Introduction

Science is about developing theories of nature based upon evidence, in particular experimental evidence. Scientific theories use mathematical tools to measure, explain, and predict natural and human-made processes. Over the last few centuries, we have begun to perceive objects across many levels of scale. We have come to understand that we live on a spinning earth, which is circling a sun, which is moving within the Milky Way galaxy, which is moving in a cluster of galaxies. And we have come to understand that we are comprised of organs and blood vessels that continually pump blood, which moves cells made of proteins that supply oxygen and nutrients to other cells. And all these cells and proteins are made up of molecules and atoms and particles that are constantly moving on their levels. Scientific theories and disciplines tend to address one or another level of scale e.g., quantum physics, molecular chemistry, human medicine, planetary ecology, stellar systems, galaxy clusters. Some address multiple levels of scale. However, nature includes all these levels operating together, in some interconnected fashion.

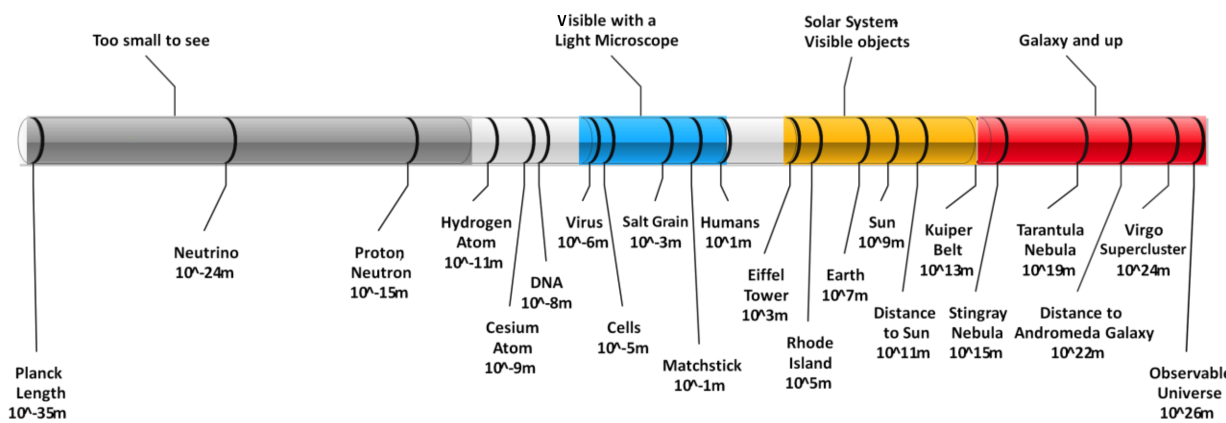
Heuristic Challenge for Science

If we are to model all of nature, then we need to include all these levels, in a single model. When we touch our finger to a pane of glass, the direct evidence is of our finger touching the glass. If we perceive the action with a magnifying glass, we would see specific ridges of our skin touching the less than smooth surface of the glass. If we perceive the action with a microscope, we would see cells touching the rough surface of the glass. We can continue indirect observations using different magnifying tools down to the protein, molecular and atomic scale levels. We could setup multiple observational tools to observe different scale levels during the same action and we would gather the observational evidence that the action occurs at all these levels together, not one or the other. Nature operates as a cohesive whole.

Some scientists have suggested that nature exists only at the lowest level and such an action only occurs on the atomic or sub-atomic level where we never actually touch the glass and that all but the lowest levels are illusory[1]. However, this subverts the direct observational evidence of our eyes and bases an explanation only on what theoretically occurs at an indirectly observed molecular or atomic level. Limiting science to such indirect evidence would seem at odds with the history of science based upon direct observational evidence. A more appropriate perspective would be to consider all levels existing together.

Another concerning item is that we have different models of nature, depending upon which level we are concerned with. There is the ‘classical’ model of what we see around us. We have medical models of our bodies, organs, and cells. Then there are the models of proteins and macromolecules that are built up from models of molecules and atoms, which build upon models of particles. Multi-Scale Modeling attempts to bridge adjacent models, and there are efforts to explain natural phenomenon at multiple levels[2]. To model nature we should be attempting to describe all levels using a single model that can explain the actions and observations at any and all levels.

If we are to efficiently model this one action of touching a pane of glass at all levels, we will need to be able to specify the actions at every level and then combine them across levels. Each level exists at a certain scale, with differing modeled objects at different scales. Since we model each level as actions in a three-dimensional space, combining them would most effectively use a four-dimensional model, identifying the scale level as one of the locators in the modeled space. This heuristic model would provide a means of locating a pen on a table, an atom of the pen, and a star in a different galaxy all in one model. It would not be limited by individual models at each scale, nor require multiple models to be integrated together across scale. Such a model could allow for actions between levels, both upward in scale as well as downward in scale.



