Causal Asymmetry *in a quantum world*

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AIP 2018

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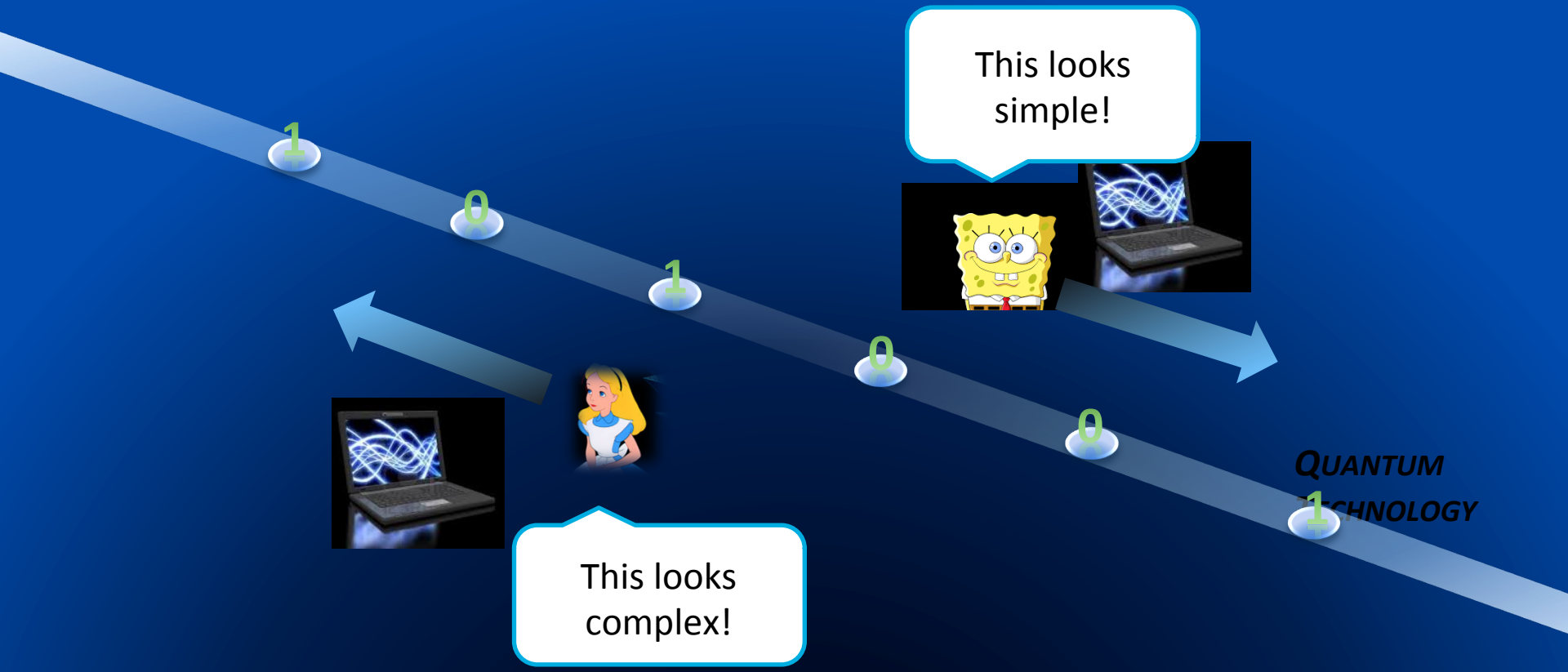
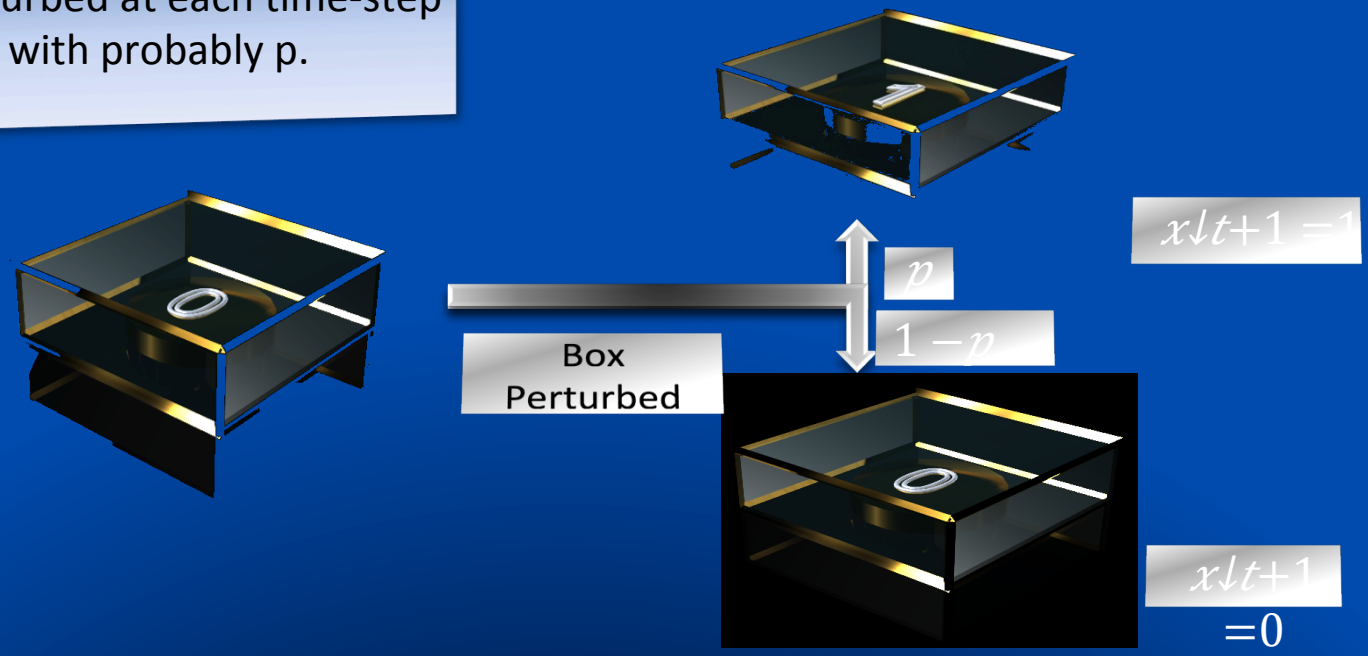
Watching a movie in reverse time

Causal Asymmetry

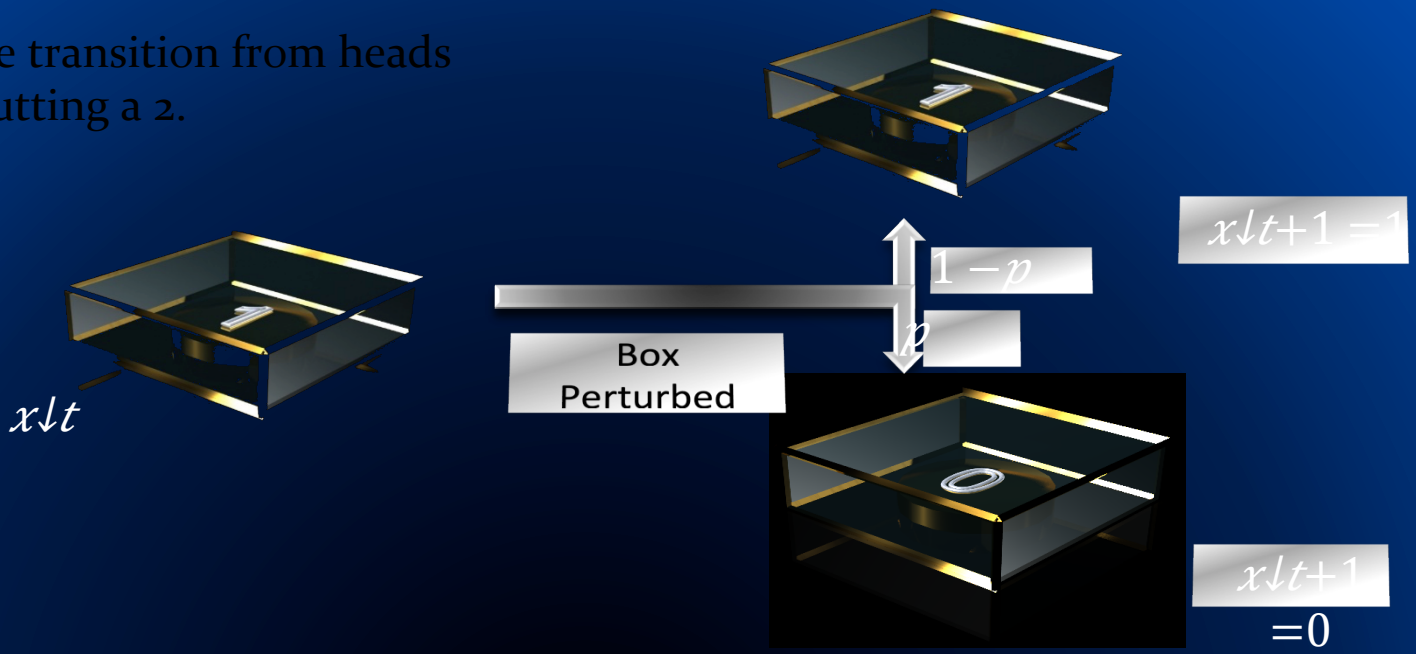


J. Crutchfield, C. Ellison, and J. Mahoney, *'Time's Barbed Arrow: Irreversibility, Crypticity, and Stored Information'* Phys. Rev. Lett 103, 094101, (2009)

Box with coin, perturbed at each time-step such that coin flips with probably p.

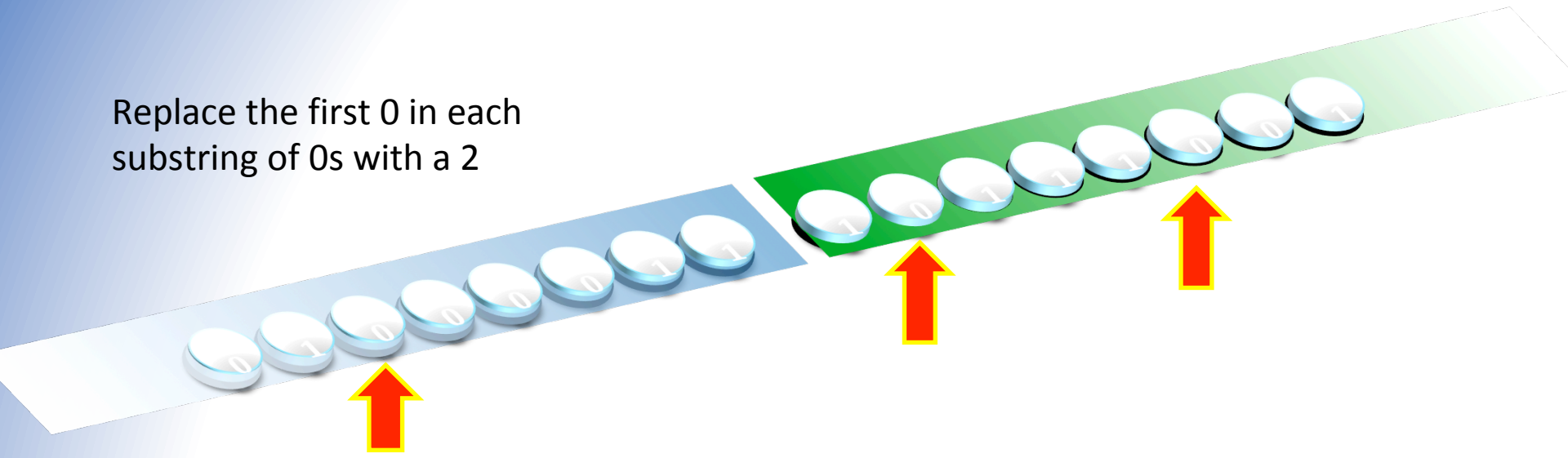


That heralds the transition from heads to tails by outputting a 2.



