I thought I was going to put an entire derivation of quantum and classical (KvN) Hilbert space theories here. It was going to show that assuming the appropriate fundamental commutation relation is (in combination with the Ehrenfest theorems) sufficient to derive the dynamical theory of both. I simply can't be bothered to fight this markdown format however, and would likely have been an exercise in ego more than education. You will just have to trust me that it's quite easy to show non-commutation is the defining element of quantum theory. Phys. Rev. Lett. 109, 190403 (2012) offers a beautiful rendering and generalisation of this idea, and the two formalisms are united in PRA 108, 052208 (2023).