1. Introduction

Our FQxI community remains deeply interested in exploring information as a fundamental building block for physics and reality. The next significant breakthrough may occur at the boundary between physical science and information science. Although no one can predict when such breakthroughs will happen, new opportunities may arise now as two major challenges converge: the scaling limits of silicon chip engineering and the elusive journey to rebuild basic science on an information-centric platform. A similar crossroad occurred some 60 years ago, but technology apparently took off without science; this time, the outcome may be different if science and technology may finally come together.

Our focus on scientific exploration has been driven by the motivation to acquire knowledge rather than pursuing technological advancements or monetary gains. On the other hand, the semiconductor chip industry, characterized by its vast economic scale and advanced device, circuit, and system design capabilities, relies on research and development to propel innovation. As the industry currently approaches a technological plateau, its eagerness to return to basic science intensifies. This symbiotic relationship between the chip industry and basic science represents unparalleled opportunities to a new era of the unprecedented integration of science and technology. The bond may arise at the interface between the universality of information processing and its physical representation in the world.

2. Semiconductor Device Scaling: Challenge and/or Opportunity

For several decades, semiconductor technology has sustained exponential improvement in cost, performance, and functionality, in accordance with Moore's law [1] and Dennard's scaling rules [2]. The seminal paper by Robert Dennard and his colleagues in 1974 described a set of rules for scaling transistor device parameters and the expected resultant circuit benefits. Dennard's scaling theory was adopted by the semiconductor industry for the next three decades as an evolutionary roadmap for obtaining simultaneous improvements in transistor density, switching speed and power dissipation, in accordance with Moore's Law. However, MOSFET scaling has deviated from this evolutionary path in recent years due to fundamental physical limitations, so that that such simultaneous improvement of performance, power and cost has been significantly slowed down and is now approaching a plateau.

The current situation resembles the happenings from about 60 years ago. Jack Morton at Bell Labs wrote in 1965, "Electronics may be approaching a plateau. To ensure a renewed growth, a new philosophy of engineering is needed. In such a philosophy, systems engineering and the physical sciences will provide complementary and lasting disciplines for our future innovators."[3] Then a few things happened in 1968, the first programmable desktop calculator, the invention of 1-transistor DRAM, the founding of Intel corporation, a graphic user interface, and more applications [4]; leading to the transformation of silicon technology for decades to come, without the anticipated collaboration with information-centric sciences. However, the ongoing crisis might result in a different outcome, as both science and technology have now advanced to the levels potentially allowing for better integration.

3. Science, Binary Information and Chinese Figures

Computer chips process information using binary numbers, which are represented as strings of 0s and 1s. The study of binary numbers by Gottfried Leibniz marked the beginning of the information age. The full title of Leibniz's 1703 paper is, "Explanation of binary arithmetic, which uses only the characters 0 and 1, with some remarks of its usefulness, and on the light it throws on the ancient Chinese figures of Fuxi” [5]. Dated from the 9th century BC and described in the *I Ching* or *Book of Changes*, these Chinese figures are shown in a set of 64 hexagrams. Leibniz noted that the hexagrams, consisting of six lines that can be either broken (yin) or solid (yang), are analogous to six-digit binary numbers.

American physicist John Archibald Wheeler talked about Leibniz in his first lecture at University of Science and Technology of China (USTC) in 1981. Mentioning Leibniz’s view of space and time as relational concepts (contrasts with Newton's belief in absolute space and time) as evidence, proceeding to the similar views of Neils Bohr and Albert Einstein, Wheeler concluded that Leibniz was one of the most influential persons who brought Chinese thought to the western world [6].

Sounded more than intriguing when Wheeler spoke out in his second lecture at USTC. He said the foundation of physics is destined to collapse and it would be rebuilt on a new foundation. This new foundation, John suggested, would be “law without law”, where almost everything in the world would come from almost nothing. The second lecture was specially prepared for his China visit in Wheeler’s three-lecture series titled “Physics and austerity: law without law”, written during his three-day river cruise at Yangtze River. Wheeler felt very excited when he watched a Chinese dance drama "Investiture of the Gods" when he learned that Chinese general’s commanding flag has only one character meaning "nothing".