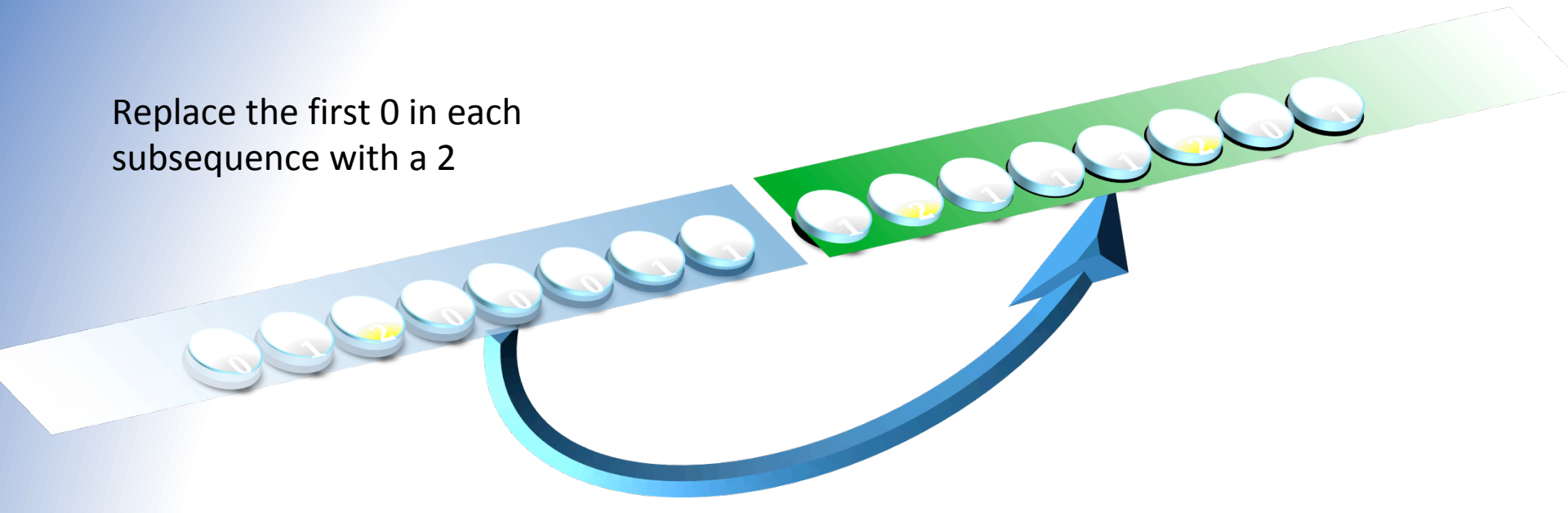
Introducing Asymmetry

Replace the first 0 in each substring of 0s with a 2



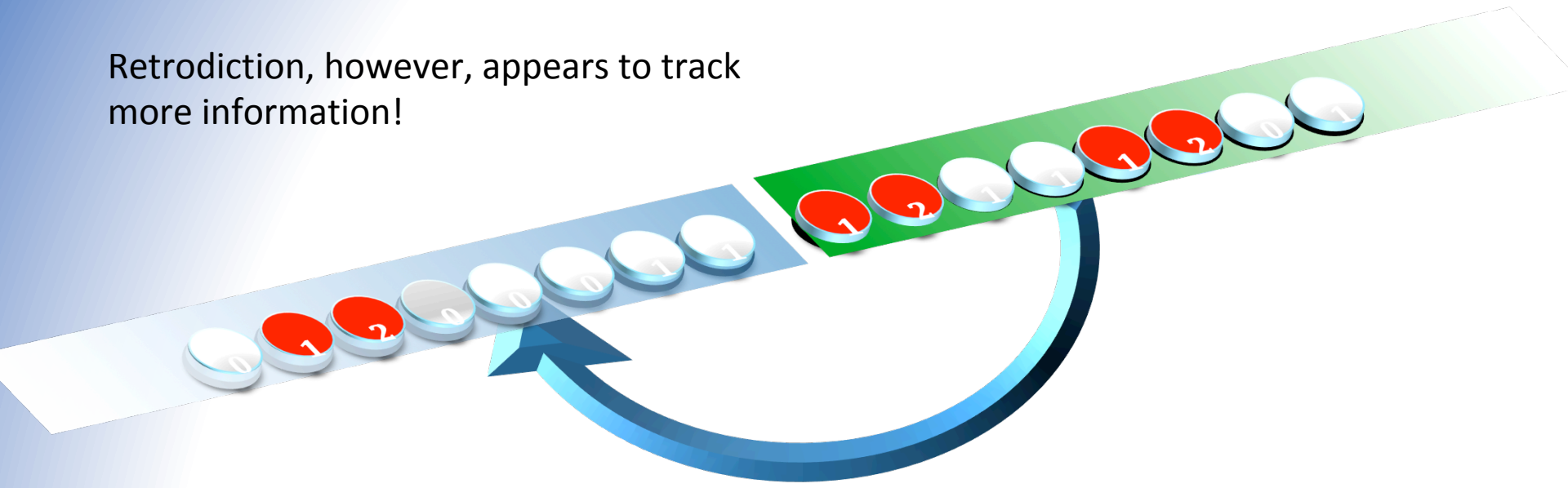
Introducing Asymmetry

Replace the first 0 in each subsequence with a 2



Introducing Asymmetry

Retrodiction, however, appears to track more information!

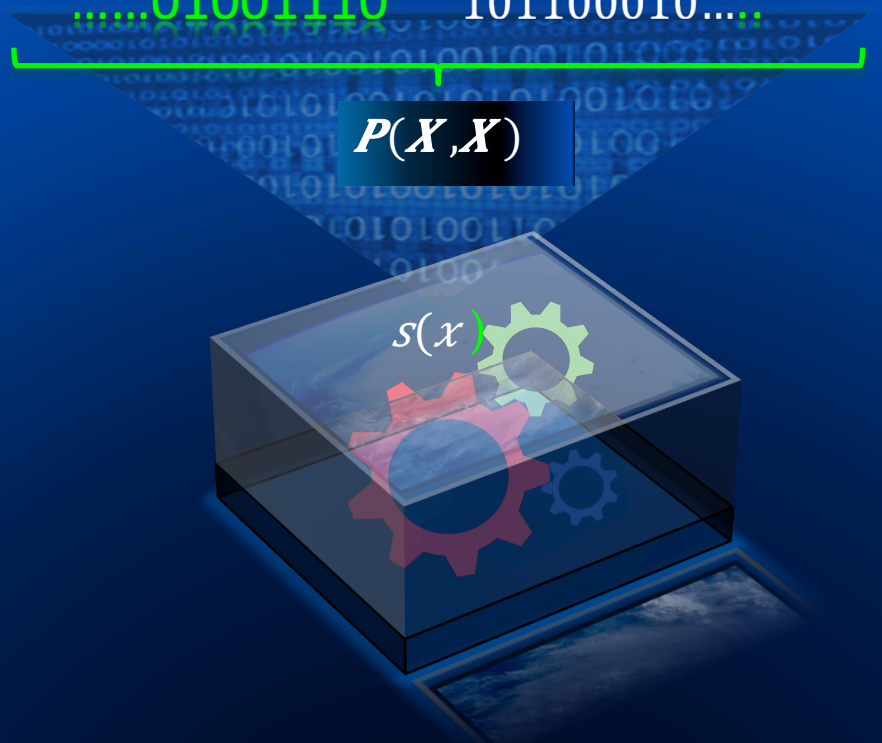


Complex Processes

We often observe complex systems as a sequence of outputs.

