the supposed impracticability of the direct one, which would use only relative distances from the start.

Mach also criticizes the concept of time. In Newton´s classical mechanics, time is a monotonic label attached to configurations of the universe that gives them an order. But, as with absolute space, there seems to be excessive structure: we only have access to time by looking at clocks, which are physical objects themselves. So time is a derived concept from that of motion of physical objects. In a relational picture, to say that time runs in an empty universe is both useless (for there is no motion to be analyzed) and meaningless (for the flow of time is unobservable). No change equals no time. If everything speeded up in absolute time, including clocks, it also seems that the flow of an independent, external time is both *useless* and *meaningless*. In Mach´s own words:

''It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction at which we arrive by means of the changes of things.''

We see that Machian mechanics presupposes a lot less structure than the traditional Newtonian mechanics, and also eliminates unobservable concepts such as absolute space and external time. Mach, however, never put his ideas into a complete testable theory, and Newton´s paradigm remained as the dominant view.

In fact, while Einstein attacked some of the old Newtonian conceptions of the fundamentals of dynamics with GR, quantum mechanics was formulated using functions defined on an absolute space and external time (x,y,z,t), which then evolved to quantum field theory and the standard model. The fact that QFT is tremendously successful is used as an argument against Mach´s conceptions of dynamics. Mach wanted to constrain the Newtonian space and time structure upon the concept of a physical material object. But matter itself is modeled as fields in QFT, which are objects defined on space or space-time. This is what makes the old absolute x relative discussion seem so out of place.

But leaving that aside for a moment, what would we find if we tried to do what Mach suggested and that Einstein thought was impractical, that is, develop physics without absolute space and external time from the start?

3. BARBOUR´S RELATIONAL PHYSICS

Julian Barbour is a British scientist who has dedicated much of his life to the study of Machian principles and their translation to physics. With the help of collaborators, he has successfully implemented Mach´s philosophy [3],[4]. I will give a quick exposition of his work and those interested on the details should look at [5] [6] [9] [10]

For setting up rigorously what a Machian theory should achieve we have to first understand the important concepts of Relative Configuration Spaces and Shape Spaces, used extensively by Barbour. When we look at N point-particles living in Newton´s Euclidean absolute space, the observable data upon that is actually the set of $N(N-1)/2$ numbers r_{ij} of particle separations. These numbers are not independent: they satisfy constraints analogous to the familiar triangular inequalities (when N=3). By performing absolute rotations and translations, the set r_{ij} does not change. Thus, in principle, they can be expressed in a set of $3N-6$ intrinsic coordinates which eliminates the Cartesian redundancy. We should also turn our attention to the set of $3N-7$ numbers $\tilde{r}_{ij}$ which are also invariant to absolute contractions and dilatations. This is because when we say a distance between two objects is ''x '' we actually mean that the ratio between this distance and a particular reference length is x, so that only ratio of distances should be significant. For each N, there are many possible sets $r_{ij}, \tilde{r}_{ij}$, and each of these sets is a point in the r*elative configuration space* (RCS) and in *shape space* (SS), respectively.

Information in SS is exactly all that is directly seen-every time we state that a physical system is represented as a point in the configuration space (for instance the Cartesian coordinates of n particles $\{x_i\}_j$)all that is observed is the projection of this point on SS. Barbour states that for a theory to be perfectly relational, *a point and a direction in SS should uniquely determine a curve in SS* [7] . A direction, rather than a velocity, is specified because there should be no unobservable time parameter t. This makes explicit ''the no change equals no time'' aspect of Machian philosophy.

But we know that Newtonian mechanics has good empirical results, for instance, when we substitute the absolute space for the distant stars and the external time with the rotation of the earth. So, by applying the requirements for a relational theory, could we recover Newtonian mechanics-that is, derive the concepts of duration and inertial frame-from data in SS?

