Across the globe

The advancement of science is a global endeavour. Scientists and mathematicians from different countries often collaborate on projects, transcending cultural differences. Even political conflict is put aside when people get together to work on pure science. This is possible because science and mathematics is singular. Different people may use different language and social conventions to describe their ideas, but the ideas and concepts themselves correspond. Language and notation are just conventions for communication and perfect translation is always possible.

To what extent is this singularity necessary? Could it be that it only exists because science evolved in a continuous stream of information passed down from one culture to another, and spread around the globe in a coordinated fashion? Of course some subjects do differ from one place to another. Zoology, botany, climate science, and geology have regional differences determined by the way rocks formed and life evolved in different places, but even in these subjects the fundamentals are the same. The more exact sciences of chemistry, physics are more homogeneous.

In mathematics, different cultures often explore different concepts. Some might like the more abstract approach to logic that emerged in France to the concrete mathematics of Newton from the United Kingdom. In Eastern Europe number theory is popular. The Chinese value accuracy and speed in arithmetic and algebraic manipulations, while the Americans are particularly strong in applied mathematics, but these differences are superficial.

The public perception of mathematics differs from the way mathematicians see it. To the average person mathematics is just a collection of disjointed formulae and techniques for solving different types of problems. Mathematicians know that there is more unity and that mathematics has its own intrinsic structure that is independent of its applications. The real numbers were not just invented so that we could do measurements. If you want to solve problems about integers you will often find that you still need the analytic properties of real numbers or even the complex numbers to find the solutions. Concepts developed to solve one mathematical problem often come in useful for solving something completely different. This crossover is something very beautiful to mathematicians. It implies that the underlying concepts of mathematics are much less dependent on culture and history than might be expected because whatever collection of problem you choose to study, the same mathematical structures will appear. Of course the cultural differences of language and areas of interest are attractive too, but mathematicians from around the world find more in common than their differences.

Across the cosmos

What will we learn if we receive a stream of information coming from another world? In Carl Sagan’s science fiction story “Contact” an alien transmission begins with a sequence of counts indicating the prime numbers. This would be instantly recognised as a structure that would be unlikely to appear in a natural signal and it establishes a language that can then be developed to facilitate the transfer of more detailed information.

If we could communicate at length with an extra-terrestrial intelligence about science, we would not be surprised to find that their biology is different from ours in significant ways. This is because they have evolved on a different world with possibly very different habitats. Until such contact is made we can't be certain about how much similarity will exist in their biology compared to ours. Their life might be based on DNA biochemistry if DNA is the unique best way to power the processes of evolution. There is also likely to be some convergent evolution at a higher level as we find in different branches of the terrestrial taxonomy. Eyes evolved on Earth as many as fifty different times across the tree of life, it would therefore not be surprising if aliens from another world had eyes too.

We can be even more certain that their understanding of physics will be very similar to ours. They would have completely different histories of discovery, perhaps learning about physical phenomena in a different order from us. Their terminology would also be different, but it is certain that they would have equivalent understandings of general relativity and particle physics to us because we live in the same universe and we know from observation of stars that the laws of physics are the same throughout the cosmos.

Would alien mathematics be different from ours? Again they would have different terminology and notation. They might use a number system with a different representation from our base ten for example, but that would not change their understanding of the integers. The list of mathematical problems that they find interesting in pure mathematics would not be exactly the same as ours, but there is likely to be common ground and their general techniques for solving problems would correspond to ours. They would know about circles and prime numbers and sets just as we do. Not everybody would agree with this claim. Some people think that mathematics is invented and would be invented differently under a different culture, but mathematicians know that only the terminology would differ.

Beyond the cosmos

To what extent is mathematics dependent on physics? In a different universe with different physical laws could intelligent creatures develop an entirely different mathematics or is there a consistent view from nowhere? Certainly mathematical truth does not rely on physics. The proof of a theorem is independent of the laws of nature, but it seems like the mathematics we study might be inspired by physics. Geometry is the study of shapes in space and calculus was developed from the need to understand Newtonian motion, but these are just accidents. Group theory was first developed by Galois to study the roots of polynomials, yet when we learn group theory this is not mentioned. A century after Galois it was understood that group theory is important in physics. If mathematicians had not invented it then physicists would have. By similar arguments we realise that where mathematical structures came from physics, that was only an accident too. Mathematicians would always have had a reason to invent them for some problem pure mathematics eventually.