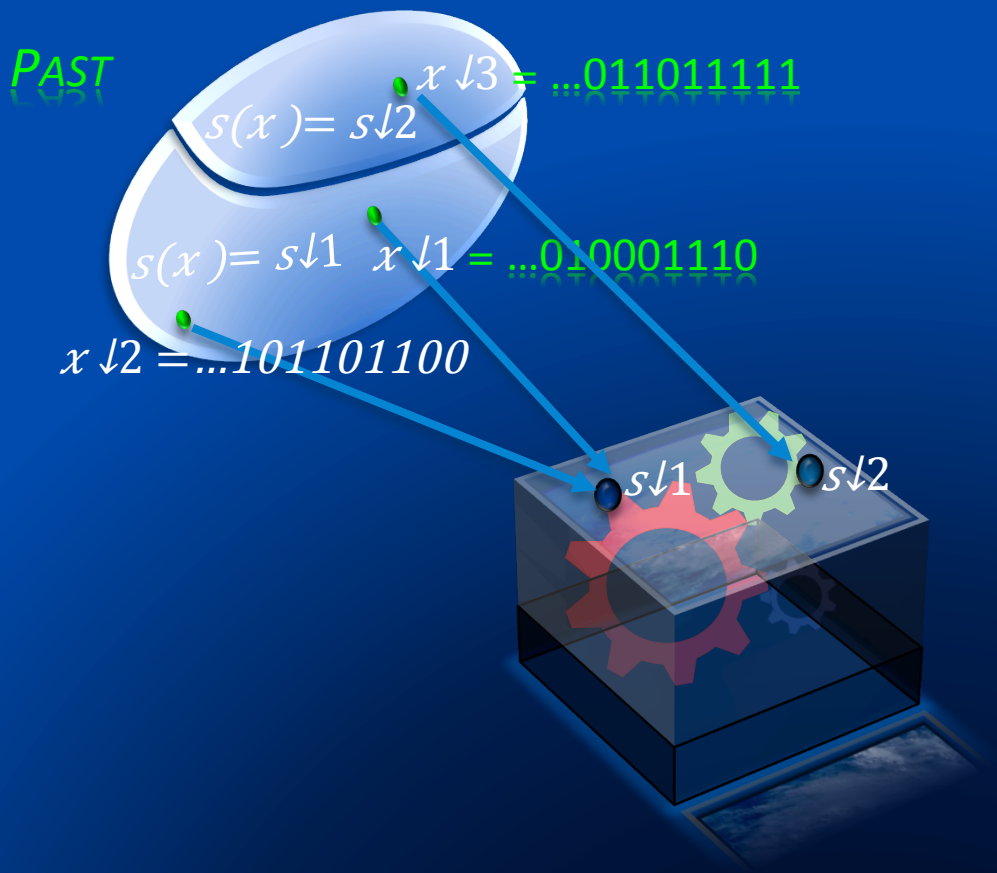
Causal Model

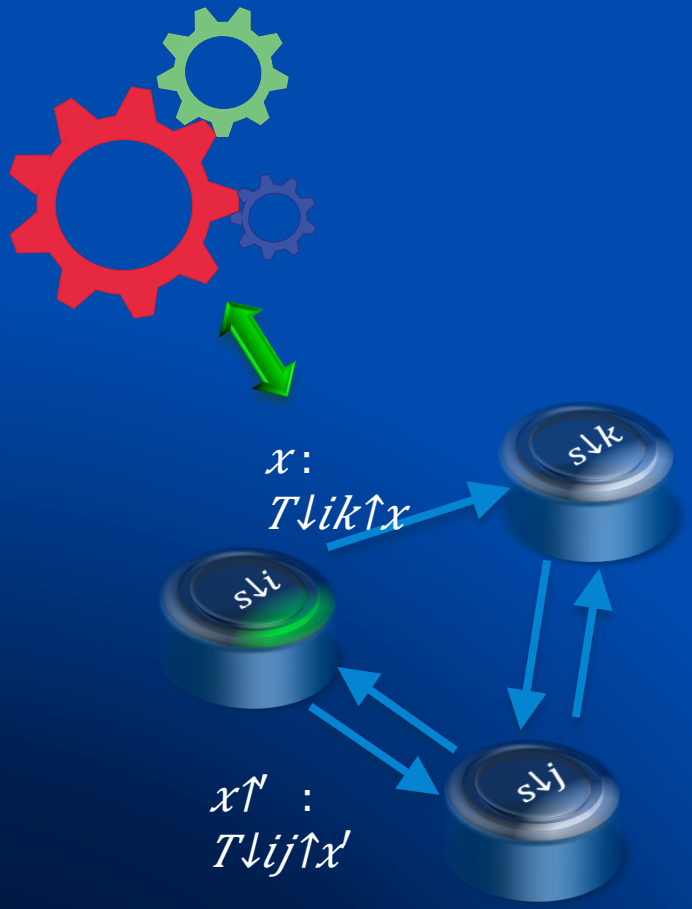
PAST ……01001110 101100010….. FUTURE



A causal model is a deterministic mechanism for taking information directly available from the past and using it to generate a prediction about the future.

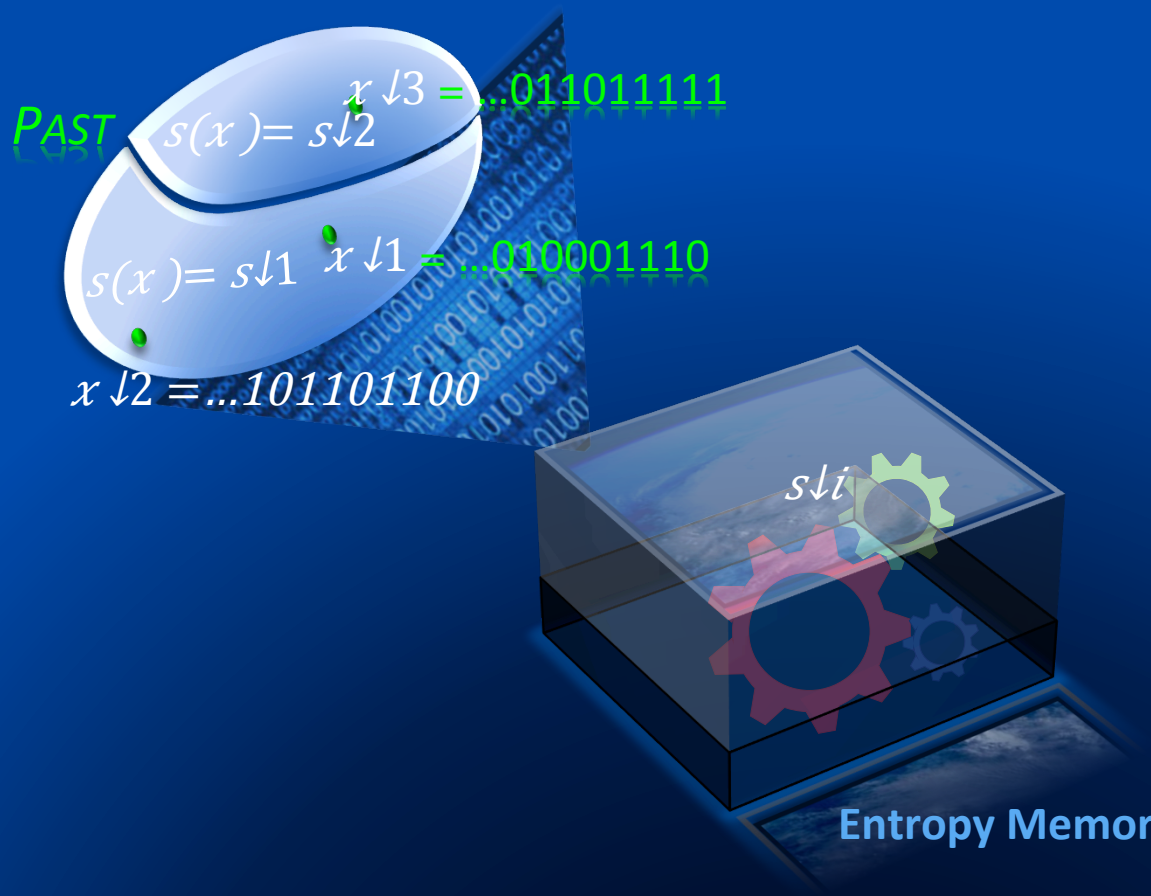
Causal Model: ε-machines





$x\downarrow 1 \sim\downarrow\varepsilon x\downarrow 2$ iff $P(X \mid X = x\downarrow 1) = P(X \mid X = x\downarrow 2)$

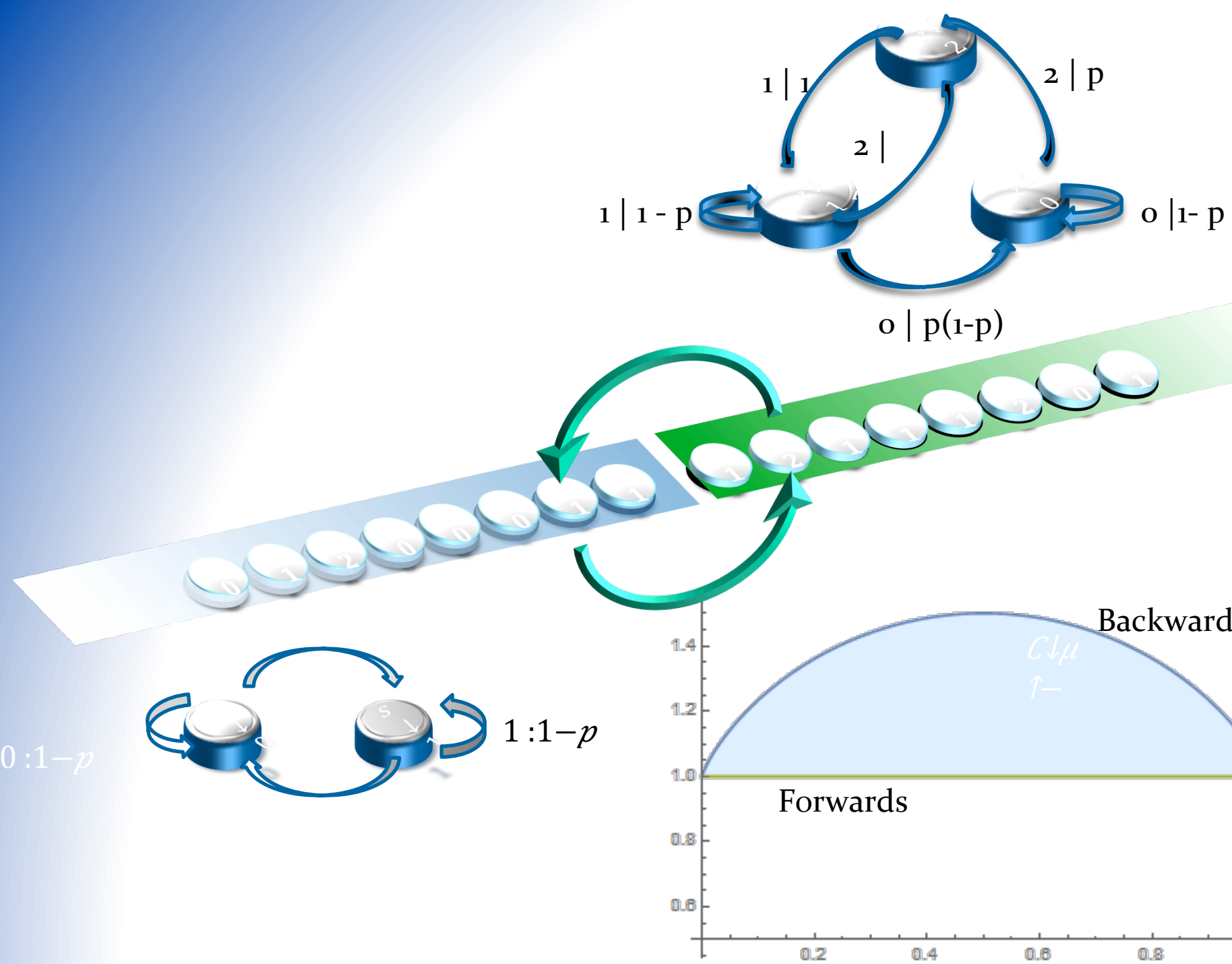
How complex is a process to predict?