Much of these ideas discussed in Wheeler’s USTC lectures in 1981 developed into the idea of “it from bit”, which could be summarized in one sentence, “Otherwise stated, all things physical, all its, must in the end submit to an information-theoretic description [7].” IBM Journal of Research and Development, Volume 32, No. 1, was published in January 1988 to honor Rolf Landauer's contributions to computation on his 60th birthday; the first paper in the issue, written by Wheeler, discusses the world as a self-synthesizing system built on observer-participancy via a network of elementary quantum phenomena, with life's explosion shaping the vast universe [8]. Wheeler’s USTC lectures were cited and quoted in length.

The second paper in the same issue, written by Charles H. Bennett, reviews the history of reversible computation, including Landauer's discovery of the thermodynamic cost of information destruction, and provides a brief survey on quantum reversible computation [9].

4. The Binary Universe: Microscopic, Macroscopic, and Holistic Scales

Binary information can manifest across different scales, ranging from the microscopic to the macroscopic and holistic levels.

The physical representation of information in computer chips mainly relies on microscopic switches, like transistors, which can be turned on or off to denote binary states (1 or 0). These switches form the foundation of digital computing and carry various computational operations such as AND, OR, and NOT gates, memory storage, and data manipulation. The layout and connections of billions of transistors on silicon-based chips illustrate the processing of digital information at a microscopic level.

Wheeler's description of the binary system, specifically his concept of "it from bit," emphasizes the fundamental nature of information in the universe. Consistent with his “principle of austerity”, often illustrated by the game of "twenty questions" as an example in his lectures, Wheeler emphasizes a minimalist approach by suggesting that every physical event can ultimately be reduced to a series of binary choices. Although the concept seems to be applicable to microscopic scale as well, it should be viewed as holistic and macroscopic.

The binary system described in the *I Ching*, also known as the Book of Changes, based on the complementary forces of yin and yang to understand the world and its transformations (aka changes), is on a holistic and philosophical perspective [10,11].

As silicon technology strives to overcome the imminent device scaling plateau, there is potential for the future of computer chips to extend beyond the traditional microscopic scale and incorporate more holistic and macroscopic approaches to integrated circuits. This will be the subject of the following discussion on holistic integrated binary circuits for statistical inference.

5. Holistic Binary Statistical Inference with Integrated Sensor Circuits

Statistical inference is essential for various studies and problems. Generally, larger sample sizes yield more accurate inference of statistical variability. Traditional methods require detailed knowledge of each sample, making the process expensive. Using a holistic binary statistical inference scheme described below, a much more efficient scheme could be implemented. Using modern transistor as ideal switches, the scheme is based on Schrödinger's pair of complementary means and its dimensionality corresponds to Fuxi's binary orders of yin-yang attributes described in the *I Ching*.

5A. Motivation

With the shrinking transistor sizes, the variability of its device characteristics grows to be a serious issue [12]. For example, the static noise margin of semiconductor memories is severely affected by the mismatch in transistor threshold voltages [13]. Even with redundant bits for repair and extra bits for error correction, less than a few raw error bits out of millions are commonly required. Instead of testing devices individually, some unconventional test structures of variability have emerged. Particularly, a switchable array structure was proposed to estimate the device variability [14].

Instead of testing many devices individually and then summarizing for statistical parameters, is there a better way? How many measurements do we really need to know the device variation of a sample of one million resistors? From the traditional thinking, to be most accurate, one may need to measure every one of the million resistors, and then calculate the sample mean and sample variance. This process could be viewed as data collection at the microscopic or atomistic level, and then analysis using statistical techniques to produce a macroscopic summary. On the other hand, the sample mean could be easily measured in a collective way. A million resistors can be connected in series or parallel, and then measured with a "one-shot" test procedure of the total resistance divided or multiplied by one million to obtain the sample mean. As this procedure to measure sample mean appears to be trivial, how about parameters for variations? It is possible with statistical inference on the macroscopic level on a two-dimensional resistor array having its internal connections configurable with transistor implemented switches. It is a holistic systematic switching while information is processed with transistors functioning as microscopic switches.