The derivation of duration is very simple. Newtonian mechanics can be cast as a timeless action principle known as Jacobi´s principle [8]:

$$\delta A_j = 0, \qquad A_j = 2\int \sqrt{\bar{T}(E-V)}\, d\gamma \;\therefore \bar{T} \equiv \frac{1}{2}\sum_{i=1}^{n} m_i \frac{dx_i}{d\gamma}\frac{dx_i}{d\gamma}$$

This can also be written as

$$A_j = \int \sqrt{2(E-V)}\sqrt{\sum\nolimits_i m_i \delta\boldsymbol{x}_i \delta\boldsymbol{x}_i}$$

Where γ is an arbitrary parameter and E is the total energy of the system. The resulting Euler-Lagrange equations are:

$$\frac{d\boldsymbol{p}_i}{d\gamma} = -\sqrt{\frac{\bar{T}}{E-V}}\frac{\partial V}{\partial \boldsymbol{x}_i} \;\therefore\; \boldsymbol{p}_i \equiv \frac{\partial L}{\partial \boldsymbol{x}_i} = \sqrt{\frac{E-V}{\bar{T}}} m_i \frac{d\boldsymbol{x}_i}{d\gamma}$$

But the parameter γ is arbitrary. Choosing it in such a way that $\sqrt{\frac{\bar{T}}{E-V}}$ =1, we get

$$\frac{d\boldsymbol{p}_i}{d\gamma} = -\frac{\partial V}{\partial \boldsymbol{x}_i}$$

This is exactly Newton´s second law, so that γ may be identified with t. The condition $\sqrt{\frac{\bar{T}}{E-V}}$ =1, which is just conservation of energy for the universe in the Newtonian view actually *defines* time in the Machian view as

$$\delta t = \sqrt{\frac{\sum_i m_i \delta\boldsymbol{x}_i\, \delta\boldsymbol{x}_i}{2(E-V)}}$$

It is usually interpreted that Jacobi´s principle gives us just the *path* of the system through configuration space, not the motion in time, which would be found after imposing energy conservation. But in the Machian view, this is the *definition of time upon motion.* The $\delta\boldsymbol{x}_i$ however are defined in a Cartesian frame which comes with redundancy as said before. To obtain a completely Machian theory we still need to eliminate absolute space.

Barbour and Bertotti proposed the method of '*best-matching*'' to resolve that. It works as follows: suppose you have two configurations of the universe, described in absolute space with coordinates $\boldsymbol{x}_i$ and $\bar{\boldsymbol{x}}_i$. Since there is no absolute space, we cannot specify $\delta\boldsymbol{x}_i$ because of the Cartesian redundancy. Instead, hold one configuration fixed and apply Euclidean translations and rotations to the other until the incongruence measure

$$\sum_i \sqrt{m_i \delta\boldsymbol{x}_i \delta\boldsymbol{x}_i}$$

gets minimized. Now the $\delta\boldsymbol{x}_i$ calculated with that specific rotation and translation is unique. This is defined as the best-matched distance between the two configurations. Notice that only data from RCS or SS is needed to determine it. We then recover Newton´s laws exactly by using a Jacobi-type action with best-matched distances:

$$A_j = \int \sqrt{2(E-V)} \sqrt{\sum_i m_i \delta \boldsymbol{x}_i{}^{bm} \, \delta \boldsymbol{x}_i{}^{bm}}$$

Actually the procedure of best-matching imposes constraints such as zero total angular and linear momentum, zero total energy for the whole universe and also forces the potential V to depend only on r_{ij}. Those ideas can be extended to field theories [9], [10], and when applied to the metric field, GR is recovered in a sense. It is truly remarkable how GR has tight correlations with Machian philosophy. The research on Barbour´s scheme is ongoing, and there are a lot of details that have not been mentioned. For us, it suffices to notice how a conceptual scheme of motion at the classical level may produce new physics and known physics from first principles.

5. AN IMPORTANT QUESTION

There is a question that is crucial for the development of physics but which is frequently overlooked: how can we conceive motion? To make such a question more tractable we could ask a simpler from a toy model: how can we conceptually modify the classical description of the motion of a system of point particles?