Adding scale as a dimension is a change in perspective of the universe that does not change our observations to date. Measurements at each level remain unchanged, so all current equations at one scale or another would remain intact. Interpretations of those equations and the theoretical impact might change. An obvious example might be the equations of gravity interpreted as a four dimensional ‘space-time’. The equations of gravity would not change only the interpretation of what the equations represent would need to be reviewed. The above discussion about touching our finger to glass suggests modeling the many levels of nature using four spatial dimensions is more direct and potentially better than our traditional three-dimensional space that still needs to account for objects at different scales. The implication is that ‘space-time’ as four dimensional might not be a good model, perspective, or interpretation. The suggestion is to expand space to four dimensions and then we could tack time onto these four spatial dimensions – conceivably modeled as a five dimensional ‘space-time’.

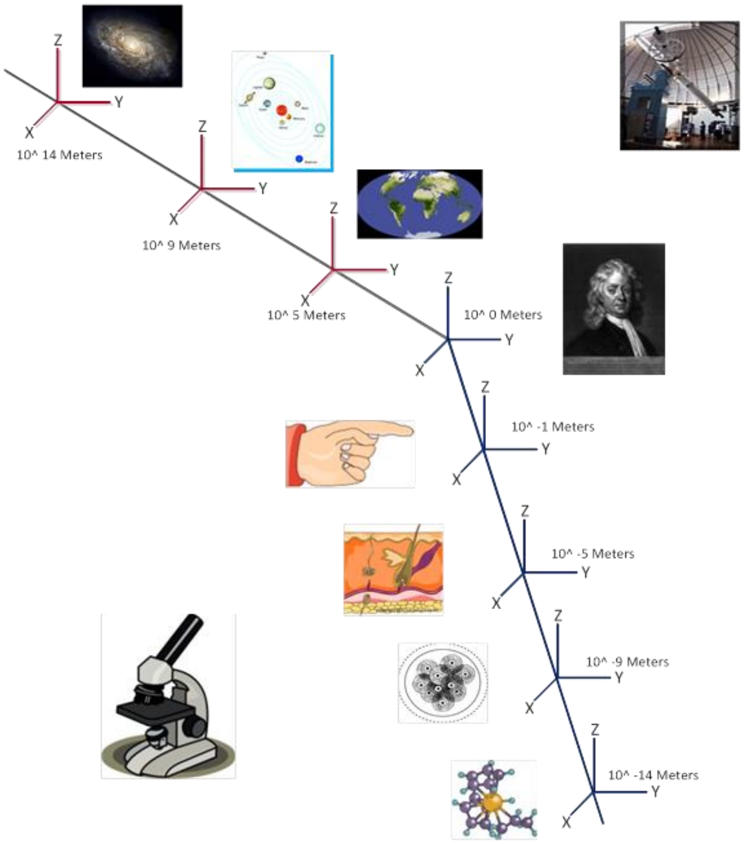
This heuristic model could provide the means to account for actions on the molecular level to affect actions on the protein or mitochondrial levels. Since it provides a means of connecting activities at different levels, its usefulness in medicine could be powerful, directly connecting actions on the chemical and protein level to those on the cellular and even organ levels – without depending upon statistical inferences. It also provides the capability for actions on a larger level, say our human scale, to affect actions on smaller scales, such as humans building the Large Hadron Collider (LHC) to guide sub-atomic particles into collisions with each other.

We could say we actually live in a four-dimensional world, not a three dimensional one – without changing anything we sense or measure about the world. It gives us a means to model all the scale levels of nature in a single model. To change an analogy from the story 'Flatland: A Romance of Many Dimensions' by Edwin A. Abbott[3] where a two dimensional being is shown a three-dimensional perspective: What if we are the Flatlanders, however instead of us living on a two-dimensional plane, we only believe we live on a two-dimensional plane, yet we really exist with three-dimensions. We can perceive objects in this other dimension with special tools, yet we believe everything that is 'below' us in scale is 'inside' us. When the Sphere visits Square and pops him out of his plane, Square sees that all the people and objects of his world actually have a 'depth' to them filled with the other objects, like organs and cells and atoms, which they do not see directly. And each Flatlander has additional surfaces to them that they do not understand as surfaces. This could be the more apt analogy (not considering the social discourse of Mr. Abbott).