5B. Mathematical Notation for Multidimensional Means

The *A/H* notation over the multidimensional variable space is used to simplify the math expressions involving the arithmetic and harmonic means. For one dimensional (1D) indexed variable X_i, where $i = 1, 2, \ldots, n$, the arithmetic and harmonic means are denoted by the following symbols of A and H, respectively,

$$A_{i=1}^{n} X_i \stackrel{\text{def}}{=} (1/n) \sum_{i=1}^{n} X_i \tag{B1}$$

$$H_{i=1}^{n} X_i \stackrel{\text{def}}{=} n / \sum_{i=1}^{n} (1/X_i) \tag{B2}$$

Extending the *A/H* notation to two dimensions (2D), we have,

$$H_{i=1}^{m} A_{j=1}^{n} X_{ij} \stackrel{\text{def}}{=} \frac{m}{\sum_{i=1}^{m} \frac{n}{\sum_{j=1}^{n} X_{ij}}} = \frac{m}{n} \left(\sum_{i=1}^{m} \frac{1}{\sum_{j=1}^{n} X_{ij}} \right)^{-1} \tag{B3}$$

$$A_{j=1}^{n} H_{i=1}^{m} X_{ij} \stackrel{\text{def}}{=} \frac{1}{n} \sum_{j=1}^{n} \frac{m}{\sum_{i=1}^{m} \frac{1}{X_{ij}}} = \frac{m}{n} \sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{m} \frac{1}{X_{ij}}} \tag{B4}$$

The notation can be further extended to higher dimensions, for example, $H_{l=1}^{p} A_{k=1}^{o} H_{i=1}^{m} A_{j=1}^{n} X_{ijkl}$ is an operation that an operator $H_{l=1}^{p} A_{k=1}^{o} H_{i=1}^{m} A_{j=1}^{n}$ is applied to a four-dimensional (4D) array X_{klij}, where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; $k = 1, 2, \ldots, o$; $l = 1, 2, \ldots, p$.

5C. Binary Integrated Structures

We will proceed with a general scheme of binary integrated system and the underlying mechanism for the statistical inference. The term "sensor" refers to any type of device where an output signal (information) is present with two terminals that could be connected in parallel and/or series. They can be configured in multi-dimensional array structures. For example, the sensors could be conductors, resistors, inductors, capacitors. When the gates is controlled separately, the transistors can be viewed as conductors or resistors between transistor's source and drain terminals.

Consider a system of these similar elementary sensors configured to output an electrical conductance signal. The output conductance of the sensors is assumed to be independent and identically distributed (*iid*) random variables, described by a probability model for a positive random variable X. Some population parameters of X are the arithmetic mean $E[X]$ denoted μ, the harmonic mean $1/E[1/X]$ denoted h, the variance $E[(X-\mu)^2]$ denoted σ^2, and the coefficient of variation $C_v = \sigma/\mu$. The harmonic mean is analytically undefined for the normal distribution and thus often overlooked, however, it is a bridge from the proposed integrated statistical inference of variability. We define a relative ratio $k \equiv \mu/h - 1$. We will show $C_v^2 = \mu/h - 1 = k$ for both the inverse Gaussian and lognormal distributions, thus the proposed schemes will yield direct measurements of statistical variance.

Schrödinger discovered the inverse Gaussian (IG) distribution for the first passage time of Brownian motion [15]. He used the harmonic and arithmetic means to derive IG distribution's shape parameter λ, *i.e.*, $1/\lambda = 1/h - 1/\mu$. Tweedie proposed the name inverse Gaussian and established some of its statistical properties [16,17,18,19]. For lognormal distribution, the relationship between the means and variability is also established [20]. Lognormal distribution is often used in the description of problems in economics and engineering, for example, the transistor's subthreshold leakage issue [21,22].

The integrated inference's architecture diagrams are shown in Figure 1 for the implementation in one- and two-dimensional arrays. The diagrams of higher dimensional orders are shown in Figure 2.