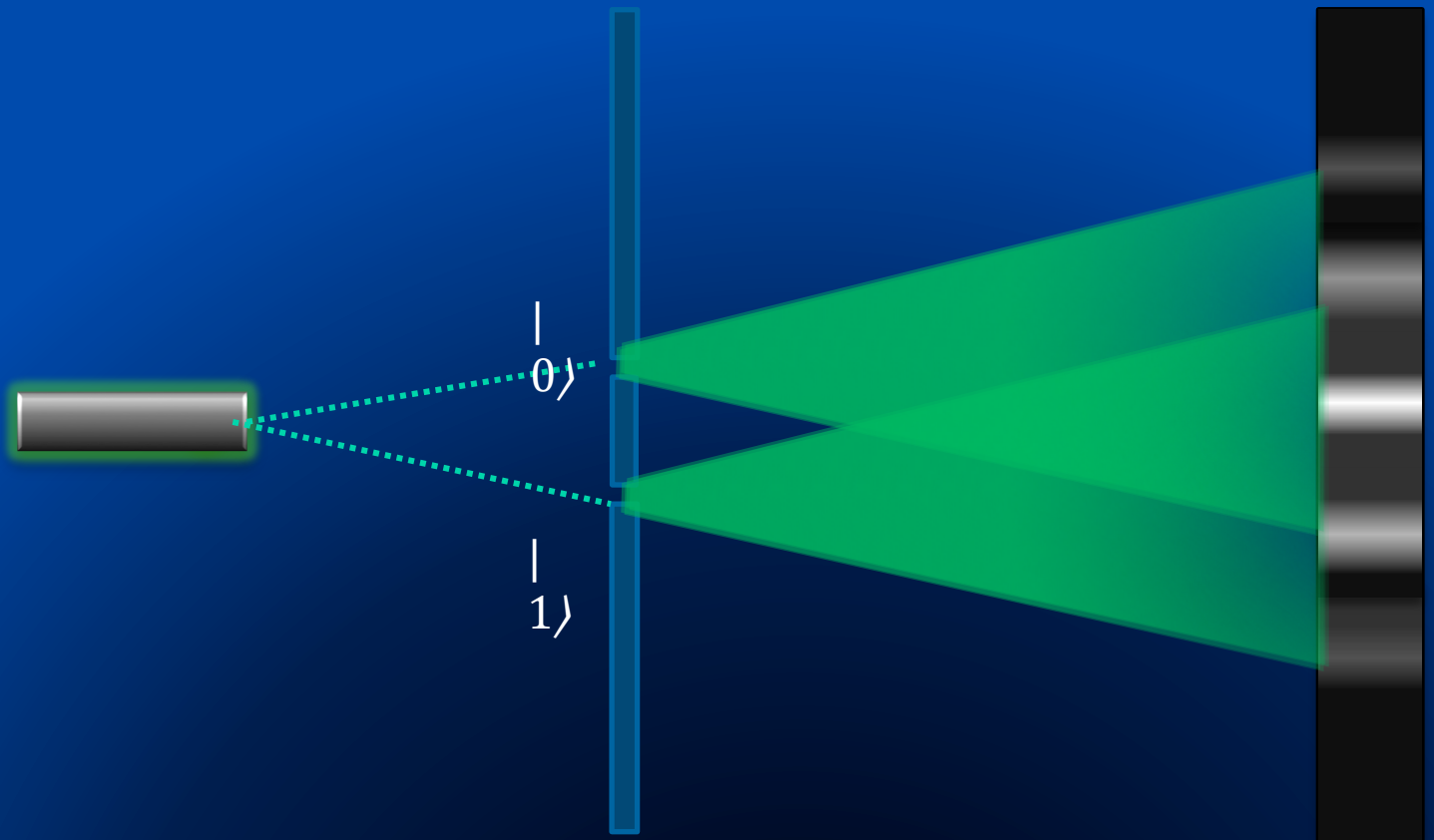
Entropy Memory Cost

$$C\downarrow\mu = H(S) = -\sum\uparrow\blacksquare p(s\downarrow i)\log\square p(s\downarrow i)$$

(Amount of information needed to communicate the causal state)


1 | 1
2 | p
2 |
1 | 1 - p
0 |1- p
0 | p(1-p)
1 :1−p
0 :1−p
Backwards
Forwards
1.4
1.2
1.0
0.8
0.6
0.2
0.4
0.6
0.8
1.0

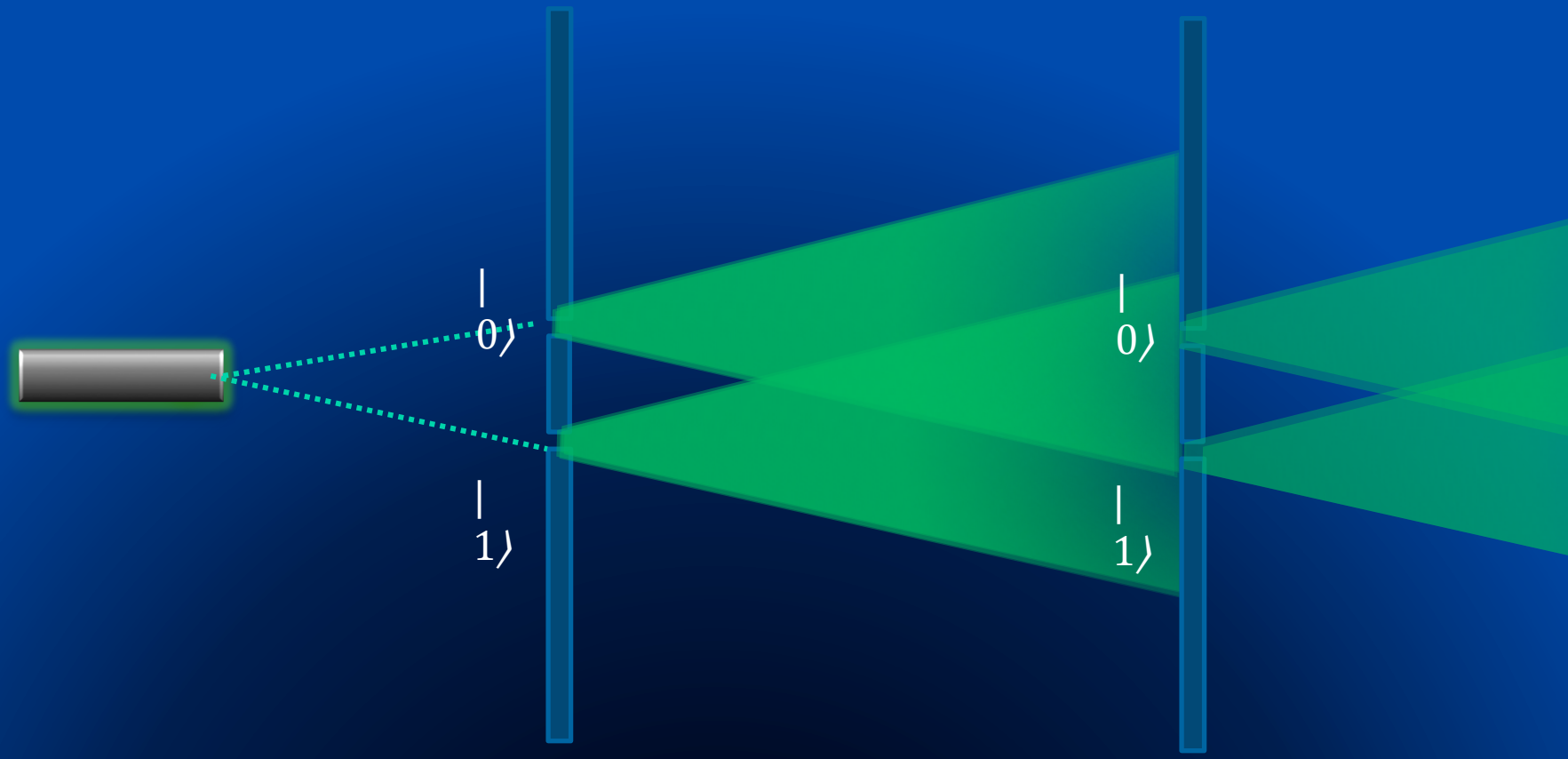
Double Slit Experiment



$|\varphi\rangle = |0\rangle + |1\rangle$

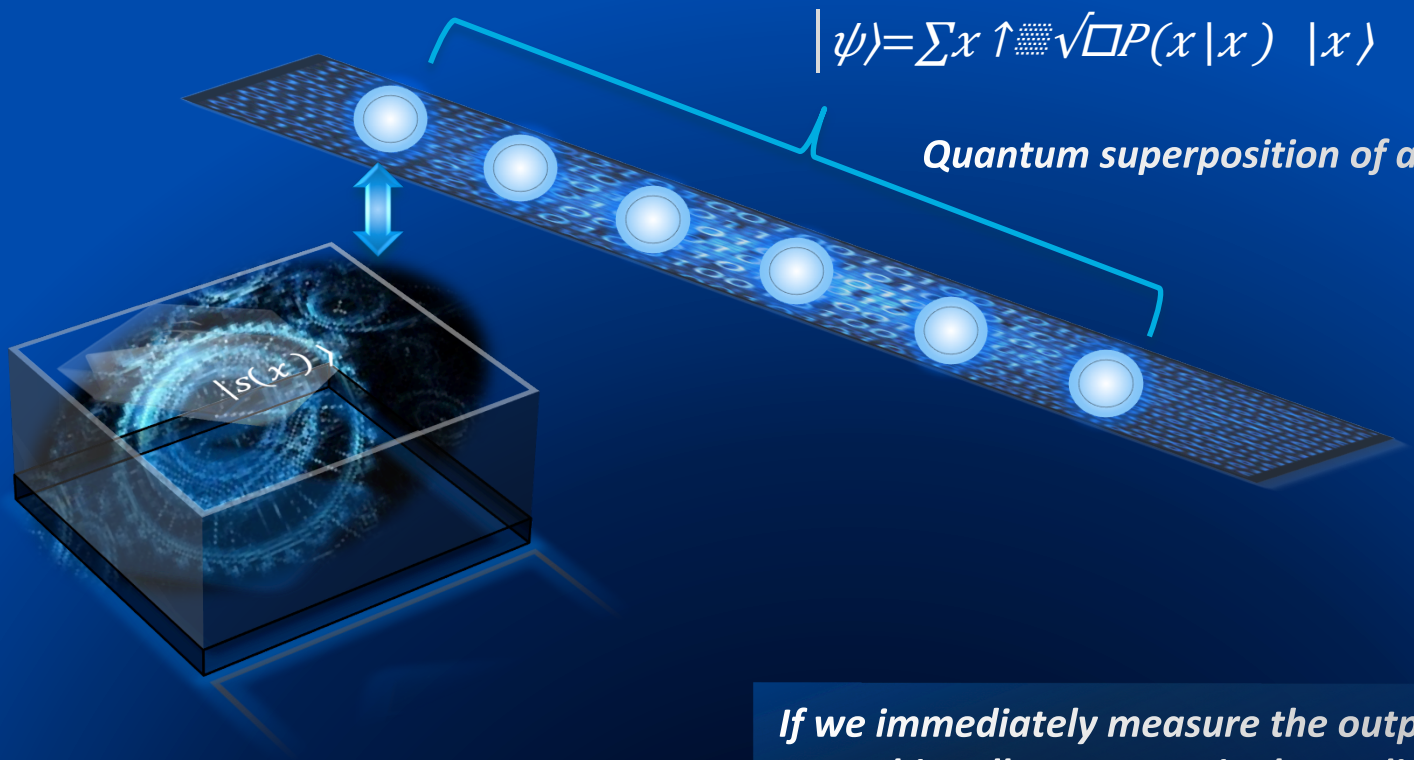
A particle can go through a superposition of both slits!