Short Critique of Nature as Three dimensional

If we only lived in three dimensions, then we should expect higher precision of measurements to produce more accurate information about the objects we are making measurements about at the original scale. This would be the expectation of a mathematical space of three dimensions – more precise measurements of objects result in more accurate models. However, nature does not always mesh nicely with mathematics, since adding more and more precision to measurements at one scale shifts us into another scale with different objects, say from our bodily organs to cells or to molecules. More importantly, the objects we can measure at one level, say our body at our level, are not the same objects we can measure at other levels, say the cellular level. Further, the volume of our body does not change at different levels of scale since our body at our level and at the cellular or molecular levels has essentially the same physical three-dimensional volume. There is a statement in physics that two objects cannot occupy the same physical space. So, how can these different objects exist in the same physical three-dimensional space? Maybe the explanation is that they do not take up the same four-dimensional space.

It appears that we have to 'travel' along scale to get to one or the other objects (see interactive website by Huang bothers[9]. What are we travelling through? How can we shift from the scale of our body to that of our cells and stay in the same physical location? If we believe we only exist in three-dimensional space, why are there multiple levels inhabited by different objects somehow at the same three-dimensional location? If we use a four-dimensional model, then this all makes sense – we do travel through a 'scale space' dimension to get from our level to that of proteins and molecules with each level encompassing the same three-dimensional volume.



Consider a three-dimensional cube with 3cm sides that surrounds the joint of your knee at our level. Now build that 3cm cube surrounding your knee at the cellular level. Should we consider this a different cube encapsulating different objects? Say we do consider it is a different cube encompassing different objects. Now connect the vertices of each cube. It should not be too difficult to imagine. This figure would constitute a four-dimensional hypercube, with a three-dimensional cube at each end and an enclosed space through the intervening scales including the objects at these intermediate scales, like bone and blood vessels and nerves. We have just visualized a four-dimensional hypercube or tesseract. This should be a clue that our world may not be properly modeled in three dimensions since we actually observe different objects at different scales. It also means we need to identify the scale of the objects we measure in order to properly locate an object in our world - constituting a fourth location axis.

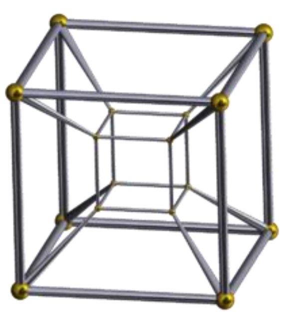


Figure from Wikipedia http://en.wikipedia.org/wiki/Tesseract

Heuristic Challenge for Measurements and Mathematics

While the above suggestion might seem a simple expansion of space to four dimensions, there is a key difference between our traditional three dimensions and that of scale. The distance (length) measurements in our traditional dimensions are all equivalent – 1 unit (e.g., one centimeter) in any direction equates to the same unit in any other dimension. These are linearly equivalent related dimensions. This is not the case with scale, which has a different length measurement or metric. Relative to the units we use in our traditional dimensions, scale has an exponential length measurement. One traditional unit converts to a unit with an exponent (> 1) along the dimension of scale. Several people have made visualizations of this exponential metric as traveling up or down in scale, usually using powers of ten as the length unit[4,5,6,7,8,9].

To locate an object in 'scale-space', we use our traditional three-dimensional measurements and add one more for the location in scale. For typical theories that apply to one level of scale, the scale location, is not addressed. This is okay as the scale value does not change for any objects under observation and can be considered a constant. However, this constancy of scale should be considered an assumption of the theory, as changes in scale could impact the experimental apparatus and even tools used for observation (consider the magnifying glass and microscopes to view our finger touching the glass pane).

Now, all locations should consist of four (spatial location) values, not the current three (and the spaces we deal with are not in $\Re^3$ but in some variant of $\Re^4$). This could require an expansion to four space vectors. Once this is accomplished, we can start to consider how an equation changes when scale is altered. However, this introduces a measurement challenge as such a model has three equivalent linear dimensions plus one that is exponential in extent (when related to the other three). This last is a challenge for measurements in science as the unit of distance is linearly defined. and could indicate a need for additional mathematics. Traditional distance measurements assume all dimensions of a space can be measured using the same units. However, to model nature, we may need a method that does not have the same distance measurement, or metric, in all spatial dimensions. Distance could no longer be defined by our standard metric of a meter (which currently works in any of our traditional three dimensions since a unit of distance is the same in all three).

What would be the distance from a pen to an atom in the pen? To include our other three dimensions, maybe we should consider the surface of the pen touching the table and an atom of the table. This gives a distance in our traditional space plus one across scale. What are the units that cross scale, where 1 scale unit could equate to 1 meter, 2 scale units could equate to 10 meters and 3 scale units could be 100 meters?

