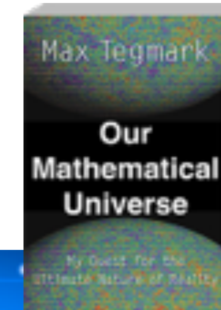
Consciousness as a State of Matter



Max Tegmark, MIT

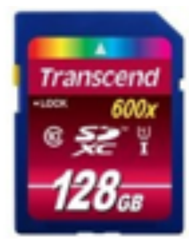
arXiv:1401.1219 [quant-ph]

Consciousness as a State of Matter

- *Viscosity?*
- *Compressibility?*
- *Electrical conductivity?*
- *Diffusivity?*

Consciousness as a State of Matter

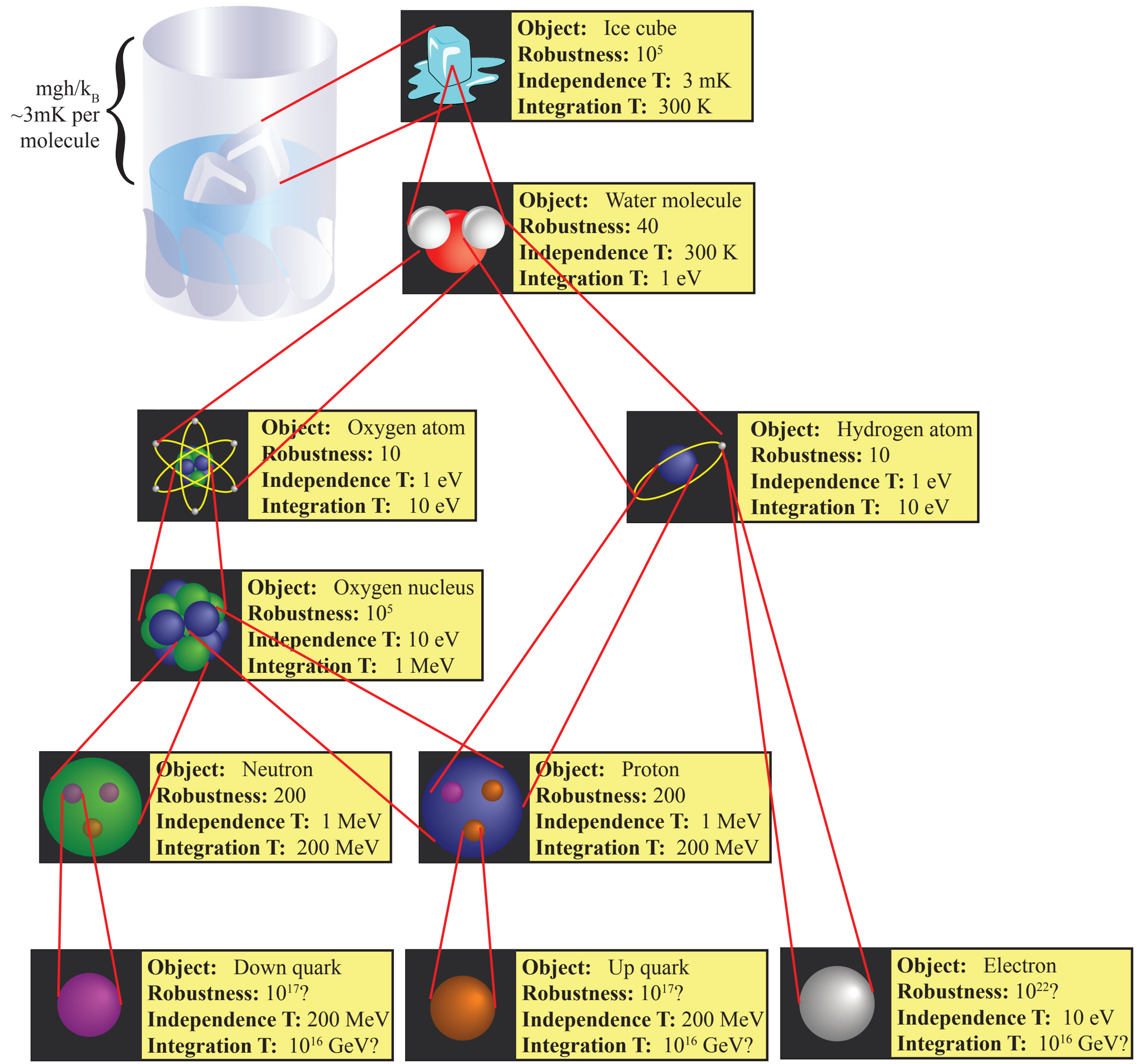
State of matter	Many long-lived states?	Information integrated?	Easily writable?	Complex? dynamics?
Gas	N	N	N	Y
Liquid	N	N	N	Y
Solid	Y	N	N	N
Memory	Y	N	Y	N
Computer	Y	?	Y	Y
Consciousness	Y	Y	Y	Y



"perceptronium"

"Physics-from-scratch" problem:

How go from ρ & H to this?