Double Slit Experiment



$$|\varphi\rangle = |0\rangle + |1\rangle$$

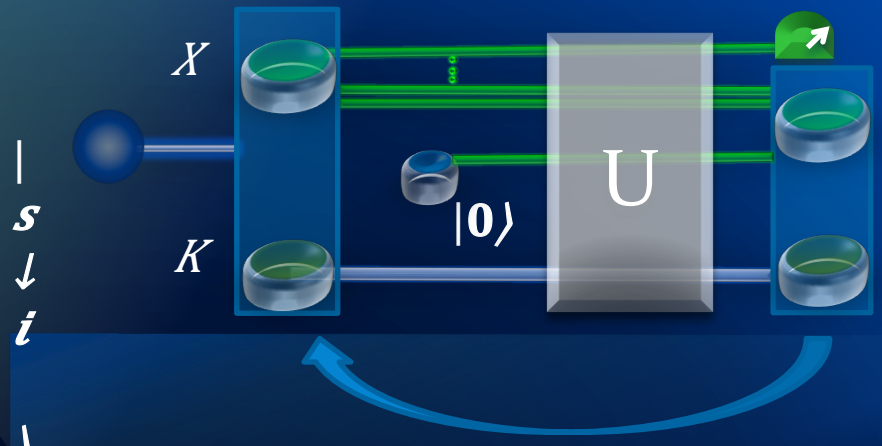
Quantum superpositions of all possible futures



If we immediately measure the output at each time step, this collapses to a single predicted future output sequence, statistically identical to the model's classical counterpart.

Quantum superpositions of all possible futures

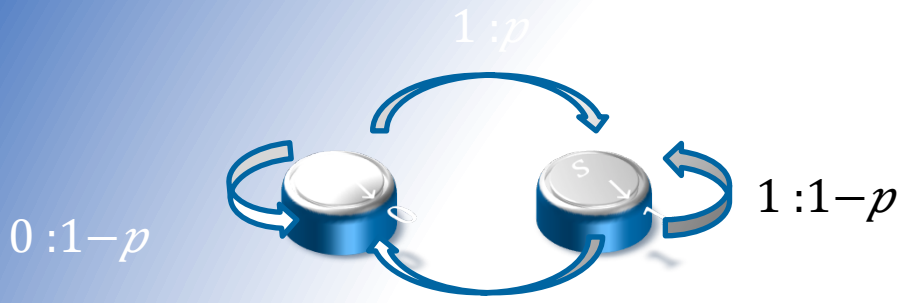
$S(\sum\uparrow p\downarrow i \,|s\downarrow i\rangle\langle s\downarrow i\,|) < C\downarrow\mu$



Repeat at next time step

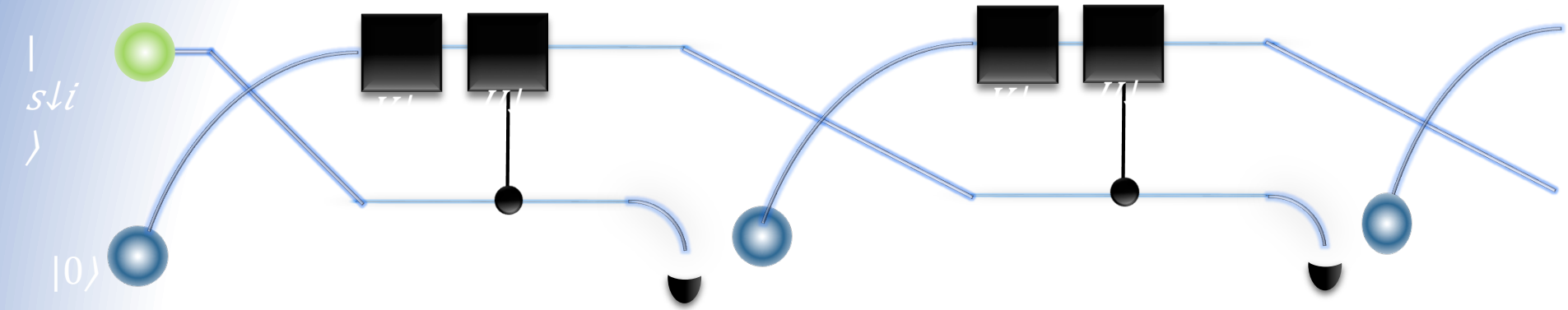
$U|s\downarrow i\rangle|0\rangle=\sum k,x\uparrow \sqrt{}T\downarrow ik\uparrow x \;|x\rangle|s\downarrow k\rangle$

Optimal Quantum Predictor

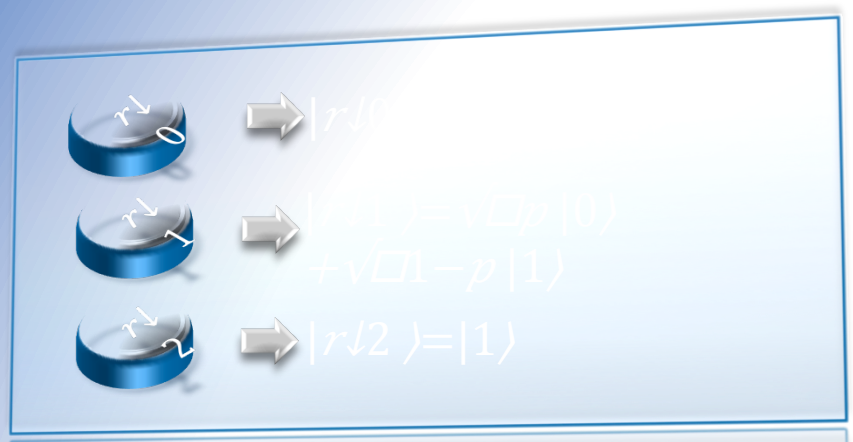


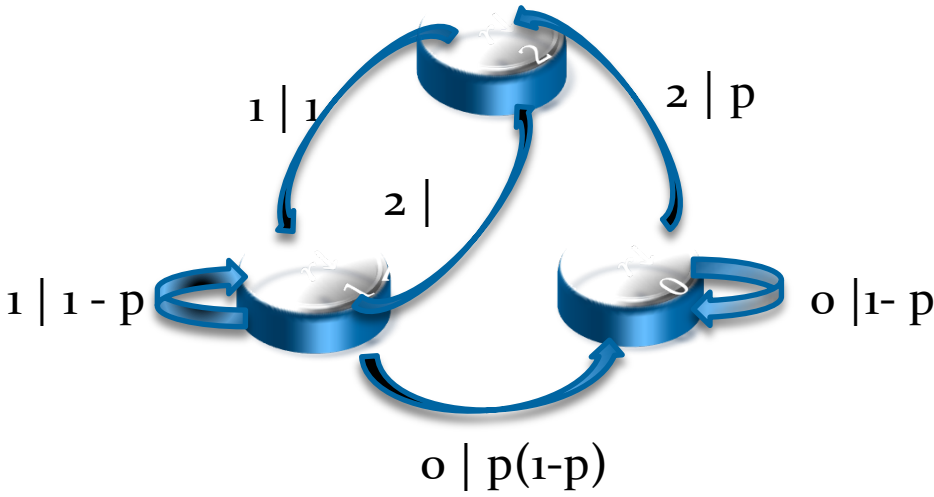
Encode as $|s\downarrow 1\rangle = \sqrt{p}\,|2\rangle + \sqrt{1-p}\,|1\rangle$

Encode as $|s\downarrow 0\rangle = \sqrt{1-p}\,|0\rangle + \sqrt{p}\,|1\rangle$

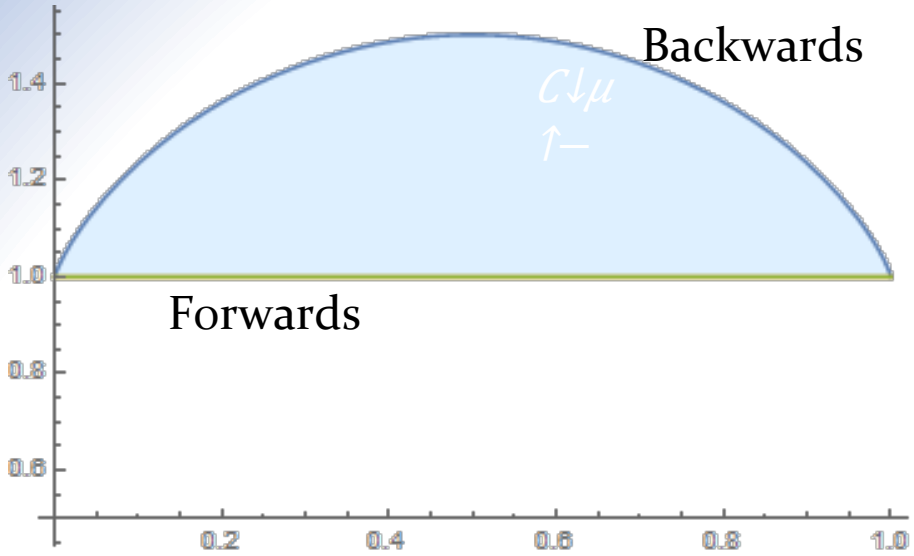


Optimal Quantum Retrodictor

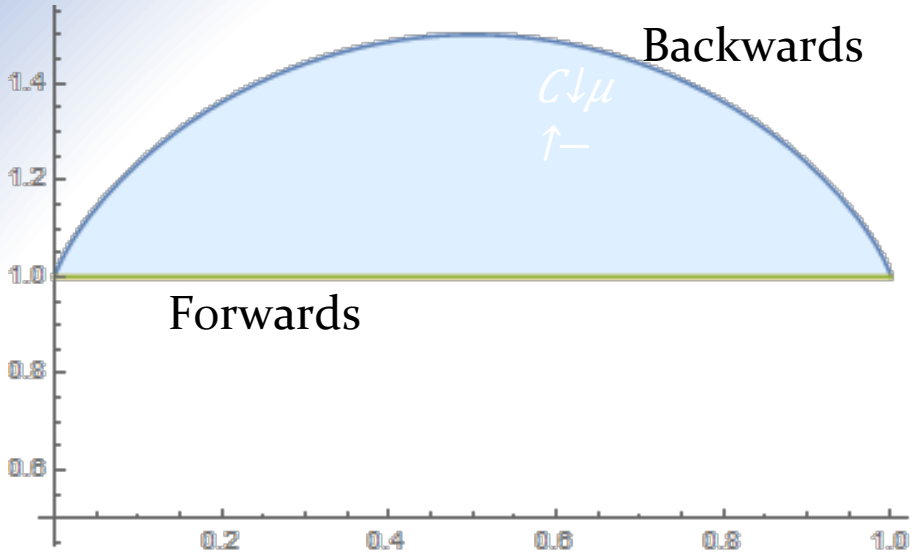




Classical

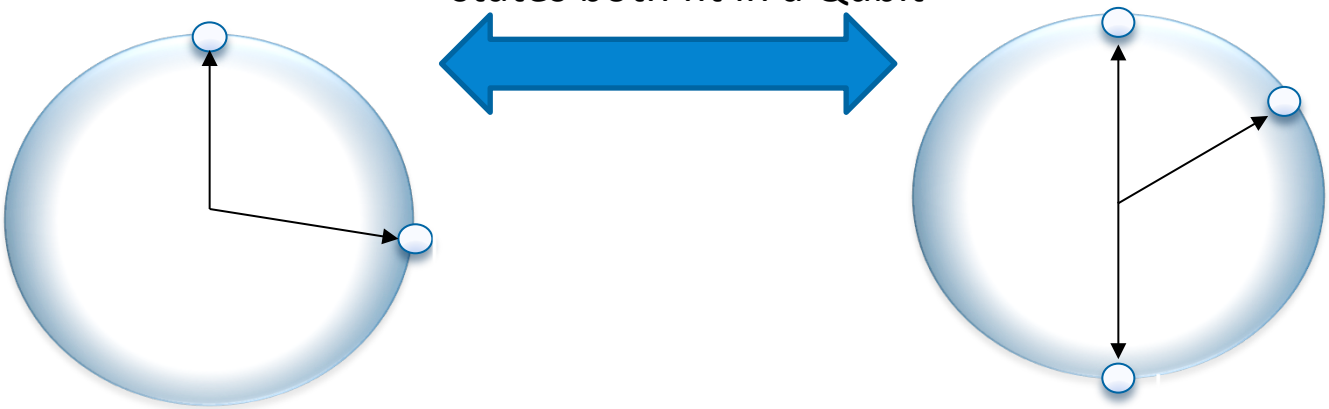

Backwards
$C\downarrow\mu$
$\uparrow-$
Forwards
1.4
1.2
1.0
0.8
0.6
0.2
0.4
0.6
0.8
1.0

