

What is it that makes undecidability proofs “tick”? What is the core principle behind them all? Some may think it is something particular to Turing machines and the Church-Turing thesis, but then we encounter the strange beast called “protosyntax” introduced by the distinguished Harvard logician W.V.O. Quine in 1940 and further developed by a student of Alonzo Church, Raymond Smullyan in 1957.

Use versus mention. Both Tarski and Quine emphasized that importance of the use-versus mention distinction in their textbooks on mathematical logic. Quine preached for over 50 years—starting from his classic *Mathematical Logic*—that failure to distinguish use and mention would prevent one from *truly* understanding Godel’s proof of incompleteness and Tarski’s proof of the indefinability. Quine gave the following example well-known to logicians:

Boston is populous. “Boston” is disyllabic.

One refers to the object and the other to the name. If linguistics seems too vague, consider the following:

Print “p”&“q”    Print “p&q”

The former example will give use two characters and the latter three characters. Anyone who can distinguish between these two lines of code can see that the difference is clear. When dealing with every day concepts it seems less interesting. Ink and the word “ink” are both physically measurable as are many things. But in the case of mathematics, we come across something much more profound.

We distinguish between number and numeral, concept and symbol. The number 5 is an abstraction that has been known for several thousands of years by many cultures but the numeral “5” was introduced in the Indian subcontinent less than two-thousand years ago. The same method of categorization can be applied to any symbol in mathematics.

What does this have to do with the self-referential proofs? If a statement is self-referential and mentioned, named, quoted, then there is a physical demonstrability about it, but since there exists a use-mention distinction, it becomes “distinct” from itself leading to contradiction. On the other hand, if we wish to preserve consistency, then we must take away the mention, name, quotation (this is equivalent to Tarski’s famous T-schema). So just with quotation alone, and with no need to appeal to the Church-Turing thesis, one must choose between *demonstrability*, i.e., the physical character of the symbols used, and *consistency*, i.e., a lack of contradiction.

Not only is this key to understanding undecidability proofs but also Wheeler’s vision of a Godelian quantum theory that *includes* the observer within the system itself. This is also the essential distinction between information and information measurement (needed for information *processing*) in living systems.