Bill Poirier

Principle	Definition
Information principle	A conscious system has substantial information storage capacity.
Dynamics principle	A conscious system has substantial information processing capacity.
Independence principle	A conscious system has substantial independence from the rest of the world.
Integration principle	A conscious system cannot consist of nearly independent parts.
Utility principle	A conscious system records mainly information that is useful for it.
Autonomy principle	A conscious system has substantial dynamics and independence.

- *Any credit should go to Giulio Tonini, Christof Koch et al*
- *Any blame should to go to me*

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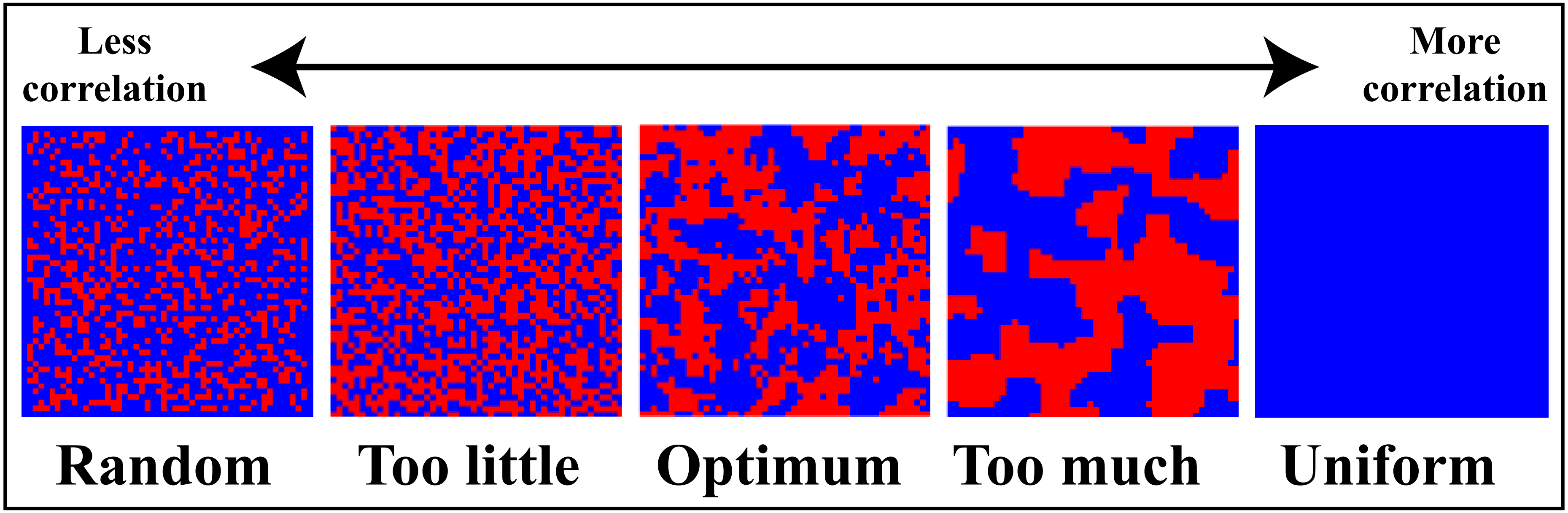
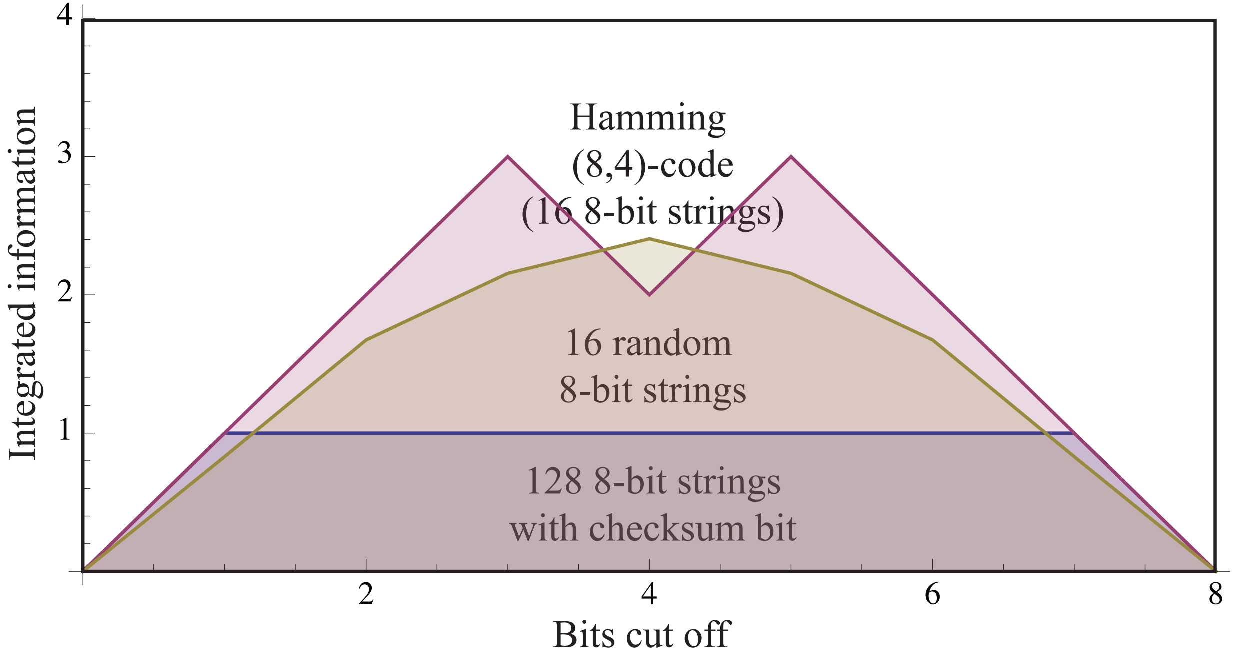
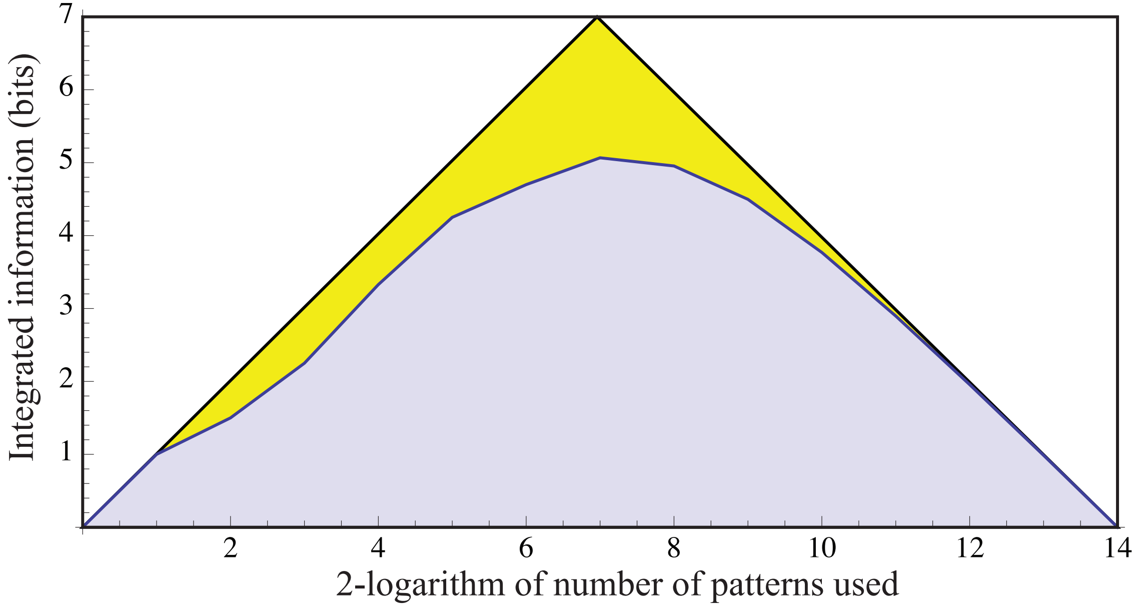
Integration