Pedagogically, the proposed scheme corresponds to Fuxi's binary order described in the *I Ching* or *Book of Changes* [10,11]. Figure 1a shows a traditional diagram of Fuxi's order. Its common explanation, attributed to Confucius and included in the commentary section of *I Ching* [10], is the following. "Therefore there is in the Changes the Great Primal Beginning. This generates the two primary forces. The two primary forces generate the four images. The four images generate the eight trigrams." The two forces are yin and yang. The four images are named as tai-yin, shao-yang, shao-yin, and tai-yang. In Chinese, tai means great, shao means lesser. Figure 1a is annotated with binary *H/A* for *H*armonic and *A*rithmetic orders, for their roles as the yin-yang of statistical inference.

Silicon MOSFET transistor (Fig. 1b) is almost an ideal switch; with a sub-threshold swing of 60 mV/decade at room temperature [23,24], its conductance in the open and closed states may span 15 orders of magnitude for 1V on the gate. Therefore, we assume the ideal switch model, *i.e.*, the conductance is zero when the switch is open and infinite when it is closed (Fig. 1c).

Figure 1d-e shows the one-dimensional (1D) array of *n* sensors, switchable between two modes with the series (d) and parallel (e) connections. The total effective conductance in the series mode is $G_S(X;n) = (H_{i=1}^{n} X_i)/n$ and that in the parallel mode is $G_P(X;n) = n\ (A_{i=1}^{n} X_i)$. Here we use the following *A/H* notation, $A_{i=1}^{n} X_i \stackrel{\text{def}}{=} (1/n)\sum_{i=1}^{n} X_i$ for the arithmetic mean, and $H_{i=1}^{n} X_i \stackrel{\text{def}}{=} n/\sum_{i=1}^{n}(1/X_i)$ for the harmonic mean. The notation is extended to higher dimensions in Supplementary Information.

Figure 1f-g shows the two-dimensional (2D) system with sensors placed in an *m*-by-*n* array. The random variables are denoted as X_{ij}, where *i* and *j* are the row and column indices. While the 1D array switches between *H* and *A*, the 2D array switches between *AH* and *HA*. In mode F, all switches are open; the conductors are connected first in series by columns, then in parallel. In mode C, all switches are closed; the conductors are connected first in parallel by rows, then in series. The total conductance in the mode-F or mode-C is denoted G_F or G_C, respectively.

$$G_F(X;m,n) = \sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{m} \frac{1}{X_{ij}}} = \frac{n}{m} A_{j=1}^{n} H_{i=1}^{m} X_{ij} \tag{1a}$$

$$G_C(X;m,n) = \left(\sum_{i=1}^{m} \frac{1}{\sum_{j=1}^{n} X_{ij}}\right)^{-1} = \frac{n}{m} H_{i=1}^{m} A_{j=1}^{n} X_{ij} \tag{1b}$$

Figure 2 shows the arrangement of the harmonic (*H*) and arithmetic (*A*) means on Fuxi's binary order up to the fourth dimension. The integrated sensor system can be constructed at higher dimensions for the application opportunities and design trade-offs. A homogeneous subset of the high dimensional arrays can be based on the *AH*-to-*HA* switching, instead of the A-to-H switching. Here the term "homogeneous" is used to describe the same system conductance without the underlying variation of sensors. Extended Data Figures 1b shows the circuit schematics of four-dimensional (4D) extension of the above subset, consisting of *AHAH*, *HAAH*, *AHHA*, and *HAHA*. Switches may be operated with multiple clock phases for the higher dimensional arrays. The 4D circuits shown in Extended Data Figure 4b is implemented with two clock phases, ϕ1 and ϕ2, where ϕ1, ϕ2 = 0 or 1 corresponding to open or closed switch positions. Since *AA->A* and *HH->H*, *HAAH* and *AHHA* may also be viewed as the 3D arrays in the forms of *HAH* and *AHA*.

For the 1D array, G_P or G_S is physically measurable. For the variability inference, K_{1D} is defined as,

$$K_{1D}(X,n) \equiv \frac{A_{i=1}^{n} X_i}{H_{i=1}^{n} X_i} - 1 = \frac{1}{n^2}\frac{G_P}{G_S} - 1 \tag{2}$$

$nG_P \geq G_S/n$, so that $K_{1D} \geq 0$, where the equality occurs if and only if $C_v = 0$.

