Consider the three events $C1$, $C2$, and $C3$, which can each take the values 0 and 1 (corresponding to the perceived orientation of the Necker cube, say). If at each point in time, there is a definite value to all three, then each run of the experiment must traverse one of eight possible histories with a certain probability $P = p$i. The table below lists all possibilities.

$C1$	$C1$	$C1$	P
0	0	0	$p1$
0	0	1	$p2$
0	1	0	$p3$
0	1	1	$p4$
1	0	0	$p5$
1	0	1	$p6$
1	1	0	$p7$
1	1	1	$p8$

Table 1. Possible 'histories' of the system weighted by their respective probabilities of occurrence.

Now define

$$\langle C_1 C_2 \rangle = P(C_1 = C_2) - P(C_1 \neq C_2) = p_1 + p_2 - p_3 - p_4 - p_5 - p_6 + p_7 + p_8,$$

and analogously

$$\langle C_2 C_3 \rangle = p_1 - p_2 - p_3 + p_4 + p_5 - p_6 - p_7 + p_8, \langle C_1 C_3 \rangle = p_1 - p_2 + p_3 - p_4 - p_5 + p_6 - p_7 + p_8.$$

The sum of these is then

$$\langle C_1 C_2 \rangle + \langle C_2 C_3 \rangle + \langle C_1 C_3 \rangle = 3p_1 - p_2 - p_3 - p_4 - p_5 - p_6 - p_7 + 3p_8 = 4(p_1 + p_8) - 1,$$

where we have used that the sum of all probabilities must be equal to 1. Now, since probabilities are always nonnegative, this entails directly

$$\langle C_1 C_2 \rangle + \langle C_2 C_3 \rangle + \langle C_1 C_3 \rangle \geq -1,$$

which is what we set out to show. Thus, the hypothesis that all of $C1$, $C2$, and $C3$ have definite values during the experiment directly entails the above inequality. By analogous reasoning, one also shows that

$$-\langle C_1 C_2 \rangle - \langle C_2 C_3 \rangle + \langle C_1 C_3 \rangle \geq -1, -\langle C_1 C_2 \rangle + \langle C_2 C_3 \rangle - \langle C_1 C_3 \rangle \geq -1, \langle C_1 C_2 \rangle - \langle C_2 C_3 \rangle - \langle C_1 C_3 \rangle \geq -1.$$

These form a complete set of Boole's conditions for the setting at hand. There is a nice geometric representation of this: each inequality represents a plane separating a three dimensional space into two half-spaces. Taken together, they delineate a tetrahedron with the property that for each of the points on its inside (or boundary), there exists a version of Table 1 with all the pi taking on definite values (see Fig. 3).

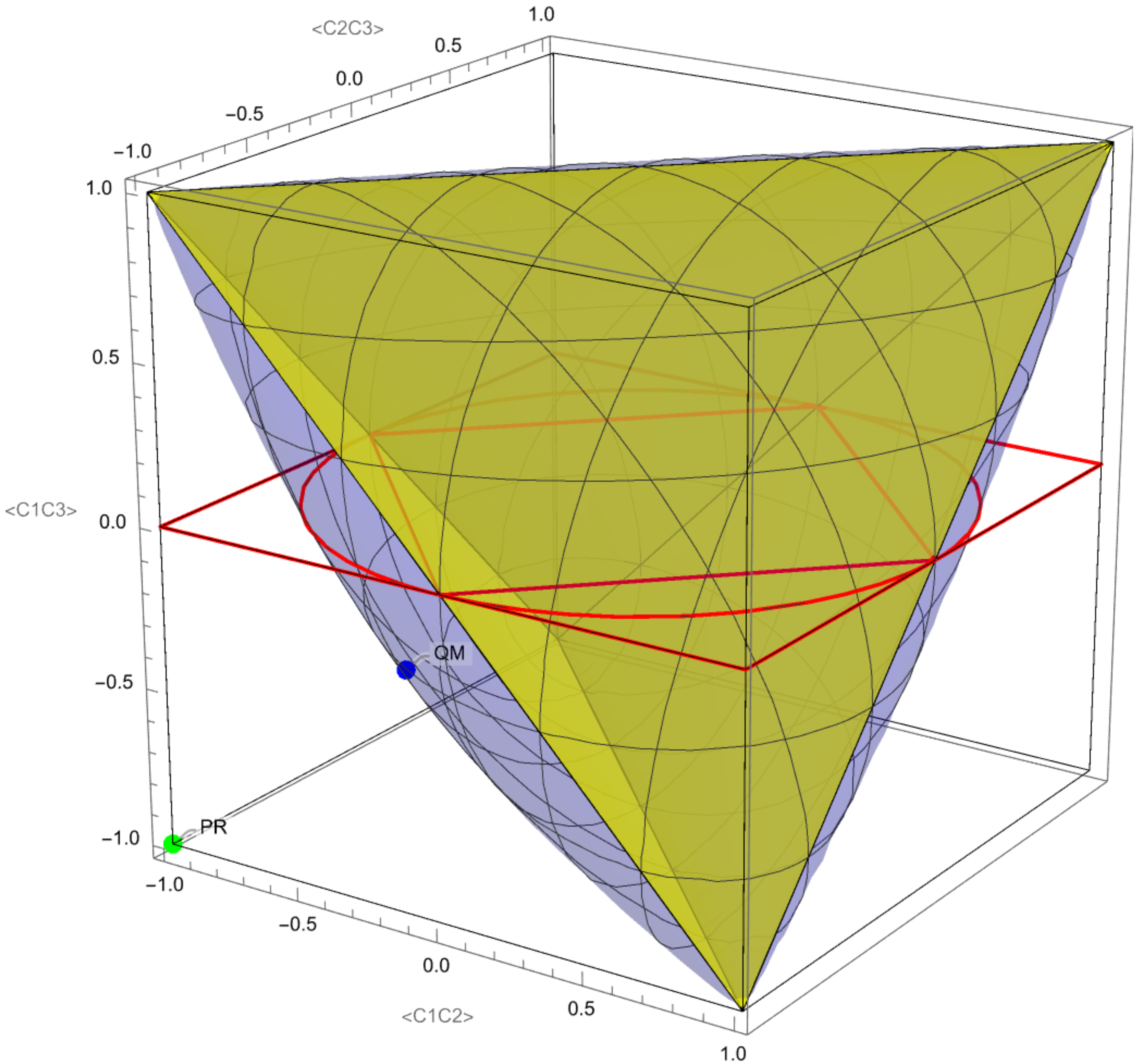

Figure 3. The set of classical (yellow) and quantum (blue) correlations for the discussed scenario with three observables. Indicated is also a reference point for a particular choice of observables in quantum mechanics (QM), as well as the point where all correlations are equal to -1 (PR, after the Popescu-Rohrlich 'boxes' [18] satisfying this limit). The cut through the middle highlights a particularly simple form of the sets, where the classically realizable correlations lie in a square, and the quantum correlations in the enclosing circle. After an example from [19].