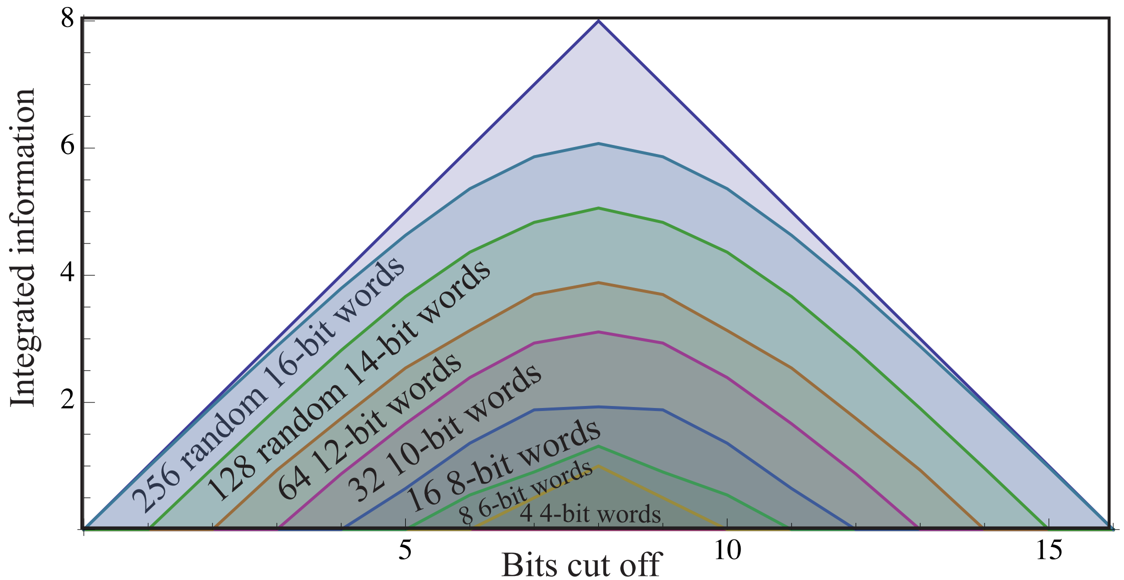


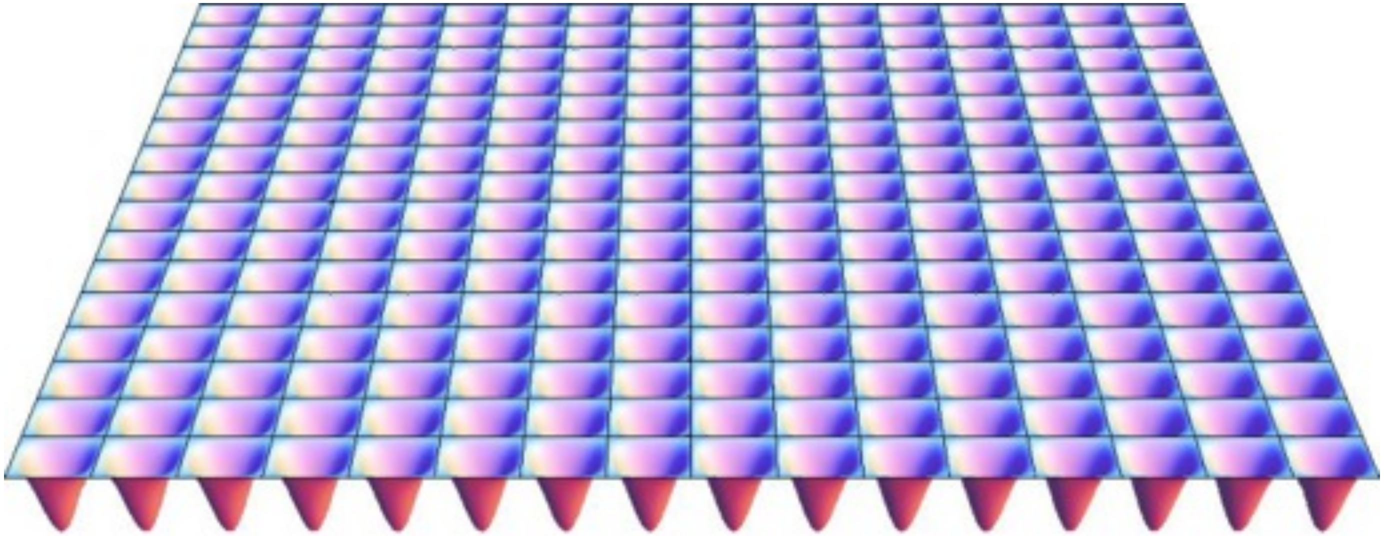
Integration of classical information:

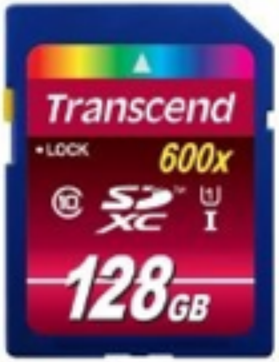
$$M = \begin{pmatrix} 0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&1&1&0&1&0&0&1&0&1&1&0&1&0&0&1 \\ 0&1&0&1&0&1&0&1&1&0&1&0&1&0&1&0 \\ 0&1&0&1&1&0&1&0&0&1&0&1&1&0&1&0 \\ 0&0&1&1&0&0&1&1&1&1&0&0&1&1&0&0 \\ 0&0&1&1&1&1&0&0&0&0&1&1&1&1&0&0 \\ 0&1&1&0&0&1&1&0&1&0&0&1&1&0&0&1 \end{pmatrix}$$