If we require a different 'measuring stick' for distances across scale, we may need to enhance our current measuring tools to address both our traditional three linear dimensions and measurements across non-linear scales. This would suggest different measuring tools that can account for, that can measure, the linear and non-linear distances. Such tools could have a direct impact upon what and how Science measures the universe.

Mathematical Recommendations to Improve Science Model

To gain a perspective on how to include the non-linear scale dimension into a single model of the universe, we could consider a simple model that collapses all three 'standard' dimensions onto a single dimension and then models scale along a separate axis. The collapsed dimension holds all linear dimensions, while the separate dimension is the non-linear dimension. We could model this using complex numbers and the complex plane as our simple model. This model might suggest that we are already making use of mathematics that differentiates our standard dimensions and measurements with a dimension that acts differently.

What is lacking is a singular value for the distance unit across traditional (Real) and complex values. A curious aspect of complex numbers is that we do not have a means of representing complex values as single numeric values – which is what would be needed for the distance unit. Complex numbers are always represented as two numeric values – $x + iy$, where x and y are numeric values, however i is not represented as a numeric value and keeps the two values separated. A single value complex number could provide a measurement for the distance of two objects at different scales. However, this might require a different (potentially more powerful) numeric system that can represent complex numbers as single values.

Potential Implications of a Four-Dimensional Scale Space

Corollary: Consider that a fourth scale dimension would suggest reasons for non-interaction of objects and particles at different scales – say a neutrino and a cell (or our body). Rather than passing 'through' the cell or body (missing all objects of the cell or body), it actually passes 'below' it at a smaller scale.

Speculation: The Flatland analogy would even suggest there is a 'lower scale' boundary to ourselves – under which other particles could pass. For two objects to 'hit' each other, they would need to be at the same level, or have objects or fields at the same common level that could interact. In fact, they would only 'hit' each other if they have objects at common scale levels. This goes to the initial example of touching our finger to a pane of glass.

Observationally, it would appear that most objects we know of in our immediate neighborhood exist with a number of essentially the same scale levels. Our bodies, the table, the book, or computer all have a surface at our level with objects at the molecular and atomic levels and structures of objects in between. Microbes would have an upper level much smaller than ourselves and stars have a surface at a much larger scale than ourselves, yet both microbes and stars have atoms and molecules.

Speculation: Particles (especially if we consider them as fields rather than particles) could move in scale, potentially revolving around a nucleus, not just at different ‘orbits’ but at different, although close, scales as well. How might we measure an object that shifts in scale? Could we pin-point its location in a 3-dimensional model of space or would we have difficulty with its location? If we shower the particle with photons or other particles and it shifts in scale, would we be less likely to determine its location? We might predict the more energy we hit it with, the more it would shift in scale and the harder it would be to locate in our three-dimensional model.

Speculation: How would an object that moves in scale appear to us? It may shrink or expand in size. Since scale is only one dimension, it might not do this equally along all three other axes. So, it might shrink in length but not width or depth. This might be the situation where the object is moving in one of our normal three dimensions at a certain velocity and moves in scale. It might then appear to shrink (or expand) in the direction of motion. This would be a relative appearance, as the object would not actually have shrunk – it would only appear to us that way.

Speculation: What if an object is stationary in our three dimensions yet only moving in scale at a constant velocity? If we were all in free space, we could all consider ourselves at rest and feel we are not moving. If we were moving in the same scale direction at the same velocity as the object, then our measurements should not change in any of our three dimensions – measurements of volume would appear constant.

Now, if we are all relatively stationary in our standard three dimensions on the earth, yet all moving in scale then we should still not observe a change in volume. There should still be some indication of the motion in scale. Constant movement in scale would appear to be moving exponentially (e.g., first 1 unit, then 10 units, then 100 units). In other words, the objects in motion in scale would appear to be accelerating. This is an important consideration, as a constant velocity in only scale would appear to an object at rest (in our three dimensions) as moving at a constant acceleration on us. Using F=ma, this would indicate we should measure a force on an object moving in scale. Since the object is not moving in the dimensions we directly perceive, it would appear as an undefined force on the object and might be indistinguishable to a known accelerative force.

Conclusion

As we move toward digitally modeling entire bodies in the universe, we will find the need for locating objects in scale, in addition to locating objects in three-dimensional space. This will require a four-dimensional scientific model of space, which will require some distance measure and units that cross non-linear scale. Since the distance metric of scale is different than our traditional three-dimensional space, we may need to develop new values of measurement (e.g., a complex numeric system) and potentially new mathematics to understand the characteristics of such a space.