Similarly, for the 2D array, K_{2D} is defined using the single measurable quantities G_C and G_F,

$$K_{2D}(X;m,n) \equiv \frac{H_{i=1}^{m} A_{j=1}^{n} X_{ij}}{A_{j=1}^{n} H_{i=1}^{m} X_{ij}} - 1 = \frac{G_C}{G_F} - 1 = \frac{G_C - G_F}{G_F} \tag{3}$$

$G_C \geq G_F$, so that $K_{2D} \geq 0$, where the equality occurs if and only if $C_v = 0$.

The integrated inference architecture is generally useful for the positive random variables. For the cases of inverse Gaussian (IG) and lognormal (LN) random variables, an analytical solution exists for the 1D array, So that $\hat{C}_{v,1D}^2$ is unbiased estimator.

$$E\left(K_{1D}(X;n)\right) = \frac{(n-1)}{n} C_v^2 \tag{4}$$

$$\hat{C}_{v,1D}^2 = \frac{n}{n-1} K_{1D} = \frac{n}{n-1}\left(\frac{1}{n^2}\frac{G_P}{G_S} - 1\right) \tag{5}$$

The estimators of C_v^2 are also established for the integrated 2D arrays.

$$\hat{C}_{v,2D}^2 = \frac{mn}{(m-1)(n-1)} K_{2D} = \frac{mn}{(m-1)(n-1)}\left(\frac{G_C}{G_F} - 1\right) \tag{6}$$

For the 2D array of IG or LN variables, when both the array dimensions go up unbounded to infinity,

$$\lim_{m\to\infty, n\to\infty} K_{2D} = \frac{H_{i=1}^{\infty} A_{j=1}^{\infty} X_{ij}}{A_{j=1}^{\infty} H_{i=1}^{\infty} X_{ij}} - 1 = \frac{\mu}{h} - 1 = C_v^2 \tag{7}$$

For the 2D system, at least (*m*-1)(*n*-1) switches are needed for the 2D *m*-by-*n* array.

In summary, $\hat{C}_{v,1D}^2 = nK_{1D}/(n-1)$ and $\hat{C}_{v,2D}^2 = mnK_{2D}/\left((m-1)(n-1)\right)$ may be used to estimate C_v^2 using the integrated 1D and 2D arrays, respectively. For IG and LN variables, $\hat{C}_{v,1D}^2$ is an unbiased estimator. $\hat{C}_{v,2D}^2$ is a consistent estimator, that is, unbiased asymptotically when min(*m*, *n*) >> 1.

5D. Additional Comments on Holistic Binary Structures

In this work, each elementary sensor is assumed to output an electrical conductance as a two-terminal device. The integrated system is a conductor network consisting of the similar sensors placed in a logical (*i.e.*, not necessarily physical) array in one, two or higher dimensions, with transistor as the connection switches. The system is switched between two physical configurations, where the total effective conductance values correspond to some binary strings of harmonic (*H*) and arithmetic (*A*) means of the elementary sensor conductance. The elementary sensors described by the independent and identically distributed (*iid*) random variables can be viewed as “microscopic” objects while the integrated system can be viewed as a “macroscopic” and synthetic object. According to equations (2) and (3), the parameters K_{1D} and K_{2D}, defined with the total conductance of the integrated system in the binary states, are physically measurable on the “macroscopic” level and then linked to the coefficient of variation of the elementary sensors at the “microscopic” level. In this sense, variability is measured as a collective behavior on the holistic scale.

One advantage of the integrated inference is the significantly reduced measurement steps on the sample. For example, for an array of 1000-by-1000 sensors, only 2 measurements are needed using the integrated scheme while one million steps are required using the conventional method. Another advantage is the technological scale. With the advanced silicon technology available now, a few billion “1-sensor 1-transistor” cells may be integrated over a small chip area, allowing the development of statistical inference for a wide range of applications.