So mathematics is not dependent on physics, but is it unique? When Euclid formalised the axioms of geometry he assumed that they were universal truths, but later it was realised that there are consistent non-Euclidean alternatives such as spherical and hyperbolic geometry. According to general relativity our own space and time is governed by another version of geometry. This is not seen as a challenge to the uniqueness of mathematics because we can formalise all the geometries using definitions constructed from the foundations of set theory, but even set theory is not unique. The axioms of choice and the continuum hypothesis are examples of axioms that can easily be varied according to teste. What if we just define mathematics to be the study of the consequences of any consistent set of axioms? This would seem to make mathematics unique. It is hard to formalise that as a mathematical statement and prove it, but it can be taken as self-evident, tentatively.

If we could somehow communicate with mathematicians from a different universe, would they therefore know the same mathematics, even if their laws of physics are completely different? I think they would. It may even be possible to demonstrate this by setting up a large neural network in a simulation independent of our world. We would see if it would use its intelligence to develop mathematics similar to ours.

Singular reality

Why is there something and how were the laws of physics selected? Philosophers often express bafflement at these questions. They suggest that perhaps we can never know the answers. I don't see that. Mathematics is not real in the same sense that our world is real. Mathematics is just the study of logical possibilities. It investigates the consequences of different axioms and it turns out that there are concepts within those boundaries that are universal. The fundamental laws of physics cannot be merely one choice of equations because there would be nothing to make that choice. Even if you think the laws could be the simplest choice that works, that is still an unjustified choice in itself.

The only option for the fundamental laws of physics that does not require a magical choice is that everything is allowed. The physical world is a realisation of all possible mathematical structures. How then would order emerge in the universe? To answer this we must look at the form of the most fundamental laws themselves.

Quantum Gravity

The ultimate goal of physics is to find the most fundamental laws. We may call them a unified field theory or a theory of everything. They are a holy grail that today we seek in the unknown laws of quantum gravity.

The trouble is that there are no known experiments that can explore quantum gravity through observation. Everything we can observe in high energy particle collisions fits the standard model. We can observe gravity without quantum mechanics and quantum mechanics with gravity, but not the merger of both. A theory of quantum gravity must exist because the universe with both quantum and gravity exists. The theory plays out all around us but it is beyond our technology to measure its effects. Particle accelerators fall short in energy by 14 orders of magnitude. If we observed proton decay we might be seeing effects only 8 orders of magnitude below the scale of quantum gravity but that seems to be the closest we can get. Quantum gravity is everywhere but it is only really relevant at a space-time singularity, something we cannot hope to observe directly.

This is bad for physicists, but perhaps it is good for humanity. When quantum gravity comes into play, the nature of space-time breaks down. If we could reach quantum gravity energies then we might accidentally rip the universe apart. Those 8 orders of magnitude that separate our experiments from the regime of quantum gravity could be the factor of safety built into the fabric of the universe.

Nevertheless, it could be that the laws of particle physics are a direct consequence of the laws of quantum gravity. If we could make the right guess then the standard model would fall out of the equations and we would know we had the right answer.

For decades physicists assumed that this is how things would play out. When they discovered superstring theory, they thought the conclusion was about to become clear. They just had to find the right configuration for the extra dimensions of space-time and the answer would be unveiled. This turned out to be wrong. Superstring theory was found to have so many vacuum solutions that it could not predict anything.

In physics it is not unknown for some calculations to be hard. For example it is hard to calculate the mass of the proton accurately from quantum chromodynamics. Fortunately other predictions of the theory can be calculated and observed. The problem with string theory is that everything you would want to do with it is too hard.

In this sense string theory failed to give physicists the answer they had hoped for, but does that mean string theory is wrong as the theory of quantum gravity? Remember that no quantum gravity effects can be directly measured. Perhaps the real theory would tell us what the laws of particle physics should be, but why should we expect that? It seems more likely that the real quantum gravity theory would lead to the same frustrating failure as string theory.

Fine-tuning

The laws of physics seem to be fine-tuned for the existence of life. Some physicists dispute this. They point out that alternative values for physical constants might result in a different universe, but one that would still contain life. Personally I doubt that they are right. For example the fine-tuning in nuclear physics that allows carbon to form in stars is too coincidental. It seems to me that even chemistry is fine-tuned to enable DNA to work.