Classical



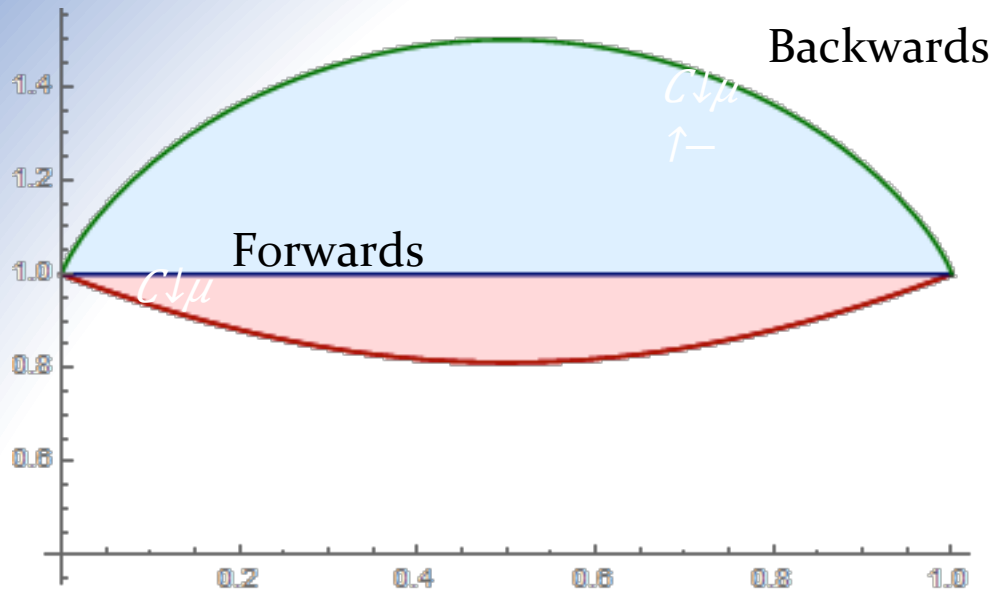
Quantum

Causal and Retrocausal states both fit in a Qubit

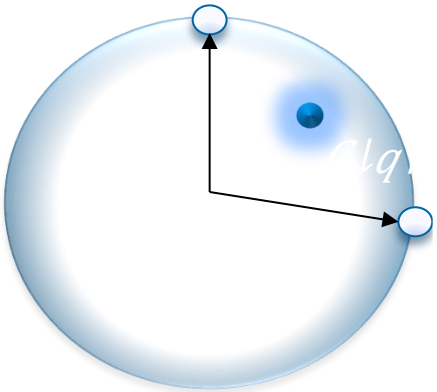


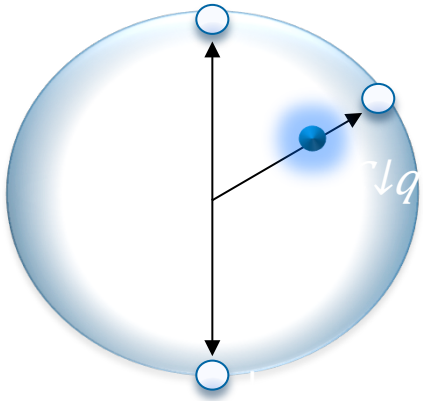
Optimal Quantum Predictor

Optimal Quantum Retrodictor



Causal and Retrocausal states both fit in a Qubit



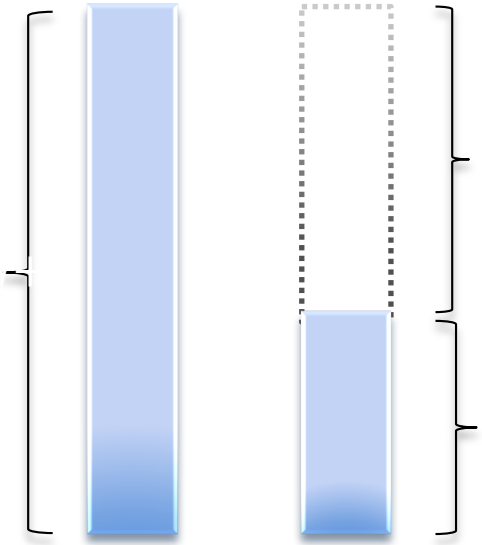


Optimal Quantum Predictor

Optimal Quantum Retrodictor

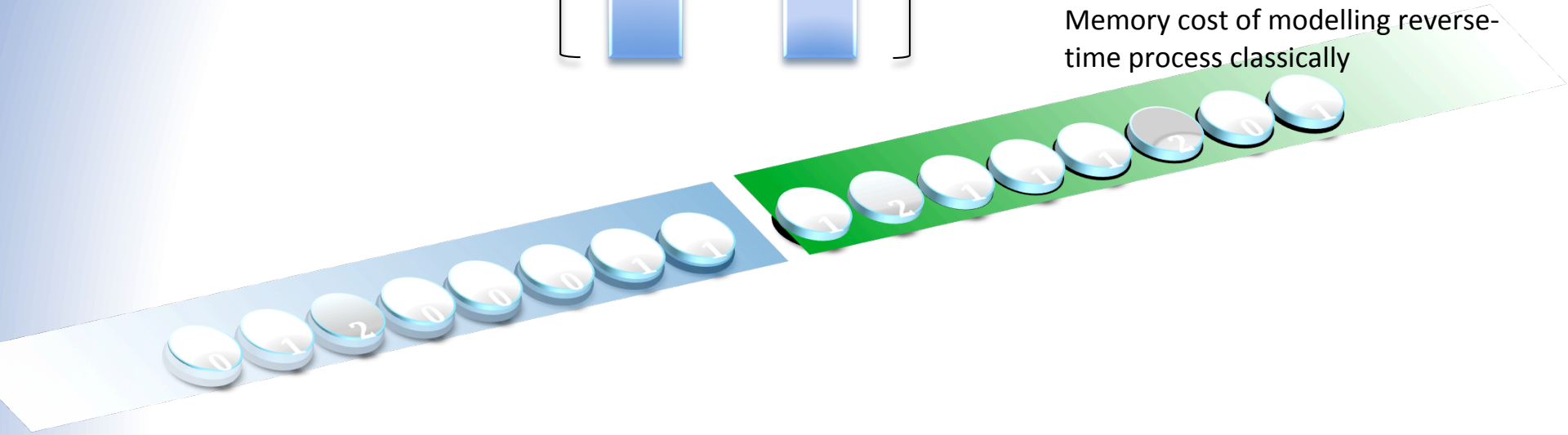
General Processes

Statistical Complexity
Memory cost of modelling the process classically

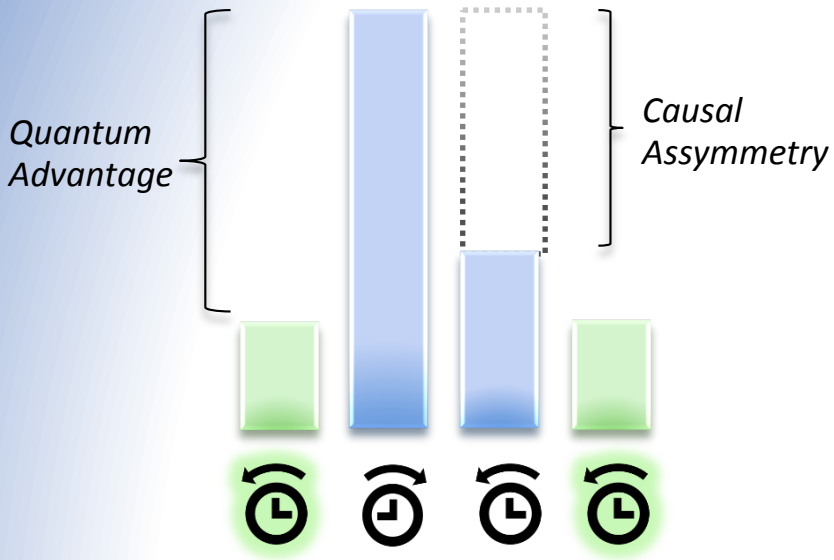


Causal Asymmetry
Memory overhead of choosing the incorrect `causal' direction is non-zero

Time-reversed Statistical Complexity
Memory cost of modelling reverse-time process classically



General Processes



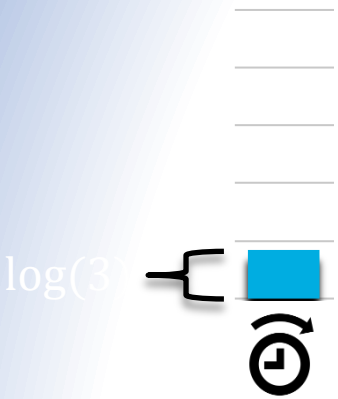
Theorem

This quantum advantage can always completely mitigate the memory overhead from causal asymmetry

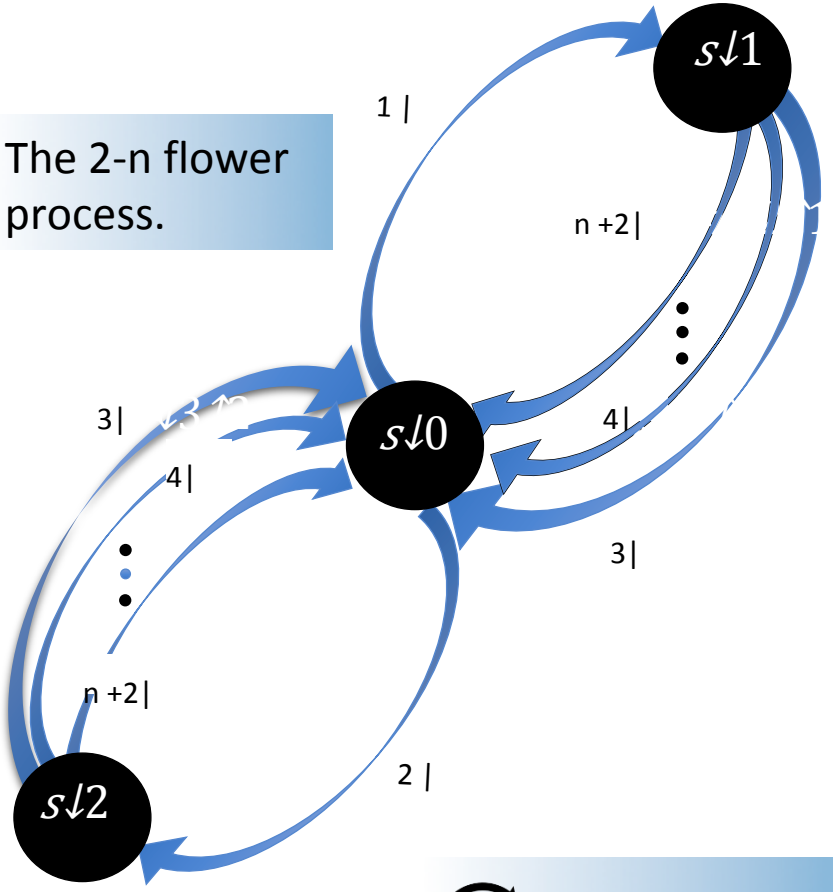
Quantum models forced to run in a particular temporal always uses no more memory than their classical counterparts – even if the latter can run any direction.