The Newtonian view of space and time, which is so ingrained in our minds, used a particular mathematical structure, for instance, that of space modeled as a flat 3-manifold and defined functions $x_i(t)$ to represent his intuition. But we can ask: what exactly is space, motion and time? If we have a conceptual change, the mathematical structure immediately changes and we can then seek for non trivial results of this new mathematical structure. For example, Mach noticed and Barbour showed that the unobservable structure of absolute space and time could be cut off from the start. That changes the mathematical formalism, for example, by exchanging the usage of configuration spaces to SS or RCS. The results that come from this new formalism can then be compared to those of the older one. Relational and absolutist pictures of the world lead to different and testable physics. This is a rather different path from what is common in physics. Usually, one looks for mathematical hints-for example, if the universe had 5 dimensions it would be possible to unify the forces of gravity and electromagnetism-which then would produce conceptual changes in our understanding of the very nature of motion. Actually, the impact QM and GR had was so powerful that led us to see the world as a bizarre place, in which there´s no room for our mundane conceptual reasoning about motion. That indeed is not strictly true: QFT came out of wave functions defined on (x,y,z,t) . If we have a different conception of motion from the start the whole procedure of *defining anything* on (x,y,z,t) could be seen as inadequate.

So how can we conceive motion after all? In an absolute or relational way? Are there other alternatives? It should suffice to begin by answering the prototype question: how can we conceptually describe the behavior of a system of point-particles? Mach attacked the unobservable data of Newtonian mechanics and Barbour translated that to new physics. Is there another method? Is there any natural conception of motion that could give us a hint about the origin of, say, QM? If the work of Barbour has shown tight correlations between a conception of motion and testable physics, shouldn´t we dig deeper in this direction? The proposal is: *find how Newton could have conceptually modified his classical mechanics, figure out a mathematical formalism for that task, search for non-trivial results of it and check the behavior or emergence of modern physics in this new language.*

5. PATTERN RECOGNITION

There is a natural way to extend relational thoughts. Newton proposed space and time so that the motion of objects could be well defined. Mach contested the invisibility of time: it should be an abstraction from the motion of objects because we can´t see the flow of time, all we can see are correlated changes of position of physical objects. The philosophical and metaphysical debates around this have lasted for centuries. But there is an important point: time cannot be *defined* without motion of objects. It lacks more than epistemic status: it lacks meaning. Every time we say ‘’n seconds have passed’’ it necessarily means that ‘’clocks are indicating that n seconds have passed’’ (or else we could never classify such a statement as true or false) and clocks are observable physical objects. Saying time cannot be defined and have any meaning without motion is stronger than saying it cannot be observed. Using the positivist criteria of meaning upon observation, this may well be the origin of Mach´s unease with Newton´s philosophy. But motion, at least in the Newtonian picture, also cannot be defined without space and time: it also lacks *meaning*.

Dynamics is about the concepts of object, motion, space and time, and Machian philosophy comes from questioning the meaning of statements that use such concepts, such as ''what do we mean by time?'' or ''what do we mean by position in space?''. But then we could also ask: what do we mean by an object? The relational solution may make the question ''what is time?'' tractable by giving meaning to time upon the motion of physical objects but the question ''what is an object?'' for instance remains nonsense. We have a *snapshot* intuition, that is, we have the tendency to start making physics by describing the motion of objects we see in a given instant, but figuring out *what´s the meaning of that snapshot seems very hard*. Actually there is no meaning; a configuration of the universe is the basic block of information for physical theories

So here is the key insight. Maybe what we really have is a pattern: Motion gains meaning from the concepts of space, time and objects in the Newtonian view. Time gains meaning from the concepts of space, motion and objects in the Machian view. Instead of struggling with opposite metaphysical positions we could be staring at a bigger fundamental structure like:

- TIME GAINS MEANING FROM SPACE,OBJECT,MOTION
- MOTION GAINS MEANING FROM SPACE, TIME OBJECT
- SPACE GAINS MEANING FROM TIME,OBJECT,MOTION
- OBJECT GAINS MEANING FROM TIME, MOTION,SPACE

This is a very natural extension of Machian thoughts. If possible, it could be a new conceptual base to reason about motion. By attaching a meaning to the concept of object, we would make the theory more *complete and at the same time drastically change its mathematical structure*. Barbour has defined ''time'' within dynamics itself using Jacobi´s principle

$$\delta t = \sqrt{\frac{\sum_i m_i \delta \boldsymbol{x}_i \, \delta \boldsymbol{x}_i}{2(E-V)}}$$

So could we also define ''object'' within dynamics itself? Could it be that the ''space gains meaning from object, motion, time'' entails the usage of shape space, or relative configuration space? What´s the exact relationship between Barbour´s models and these extended Machian thoughts? There are several interesting aspects for demanding dynamics to be cast in this *semantically complete* formalism if it proves to be definable and possible.