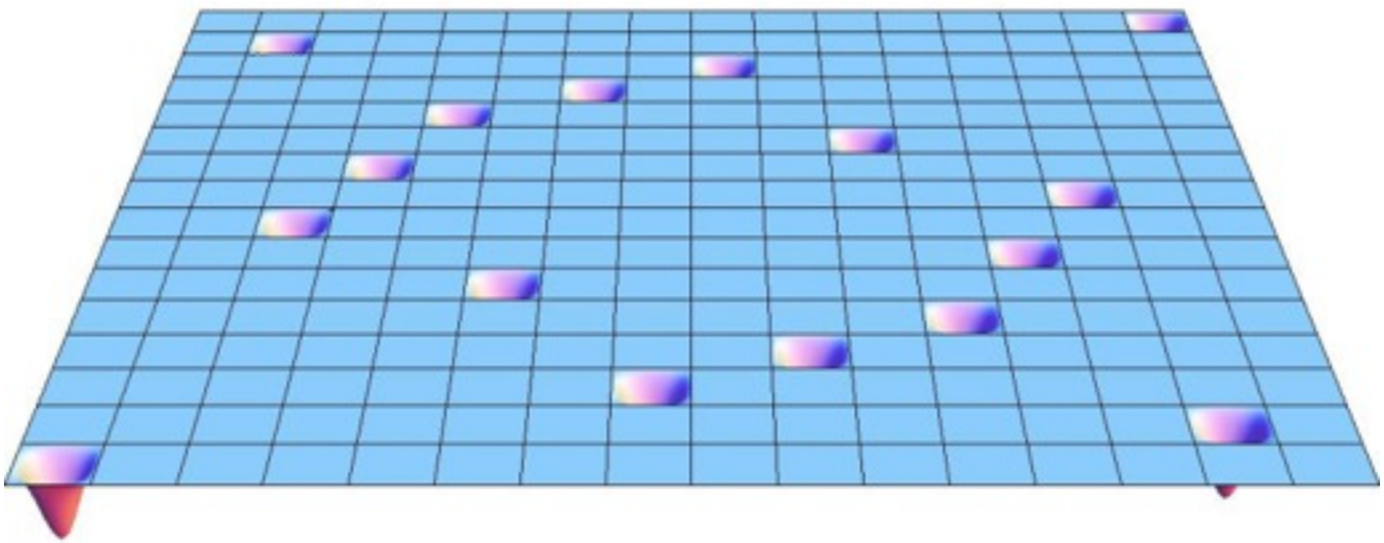


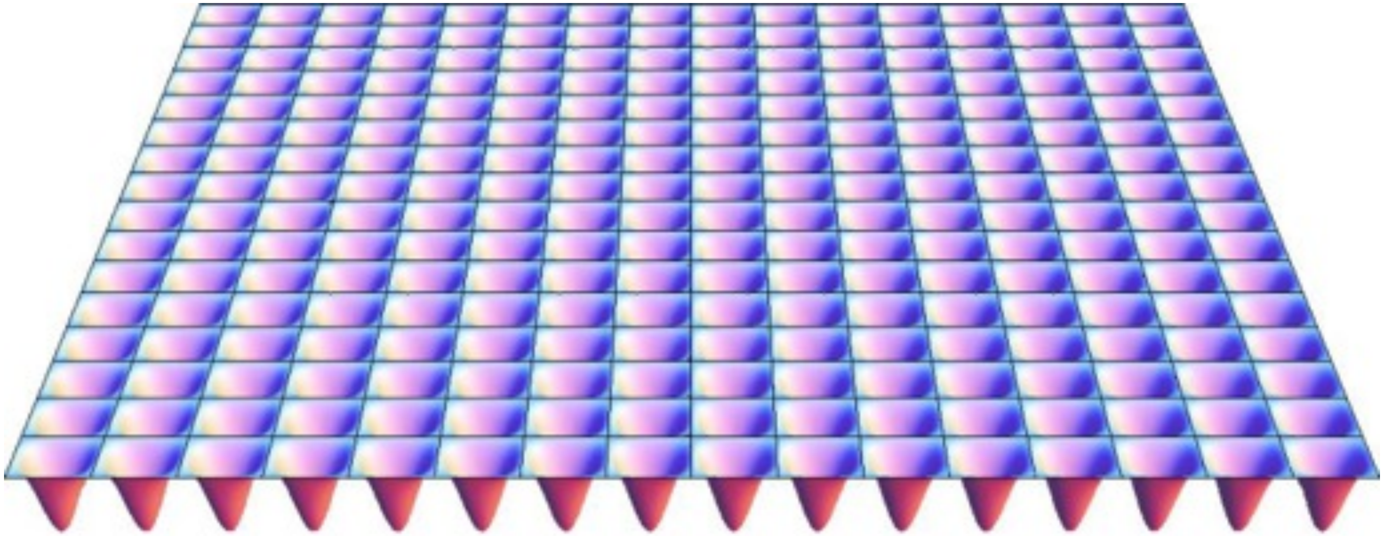

Integrated information (bits)
2-logarithm of number of patterns used
1
2
3
4
5
6
7
2
4
6
8
10
12
14

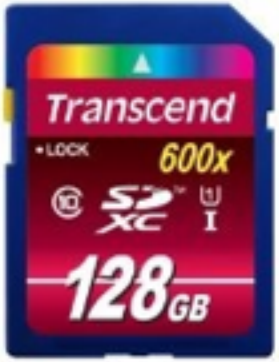

Integrated information
Bits cut off
2
4
6
8
5
10
15
256 random 16-bit words
128 random 14-bit words
64 12-bit words
32 10-bit words
16 8-bit words
8 6-bit words
4 4-bit words