Unbounded Causal Asymmetry

Causal asymmetry can become unbounded.



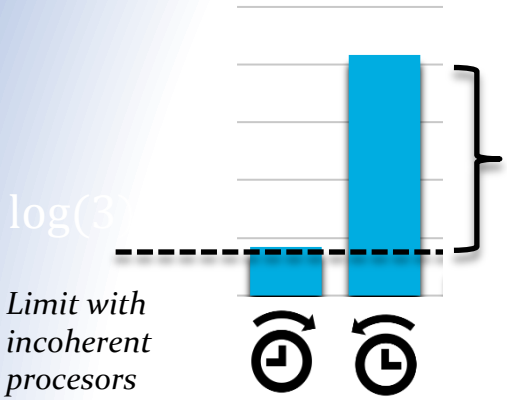
The 2-n flower process.

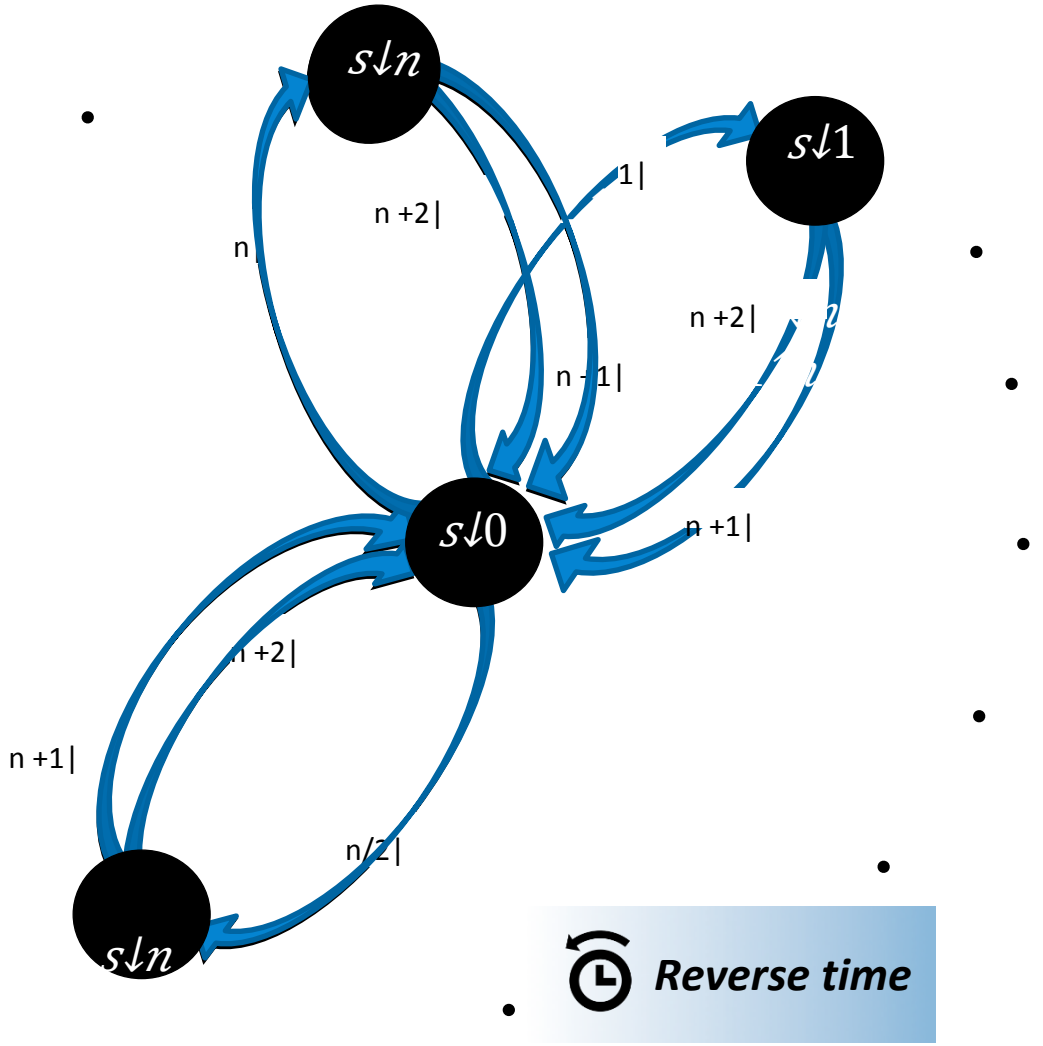


Forward time

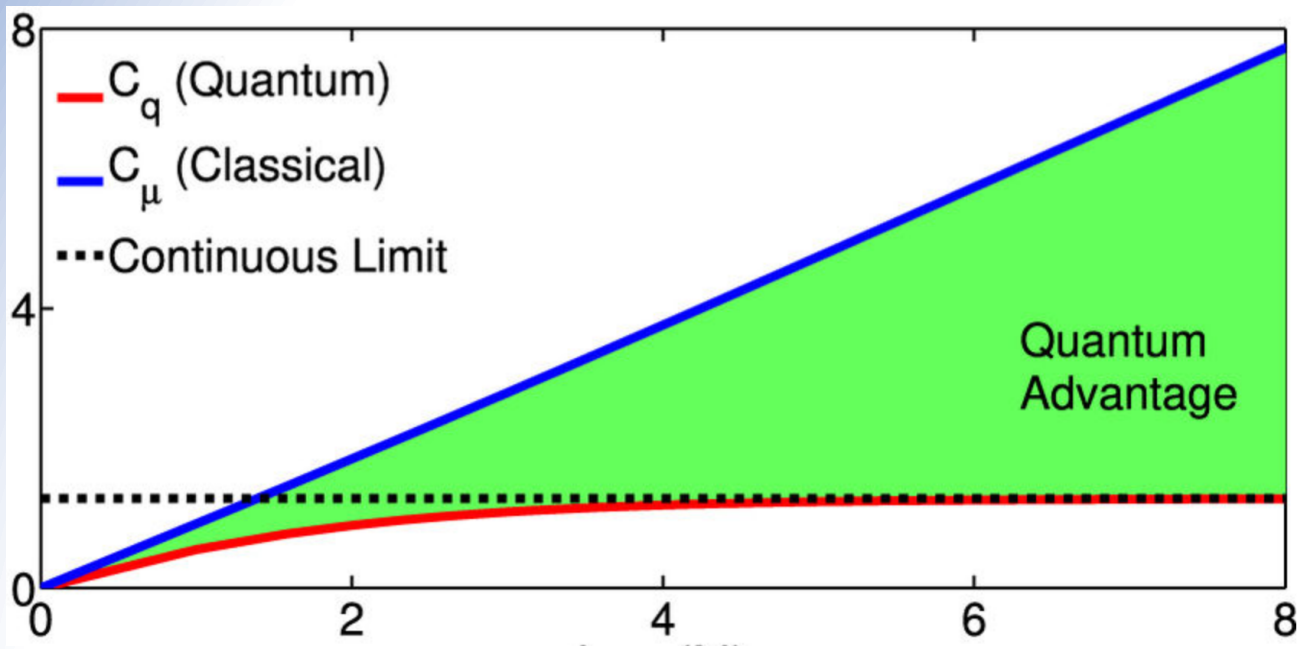
Unbounded Causal Asymmetry

Causal asymmetry can become unbounded.