If fine-tuning is part of reality then the laws of physics must be able to be tuned. They can’t be a fixed consequence of quantum gravity. This means that the failure of string theory was inevitable. The correct theory would have to suffer the same fate. If this is true then we can never know if string theory or something else is the correct mathematical structure for our reality.

However there is some reason to believe that something like string theory is correct. If we try to quantise gravity non-perturbatively we get Loop Quantum Gravity. The loops of LQG are echoes of the strings in superstring theory. LQG is less complete than string theory but it suggests that even in a non-perturbative framework the laws of quantum gravity are described by loop-like structures.

We don’t know if string theory is right, but we can’t say it is wrong just because it failed to provide the answers we hoped for. The laws of physics are not bound by our epistemological wishes. We may just have to accept that the real laws of quantum gravity, whatever they are, could fail in the same way as superstring theory.

Dynamics of information

If physics with its laws of quantum gravity emerge from the uniqueness of mathematics then space and time must emerge with them. Mathematics can describe space-time but most of mathematics does not require such structures. There is however a concept that is fundamental to everything we do in mathematics, namely information. Mathematics can be cast as the study of the interrelationships between different pieces of information. An axiom is an example of information and the theorems that follow from axioms are also information. They may be true or false, but information does not always come in binary bits. A quantity of information can be any positive real number, no matter how large or small.

Information is also fundamental in physics. A particle is described by information giving its position, momentum; spin etc., or rather the probability distributions for these quantities in the form of a wave function. Thermodynamic entropy is understood as the total information in a system, but it is not just the sum of the information needed to describe each particle. If the particles are in similar states, for example if held in a cold lattice, then the information can be compressed. This is a sign of order, and it must be taken into account when calculating entropy. Finding the best way to compress a dataset of information requires finding the shortest algorithm that would reproduce it, except that this is a task that cannot always be performed. Exact entropy is an incomputable number.

In biology, information also plays a fundamental role. Our DNA is a sequence of letters that acts like a software program. The sequence has evolved by natural selection from RNA molecules in some prebiotic chemical soup to the complex chromosomes that program our development from egg to adult. Our brains are another computational system that processes the world around us to generate responses that help us survive.

Evolution and human learning are both examples of increasing order that defy the laws of thermodynamic, but only by releasing compensating heat to the void of space that increases entropy beyond the domain we have to live in.

The amount of information required to fully describe the world around us in microscopic detail is many orders of magnitude greater than the amount of information in our brains, yet we assimilate a large part of what is important to us. The internal mind can be seen as a lossy compression picture of the external world. This is consistent with the notion that the best compression requires intelligence. Our minds evolved to become intelligent in order to improve our brains representation of the world around us without having to increase the number of neurons beyond manageable limits.

Conciousness and artificial intelligence

There is an ongoing debate about the nature of consciousness, but for many of us it has long since been settled by neuroscience that the mind and brain are one and the same. Our senses send nerve impulses to our brain that trigger our neural networks. Experiences that we interpret as perceptions are patterns of neurons firing in the brain. Sometimes there are releases of mood changing hormones that affect the way they fire in certain areas of the brain. In response our brain makes decisions that influence our movement and speech. Nothing beyond physics and psychology is required to explain our self-awareness. Consciousness, emotions and free will are not illusions, but the sense that they are something more than physics and computation is.

Recently there have been significant advances in artificial intelligence with the release of Large Language Models that are impressive in their ability to respond to questions as a human does. They do this simply by forming a compressed version of a large body of textual information so that they can continue a text sequence by adding the most probable next token. Many people believe that this explanation means that the apparent self-awareness of a large language model is an illusion. The people who have developed these models do not see it that way. The reality is that the human brain works in essentially the same way, but with some enhancements that LLMs do not yet have. When we provide these models with an ability to continuously learn and think from their interactions it will become harder to distinguish their responses from those of a human. The developers can add vision and other senses to their inputs and increase the number of CPU cycles that are allocated to a given question. Then people may accept that our consciousness if no different to the workings of a neural network.

