## Notes

1. A mathematical structure consists of a collection of sets and relations between these sets (subsets of Cartesian products of these sets), see *Universal Algebra* [10] and *Model Theory* [4].
2. The possible values of an observable is the *spectrum* of the operator representing it.
3. For a state vector |?> and the position eigenvectors |x>, the wavefunction is ?(x) := <x|?>.
4. The dimension of the unitary group U(n) is n2. The dimension of the subgroup of unitary transformations preserving the Hamiltonian is between n and n2.
5. The unitary group acts transitively on the space of unit vectors.
6. Transformations preserving the form of the dynamical law must commute with the Hamiltonian. In the *standard measurement model* (see [2], ?II.3.4, and ?VII for real-world examples) the Hamiltonian is, during the measurement, H = -gA ? pZ where A is the observable and pZ is the canonical conjugate of the pointer observable Z. Then, if for any eigenvalue ? of A -? is an eigenvalue of equal degeneracy, the transformation |?, a>|outcome = ?> ? |-?, a>|outcome = -?> is unitary and commutes with H . If the spectrum of A is R and the degeneracy is the same for all eigenvalues, the transformation |?, a>|?> ? |?2/?1| |?2/?1 ?, a> |?2/?1 ?> swaps the two worlds and commutes with H. These symmetries relabel a world in which the measurement has an outcome as a world in which the measurement has any other outcome, for a previously unknown observed state.
7. These observables can be obtained by unitary transformations that commute with the Hamiltonian.
8. It is often believed that a preferred decomposition into subsystems emerges from conditions like the locality of the interactions [6]. But even for relabelings that don't change the dynamical law and preserve this kind of locality, for a 22-dimensional vector space, the ways to decompose it into n qubits (vector spaces with 2 complex dimensions) form a space whose number of dimensions grows exponentially with n (to see this, factor the group of unitary transformations that preserve H through the group of local unitary transformations on the factor spaces).
9. The Kochen-Specker theorem shows that, at least for three or more dimensions, the observed property could not have a definite value before the quantum measurement [14].
10. Unlike value-indefiniteness9, in our approach5.9-5.10, not the state was indefinite, but the observable associated to the measured physical property. This can avoid the superposition of different outcomes.
11. We call the measurements of a system by its environment *decoherence* [12, 18]. It is hoped to explain the emergence of classicality at the macro level and resolve the measurement problem. There are serious unsolved questions in the decoherence program, but in any case it can't resolve the measurement problem by itself, but only combined with one of the proposals 5.7-5.10.