1) It seems to have the absolute and relational conceptual frameworks as particular cases of a deeper formalism. QM has tight roots with the absolutist picture and GR with the relational picture.
2) It could solve very difficult metaphysical questions such as: ''What is time?'' ''What is motion?''. ''What is space?'' In a very compelling way.
3) The physicist Max Tegmark has argued[13],[14], that
   ''If we assume that reality exists independently of humans, then for a description to be complete, it must also be well-defined according to non-human entities — aliens or supercomputers, say — that lack any understanding of human concepts. Put differently, such a description must be expressible in a form that is devoid of any human baggage like "particle", "observation" or other English words.''

[…] ''When we derive the consequences of a theory, we introduce new concepts — protons, molecules, stars — because they are convenient. It is important to remember, however, that it is we humans who create these concepts; in principle, everything could be calculated without this baggage.''

[..]''All of this raises the question: is it possible to find a description of external reality that involves no baggage? ''

He then concludes that the universe is not merely described by mathematics: it is mathematics. One should read his full paper to truly appreciate his proposal, but if we state that physics must be cast in the semantically complete formalism above, it also seems that there would be no *baggage.*

4) It could be used as a first tentative towards the question of ''new conceptions of motion of n particles''. Maybe there are many different ways of conceiving motion: absolutist, relational, semantically complete… A systematic study of this is obviously helpful for constructing new physical theories, especially quantum gravity. It is a surprise that this has never been addressed.

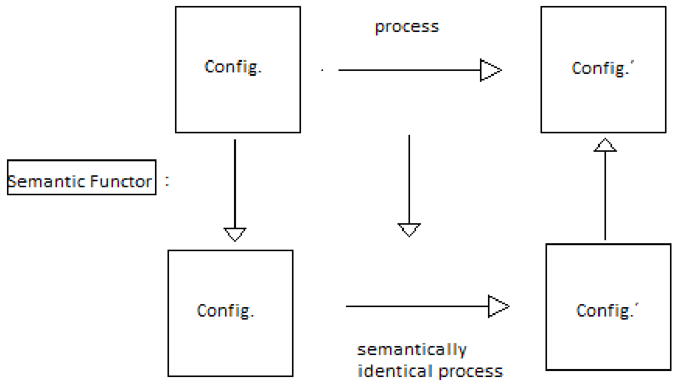
6.CATEGORY THEORY

To make any advances we have to express all these thoughts in a more rigorous, mathematical way. Fortunately, there is a branch of mathematics called category theory which seems just perfect for this task. A category is a collection of objects A, B, C… together with a set hom(A, B) for each of pair of objects A, B of the category (If ∈ hom(A, B) then we write $f: A \to B$ or $A \xrightarrow{f} B$) such that :

- For every object X, there is an identity morphism: $Id_x: X \to X$
- Morphisms are composable, that is, given $A \xrightarrow{f} B$ and $B \xrightarrow{g} C$ there is $A \xrightarrow{gf} C$ and the composition is associative.
- Any identity morphism satisfies, for any morphism f: $A \xrightarrow{f} B$, $f Id_A = f = Id_B f$

These definitions make Category theory a highly abstract, generic formalism for mathematics. It has been shown by Baez[13] that the formalism of category theory makes a number of analogies between physics, logic, topology and computation become natural. For instance, in quantum theory, objects are *Hilbert spaces* and the morphisms are *linear operators*. In logic, objects are *propositions* and morphisms are proofs. Baez argues that these analogies suggest that category theory is a language suitable for developing *a general science of systems and process.* To see how it is natural to talk about the ideas about motion of the last sections, it will be useful to introduce the concept of *functor* between categories. Given two categories A, B, a functor $F: A \to B$ is a map sending each A-object a to a B-object $F(a)$, and each A-morphism f, to a B-morphism $F(f)$ such that F preserves identity and composition ($F(Id_a) = Id_{F(a)}$, $F(gf) = F(g)F(f)$).