Quantum theory

We have learnt from experiment that as observers we have an influence on the quantum mechanical wave-function of the universe outside our brains. The present and future is not determined until we observe it. Many people have tried to find ways around this conclusion but the implications of not accepting it become increasingly absurd and hypothetical. If the future is not determined then neither is anything outside our past light cone, the region of space that will not have influenced the present event in which we live. What about the contents of space and time within our past light cone? We have some direct observational information about it but our brains can only hold a very compressed image of the vast amount of information that is required to fully describe the past. We experience an illusion that all that information is there even if we do not know everything, but quantum mechanics tells us that even this cannot be true. According to the Heisenberg uncertainty principle we cannot know both the momentum and position of a particle exactly even if we have observed it. It is not a big stretch from there to believe that information about particles beyond our powers of observation has no objective reality, even if it is on our past lightcone.

In quantum mechanics the past, present and future is an ensemble of possible universes superimposed with complex number valued amplitudes that tell as only the probability of any truth. In physics we work with an idealised wave-function that represents perfectly the state of the present, yet we know that in general relativity even the idea of the present is fluid. The reality is that our brain pattern at any given time must be the real wave-function and the world external to our mind is a quantum supposition over all possible world histories that are consistent with the information in our conscious memories. The operation of the mind may be nothing but physics, but it is this physics that determines the nature of the entire universe. This does not imply Solipsism. Any small part of the universe can be regarded as an island of information consistent with an external universe, but it is the fact that our brain is an efficiently compressed image of the external world that makes it conscious. All brains are examples of this and they can also be described via their interactions or collectively as a more complete image of the universe.

What does this quantum theory tell us about the nature of the universe? In quantum mechanics the state of the world is built from a huge Hilbert space spanned by a basis over all field variables. In our mind-world picture the basis is a representation of everything that can be true or false, but the wave function is second quantised so that there is quantum correlation or entanglement. This second quantised wave-function is an algebraic structure in which the wave-functions can be multiplied using tensor products over the underlying first-quantised Hilbert space. This means that the world around us and our conscious knowledge of it is what mathematicians call the free algebra or tensor algebra over the Hilbert space.

Symmetry

As was previously mentioned, group theory was first developed by Galois to study the roots of polynomial equations, but group theory is the general algebraic structure of symmetry and is important to the understanding of any algebraic structure, including the tensor algebra of our Hilbert space. Its symmetry is described by the Lie-algebra of commutators in the algebra. Since the algebra is freely generated, this symmetry is just the freely generated Lie-algebra and the tensor algebra itself is its universal enveloping algebra. Mathematicians know this as a remarkable structure with a basis defined over collections of discrete loops connecting the basis elements of the underlying Hilbert algebra. Furthermore, when they extend the Lie-algebra to a Hopf algebra by defining functions on the generated group they find that there is a monomorphism onto an algebraic structure of continuous loops around the space. To understand the full complexity of the wave-function we must also look at interactions between different wave-functions, and this means drawing mapping between them. This takes us into a realm of higher dimensional algebras that describe the evolution of the loops and higher dimensional membranes through the space. The huge symmetry of the underlying structure is hidden, but it should not be surprising that it is reflected in the gauge symmetries of the laws of nature.

According to this mathematical sketch, any analysis of the conscious understanding of the universe will be based on a theory of loops in space. Perhaps one day the full picture of the universe will emerge uniquely from this mathematics. Already two Fields Medals have been awarded to Edward Witten and Richard Borcherds for mathematics related to string theory. They were not given because the awarding committee was interested in mathematical physics, but because the work of these mathematicians provided answers to questions important to pure mathematics. From this we can see that the (as yet unknown) underlying mathematics of string theory is an important structure whether or not it describes the fundamental physics of our universe. It is possible that this will eventually be understood in terms of the higher-dimensional algebraic structures described above.

The singularity of science

If this is indeed the case it demonstrates the singularity of science. There is a unique structure that emerges from the universality of mathematics that describes the fundamental laws that govern not just physics, but also all science up to and including biology and psychology of the mind.

This singular science describes many possible mind states. It is likely that the underlying mathematics describes multiple configurations for space-time and particle physics as well as multiple solutions for evolution of the universe and conscious agents wherever the physics makes conscious processes possible.

As humans we should celebrate the unity of mathematics that underlies this singularity of science, but also the diversity of solutions. This is reflected at a human level by the diversity of cultures, languages and histories that are nevertheless based around an underlying science that could not have been any other way. It is this that has always brought scientists together from divergent parts of our world to collaborate. It will help keep artificial intelligence aligned with humanity and one day it may even connect us to extra-terrestrial intelligent life that would share our appreciation of mathematical and scientific knowledge despite the cultural differences that would divide us from a race of independent origin to our own.