Scaling Advantages of Quantum Models

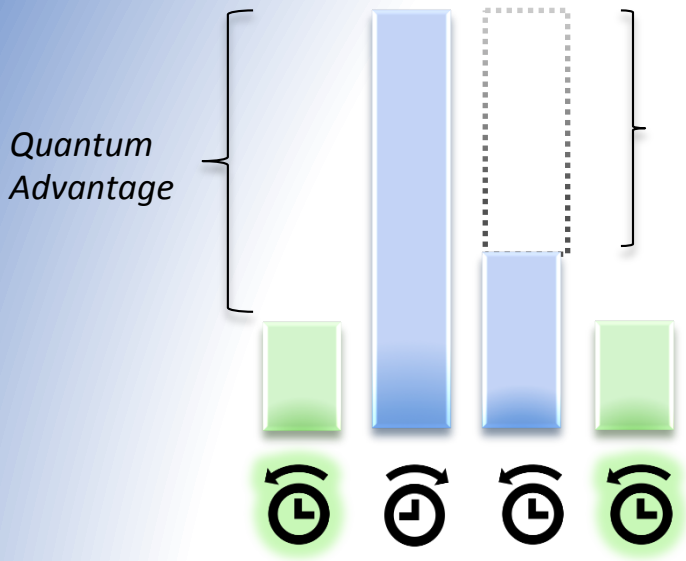


Growing Classical Complexity

Bounded Quantum Complexity

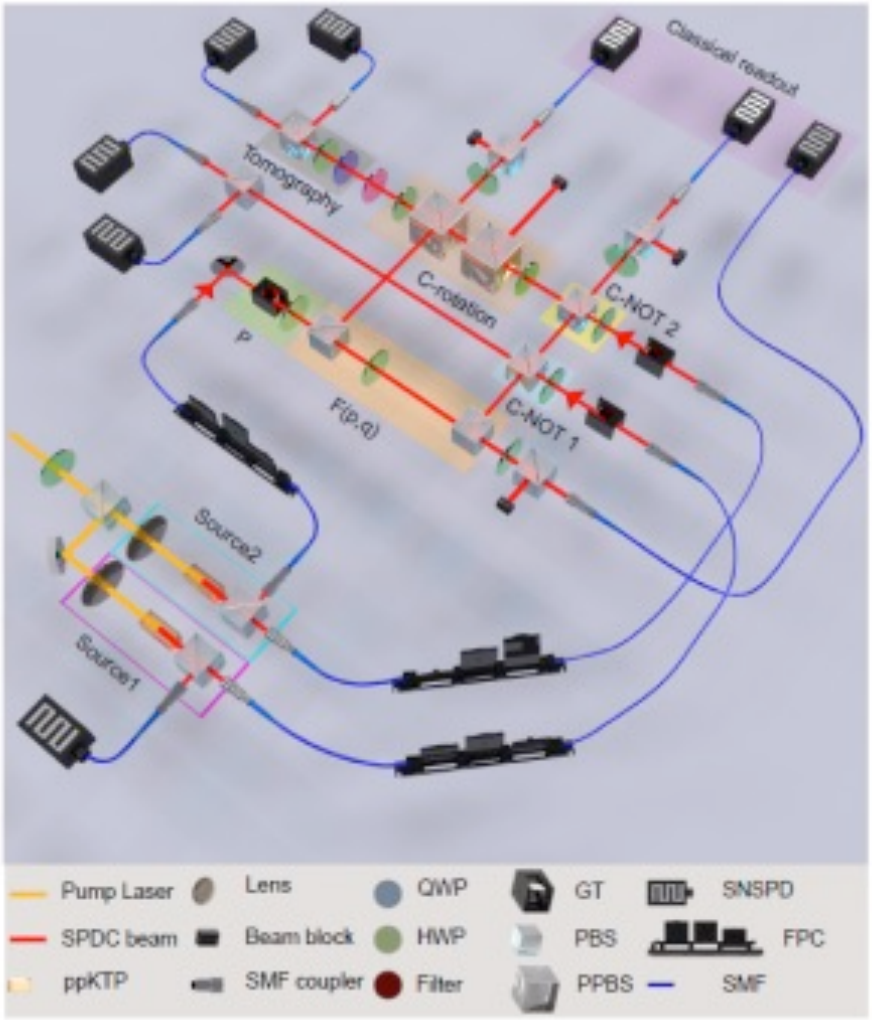
1. ***Unbounded Memory Reduction when Simulating Continuous-Time Stochastic Processes With Quantum Devices.*** *npj Quantum Information 4, 18*
2. ***Unbounded memory advantage of Stochastic Simulation,*** *New J. Phys. 19 103009*

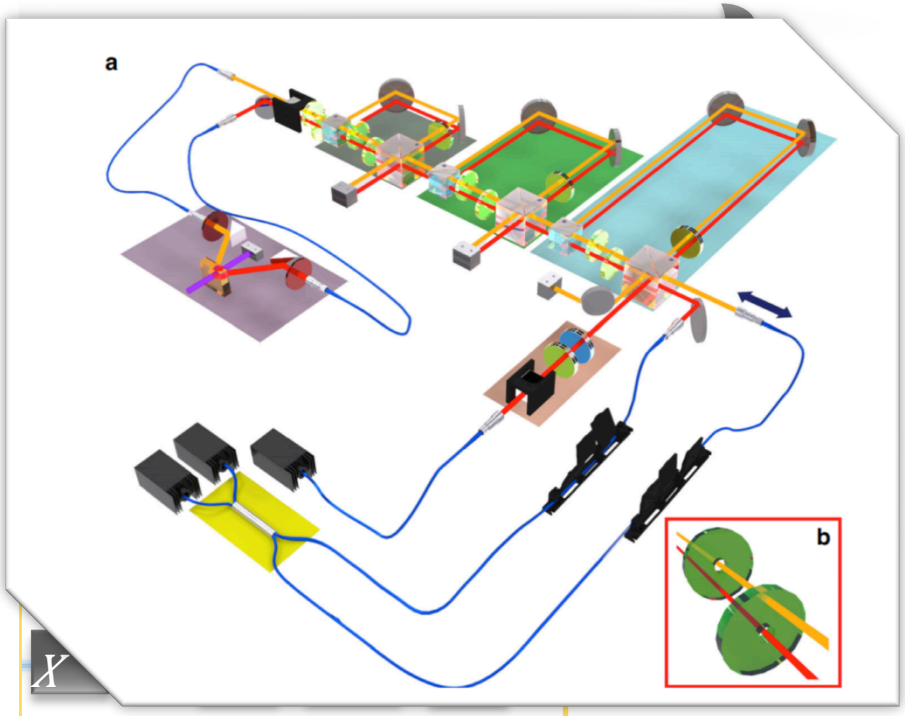
These results also apply to topological complexity



The also holds for topological complexity.

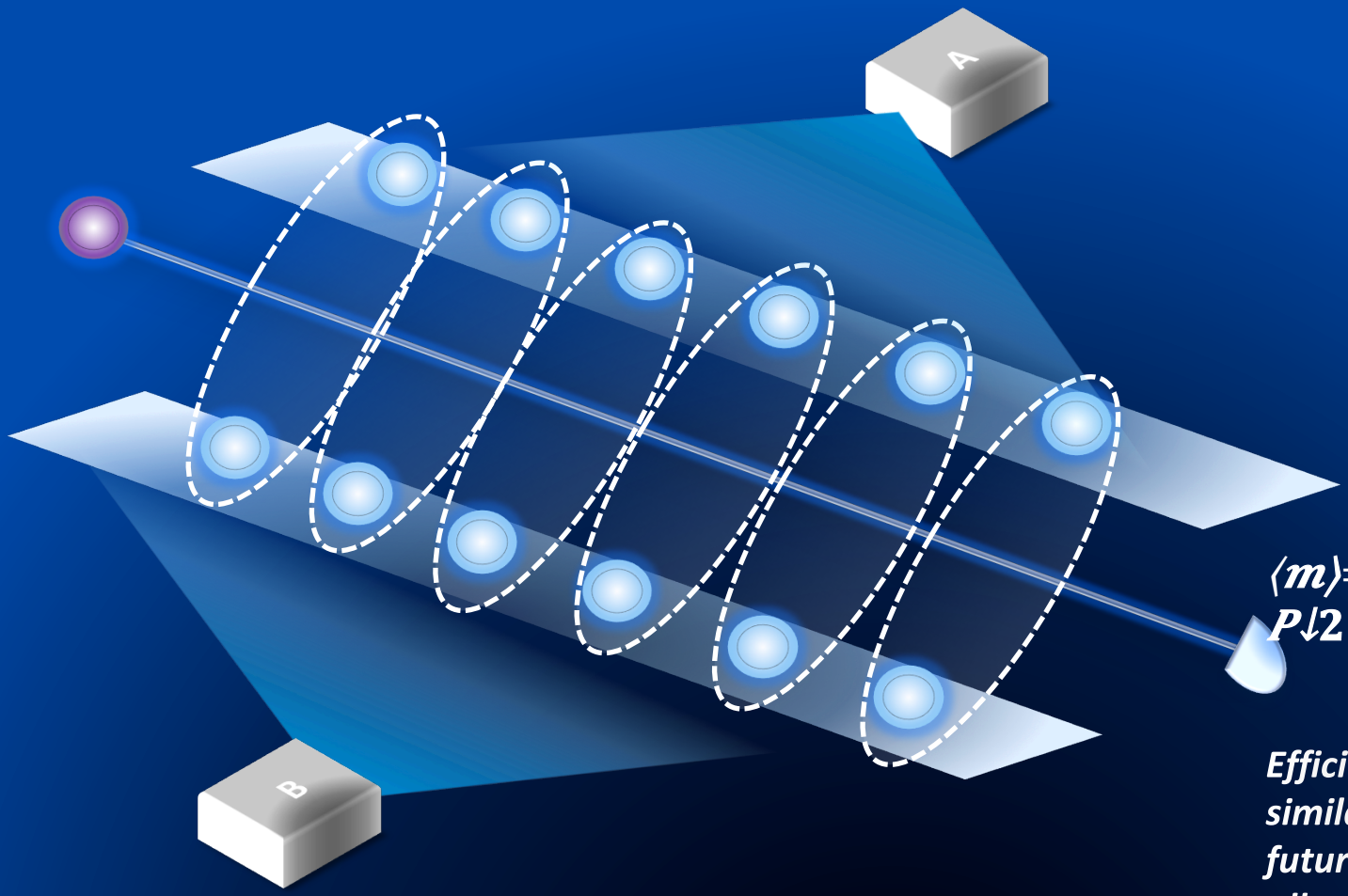
Experimental work on dimensionality reduction





C-rotation

Superposing All Possible Futures

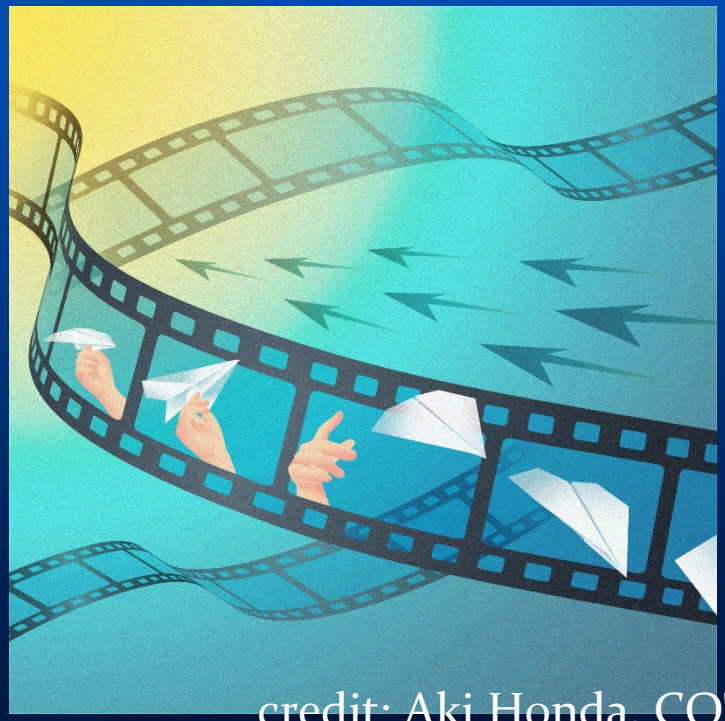


$$\langle m \rangle = \sum x \uparrow \blacksquare \sqrt{\square} P\downarrow 1\,(x|x)\, P\downarrow 2\,(x|x)$$

Efficiently estimates the similarity of two different futures – taking account of all possible path!

Conclusions

Can we ascribe a preferred causal order to two events based on their measurement statistics alone?



credit: Aki Honda, CQT

There may be more than one arrow!

It depends on how the process is generated and viewed.

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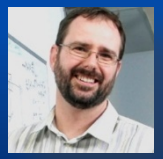
Causal Asymmetry in a Quantum World

Jayne Thompson,[1,*] Andrew J. P. Garner,[1,7] John R. Mahoney,[2] James P. Crutchfield,[2] Vlatko Vedral,[3,1,4] and Mile Gu[5,6,1,†]


N. Tirichler


F. Ghafari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Interfering trajectories in experimental quantum-enhanced stochastic simulation

Farzad Ghafari, Nora Tischler, Carlo Di Franco, Jayne Thompson, Mile Gu & Geoff J. Pryde

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ARTICLE

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Quantum mechanics can reduce the complexity of classical models

Mile Gu[1], Karoline Wiesner[2], Elisabeth Rieper[1] & Vlatko Vedral[1,3,4]

Andrew Garner

http://www.quantumcomplexity.org/

Centre for Quantum Technologies
National University of Singapore

John Templeton Foundation

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