Furthermore, the integrated scheme operates simultaneously among sensors, allowing aggregated and dynamic detections when the tested objects move around sensors. Additionally, $\hat{C}_{v,2D}^{2}$ is robust against the outliers up to its dimensional size of the 2D array. The robustness could be further improved with higher dimensional arrays.

6. Discussion

In 1703, during a comprehensive discourse on binary arithmetic, Leibniz acknowledged "the ancient Chinese figures of Fuxi." He wrote, "As these figures are perhaps the most ancient monument of science which exists in the world, this restitution of their meaning, after such a great interval of time, will seem all the more curious." We note that the *I Ching* not only utilized binary strings symbolizing yin-yang attributes but also applied them on a holistic level, often in pairs representing states before and after Change. These features resemble the integrated binary structures we have discussed.

Curiously, Leibniz observed that the Chinese might have lost the meaning of these diagrams a thousand years ago. However, this may only be part of the story, as there could be more significance to these figures. The *I Ching*'s use of binary strings, holistic levels, and paired states offers insights into the integrated inference architecture with transistor switches. It seems that the understanding of these diagrams has evolved over time, and now, after 400 years, we are discovering new meanings related to the binary system and Fuxi's diagrams. It is noted that some researchers have started and explored its significance relating to Landauer’s principle (aka, “information is physical”) [25,26].

Additionally, our current scientific approaches may be overly linear, atomistic, and lacking in nonlinear holistic analytical methods. In our view, the holistic binary inference method could capture the interest of a broader scientific audience in the future. Although Schrödinger's variability formula remains largely unnoticed in today's scientific community, it is integral to the binary scheme. Drawing parallels with thermodynamics and statistical mechanics, variability can be measured at holistic or "macroscopic" scale, without the need to examine the details of each individual sensor at the "microscopic" scale. Beyond the variability study, future works could explore new engineering opportunities as well as fundamental questions.