Imagine we have a category whose objects are configurations of the universe and whose morphisms are process between these configurations. Suppose we have a functor S that we shall call the *semantic functor,* which sends any configuration of the universe to another one which *means* the same (have the same informational content), and sends any process between two configurations to the process connecting the two *semantically identical* configurations. For example, a configuration of the universe given by a set of positions x_i is semantically identical to the configuration $x'_i = x_i + a$ where $\in \mathbb{R}$. Equipped with the definitions above, the principle for relational physics is just ''the square commutes'':



Of course there is much more to be done if we want these ideas to be less speculative and more concrete, especially if we want to define and impose the requirement of semantic completeness proposed above. There are some nice notes by Derek Wise [14] where it is shown that there are very simple categorical ways of defining ‘’Stuff, Structure and Property’’, and that should be helpful since the relationship between the whole mathematical gadget where the configurations are defined and the configurations and morphisms themselves would be explored. There should be a formal way of constructing the semantic functor upon a criterion of meaning. Notice that the concept of observation is *crucial* for that: in the Machian picture, two configurations mean the same if they are the same upon direct observation. For now, one can only wonder if the appearance of *observation* in the semantic functor could bring us any closer to quantum mechanics.

7. CONCLUSION

So we see that, at the time of Newton, there could be other ways to describe motion than the absolute or relational pictures. Then maybe the whole mathematical structure of classical mechanics would be different, and the consequences could be very drastic. It remains to be shown if the concept of ‘’semantic completeness’’ can produce any testable hypothesis and even if it can be rigorously defined and implemented. Category theory may help us here, but even if this idea turns out not to be fruitful, it looks interesting to find other ways to define motion and the structures used to make sense out of it, and that seems fundamentally a work of finding the ways one can *meaningfully* describe the universe.

ACKNOWLEDGMENTS

I would like to thank Julian Barbour for helpful comments and my friends Davi Mustafa, Maurício Kleinberg, Felipe Lemos and Mateus Fernandes for revising the text. I would also like to acknowledge the financial support provided by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

REFERENCES

[1] L. Smolin, *The trouble with Physics*, Houghton Mifflin Harcourt (2006)

[2] Huggett, Nick and Hoefer, Carl, *Absolute and Relational Theories of Space and Motion*, The Stanford Encyclopedia of Philosophy (Fall 2009 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/fall2009/entries/spacetime-theories

[3] J. B. Barbour and B. Bertotti, *Mach's Principle and the Structure of Dynamical Theories* Proc. R. Soc. A 382 (1982) no. 1783, 295–306.

[4] E. Anderson, *Foundations of Relational Particle Dynamics*, Class. Quant. Grav. 25 (2008) 025003 [arXiv:0706.3934 [gr-qc]]

[5] J. Barbour, *Shape Dynamics: An Introduction,* To be published in the refereed proceedings of the conference Quantum Field Theory and Gravity (Regensburg, 2010)(arXiv:11.05.0183v1[gr-qc])

[6] E. Anderson, J. Barbour, B. Foster, N. O'Murchadha, *Scale invariant gravity: Geometrodynamics*, Class. Quant. Grav. 20 (2003) 1571. [gr-qc/0211022

[7] J.Barbour, *The Definition of Mach´s Principle,* (2010) to be published in Foundations of Physics as invited contribution to Peter Mittelstaedt´s 80th birthday Festschrift

[8] C. Lanczos, *The Variational Principle of Mechanics,* Toronto University Press (1949)

[9] Barbour J, Foster B, and O Murchadha N, *Relativity without Relativity*, Class. Quantum Grav. 19, 1571 (2002) (arXiv: gr-qc/0012089).

[10] H. Gomes, T.Koslowski, *The link between General Relativity and Shape Dynamics,* Class.Quant.Grav. 29 (2012) 075009 (arXiv:1101.5974v3[gr-qc])

[11] M. Tegmark, *Shut up and Calculate,* (2007) (arXiv:0709.4024v1 [physics.pop-ph])

[12] M.Tegmark, *The Mathematical Universe,* Found.Phys.38:101-150 (2008) (arXiv:0704.0646v2[gr-qc])

[13] J.C.Baez, *Physics, Topology, Logic and Computation: A Rosetta Stone,* Physics, topology, logic and compIn New Structures for Physics, *ed. Bob Coecke, Lecture Notes in Physics vol. 813, Springer, Berlin, 2011, pp. 95-174* (2009) (arXiv:0903.0340v3 [quant-ph])

[14] D.Wise, J. Baez, *Quantum Gravity Seminar, Spring 2004, Properties,Structure ´n´ Stuff!, http://math.ucr.edu/home/baez/qg-spring2004/s04week01.pdf*