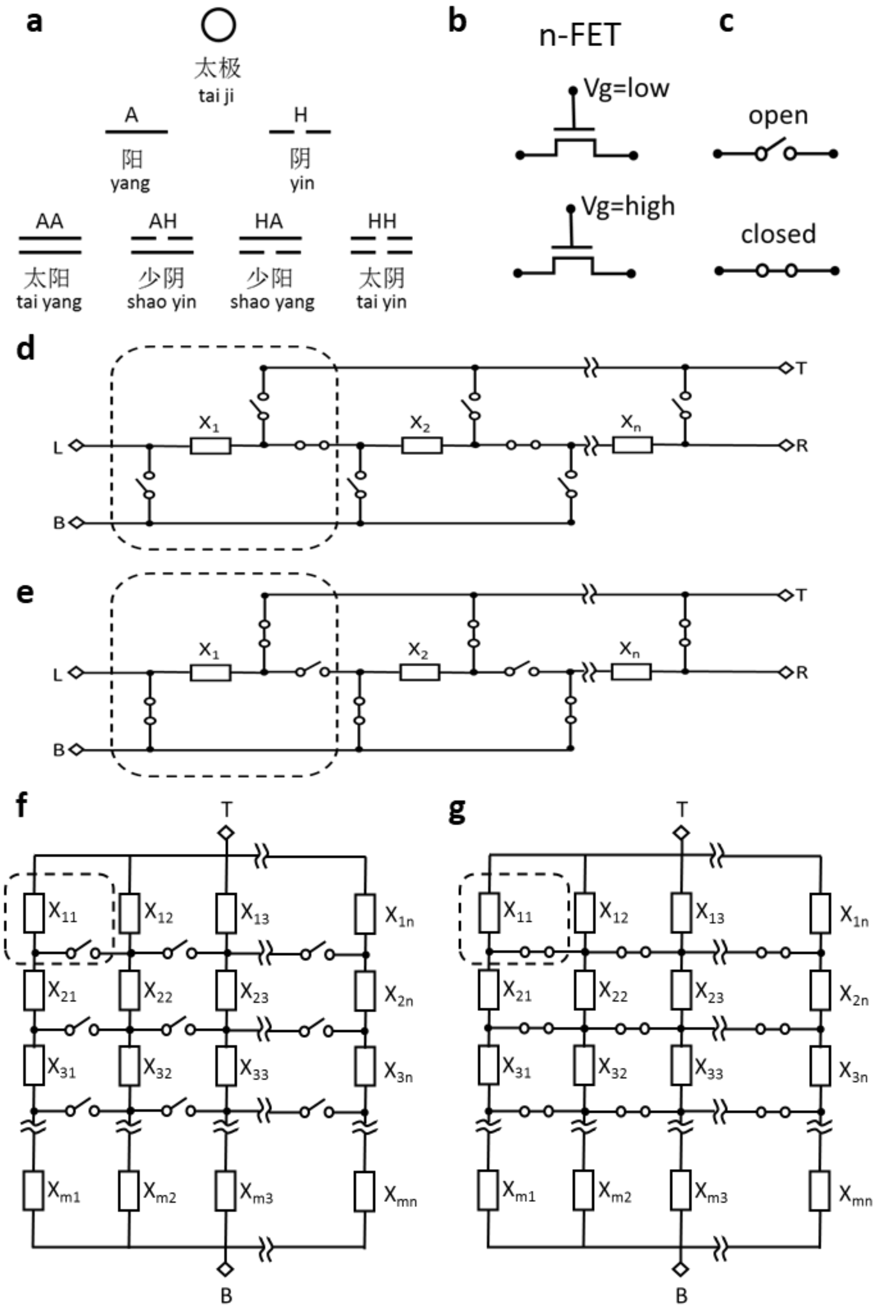

Figure 1 | Schematic diagrams of integrated sensor circuits. **a**, Fuxi's binary system. **b,** n-FET in the open and closed states with a gate control. **c,** symbols for switches in the open and closed states. **d,e,** 1D switchable circuit in mode-S and mode-P, with the series and parallel connections, respectively. **f, g,** 2D switchable circuit in mode-F and mode-C, where the switches are all open or closed, respectively. The "2-transistor 1-sensor" (2T1S) and "1-transistor 1-sensor" (1T1S) unit cells are shown by the dashed boxes in (**d-g**), for the 1D and 2D schemes, respectively.

a

O

H A

HH AH HA AA

HHH AHH HAH AAH HHA AHA HAA AAA

HHHH AHHH HAHH AAHH HHAH AHAH HAAH AAAH HHHA AHHA HAHA AAHA HHAA AHAA HAAA AAAA

b

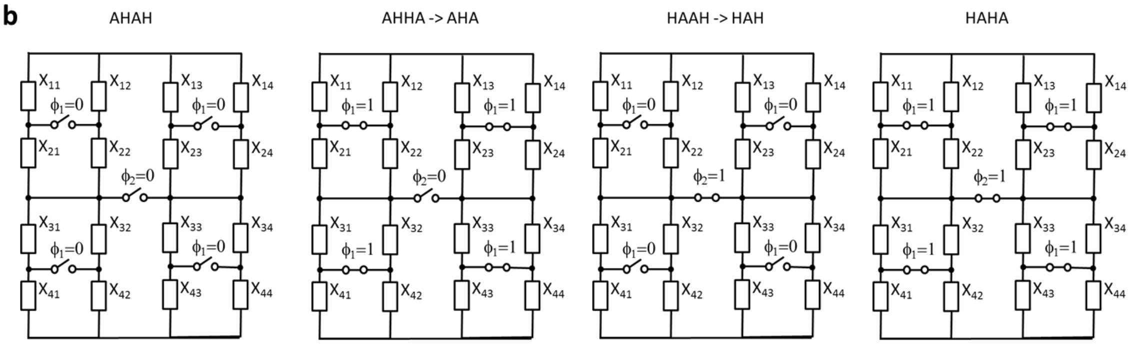

Figure 2 | Schematics of higher dimensional arrays. **a**, Fuxi's binary order up to the fourth dimension. **b,** The circuit diagrams of the 4D arrays based on the AH-to-HA switching. The switches are operated with two clock phases, ϕ1 and ϕ2, where ϕ1, ϕ2 = 0 or 1 corresponding to open or closed switch positions.