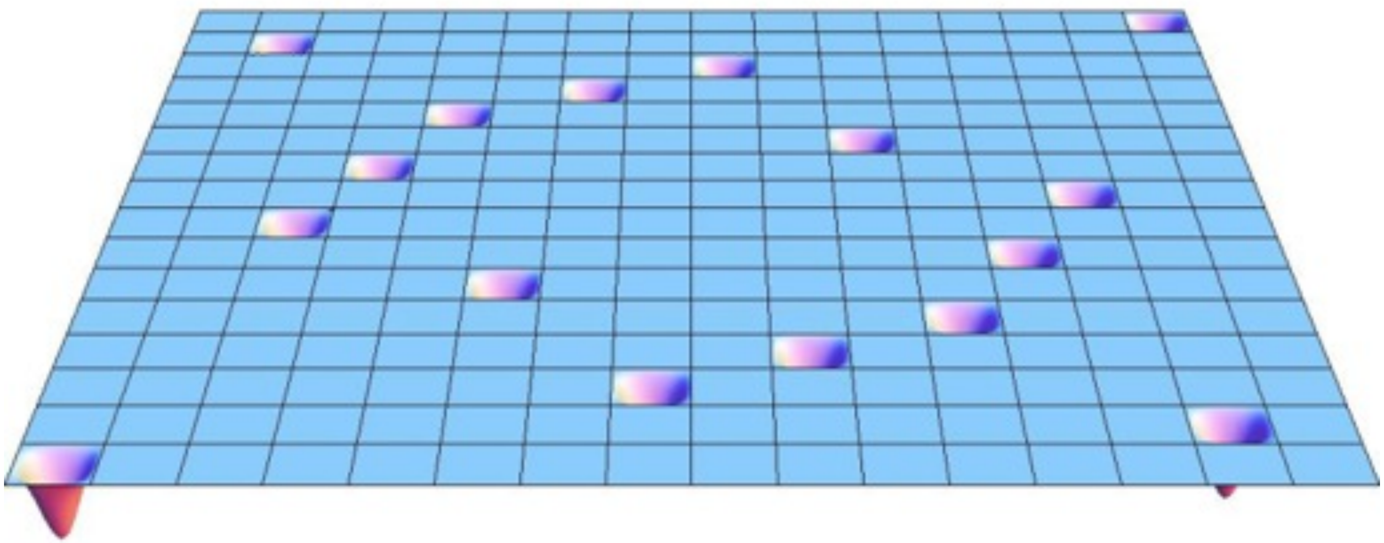



Transcend
LOCK
600x
SD XC
128GB



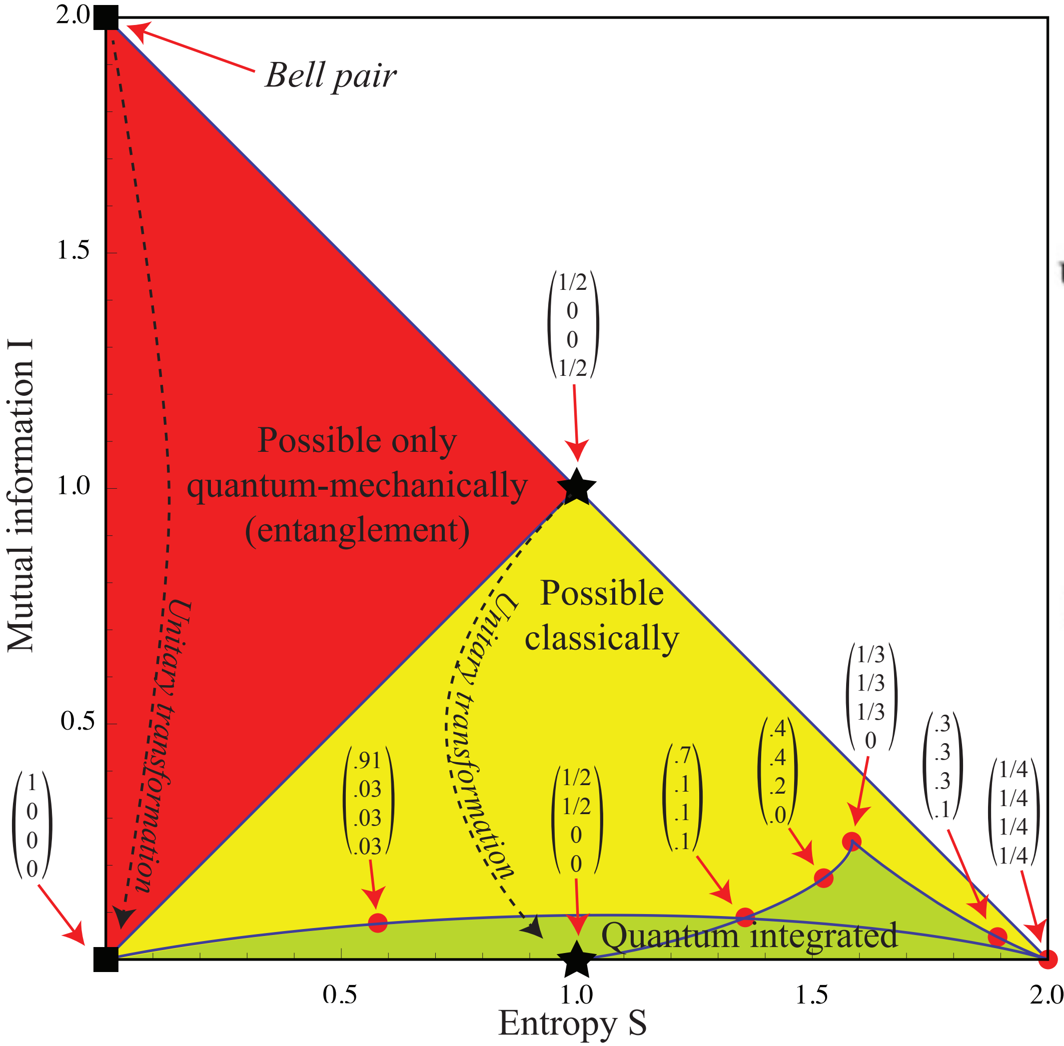






Classical integration paradox:

Hopfield network of 10^{11} neurons can store only 37 bits of integrated information



Perfect classical integration:

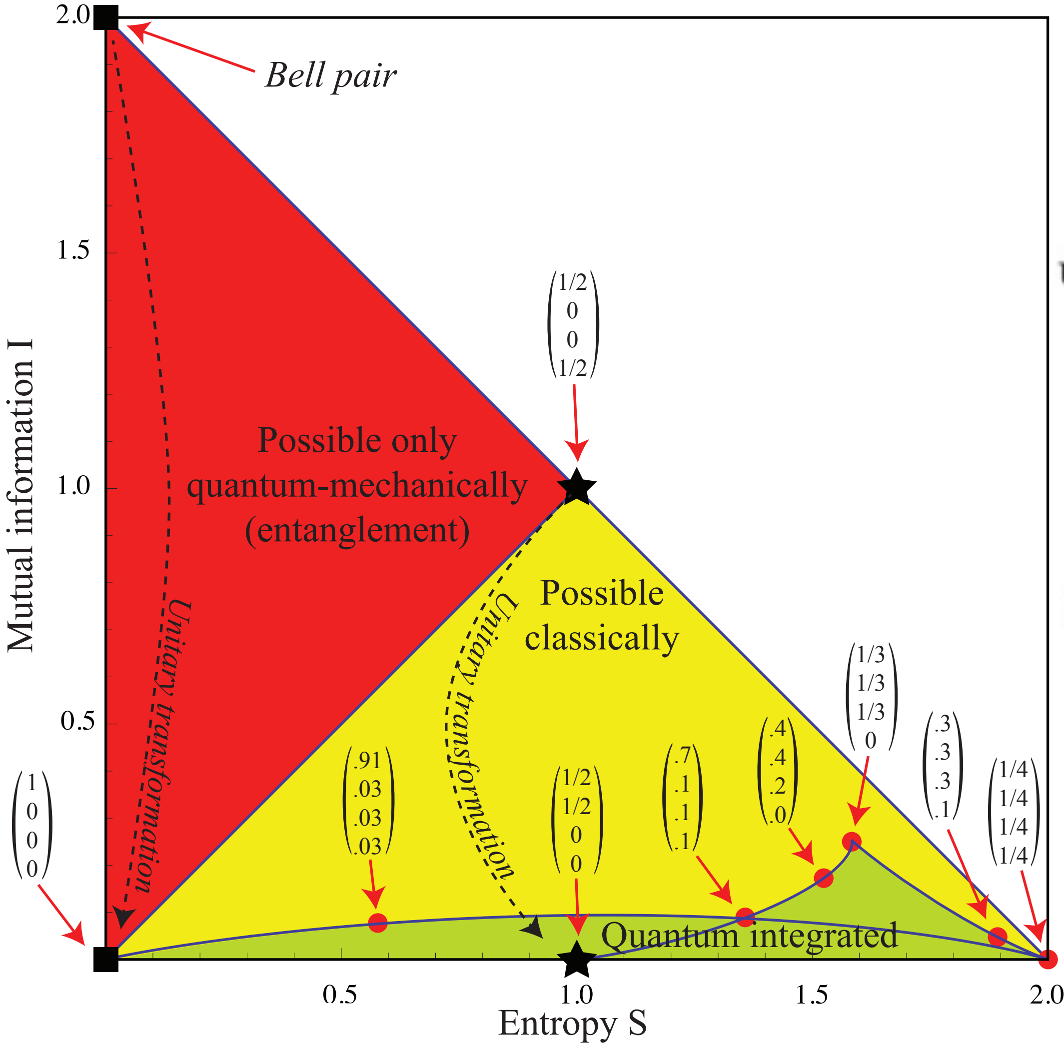
$$\mathbf{U}\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}\mathbf{U}^\dagger = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Bell pair:

$$\mathbf{U}\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}\mathbf{U}^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Quantum integration paradox:

No quantum system, no matter how large, can store more than about 0.25 bits of integrated information



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Bell pair:

$$\mathbf{U}\begin{pmatrix}\frac{1}{2}&0&0&\frac{1}{2}\\0&0&0&0\\0&0&0&0\\\frac{1}{2}&0&0&\frac{1}{2}\end{pmatrix}\mathbf{U}^\dagger=\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}=\begin{pmatrix}1&0\\0&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&0\end{pmatrix}$$

Conclusion: we also need dynamics principle

Quantum integration paradox:

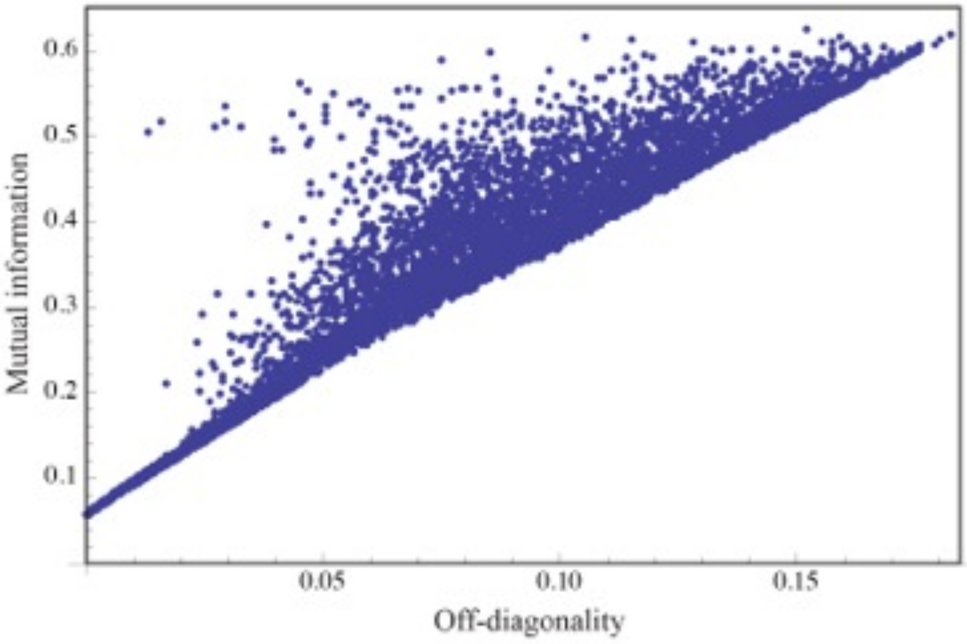
No quantum system, no matter how large, can store more than about 0.25 bits of integrated information

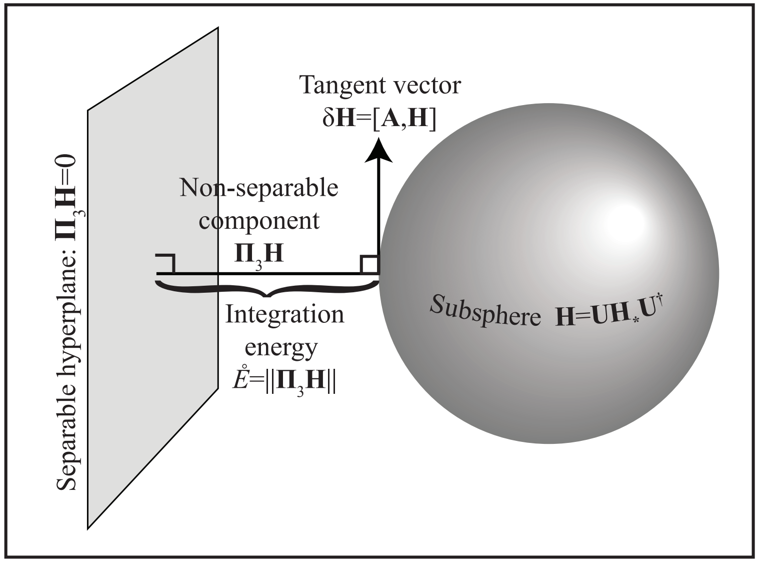
$$\mathbf{H} = \mathbf{H}_1 \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{H}_2 + \mathbf{H}_3$$

Maximizing independence gives…

$$\mathbf{H} = \mathbf{H}_1 \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{H}_2 + \mathbf{H}_3$$

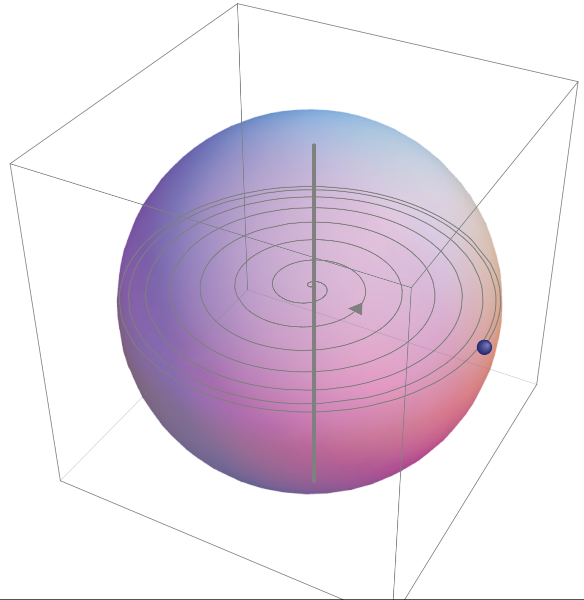
Maximizing independence gives…

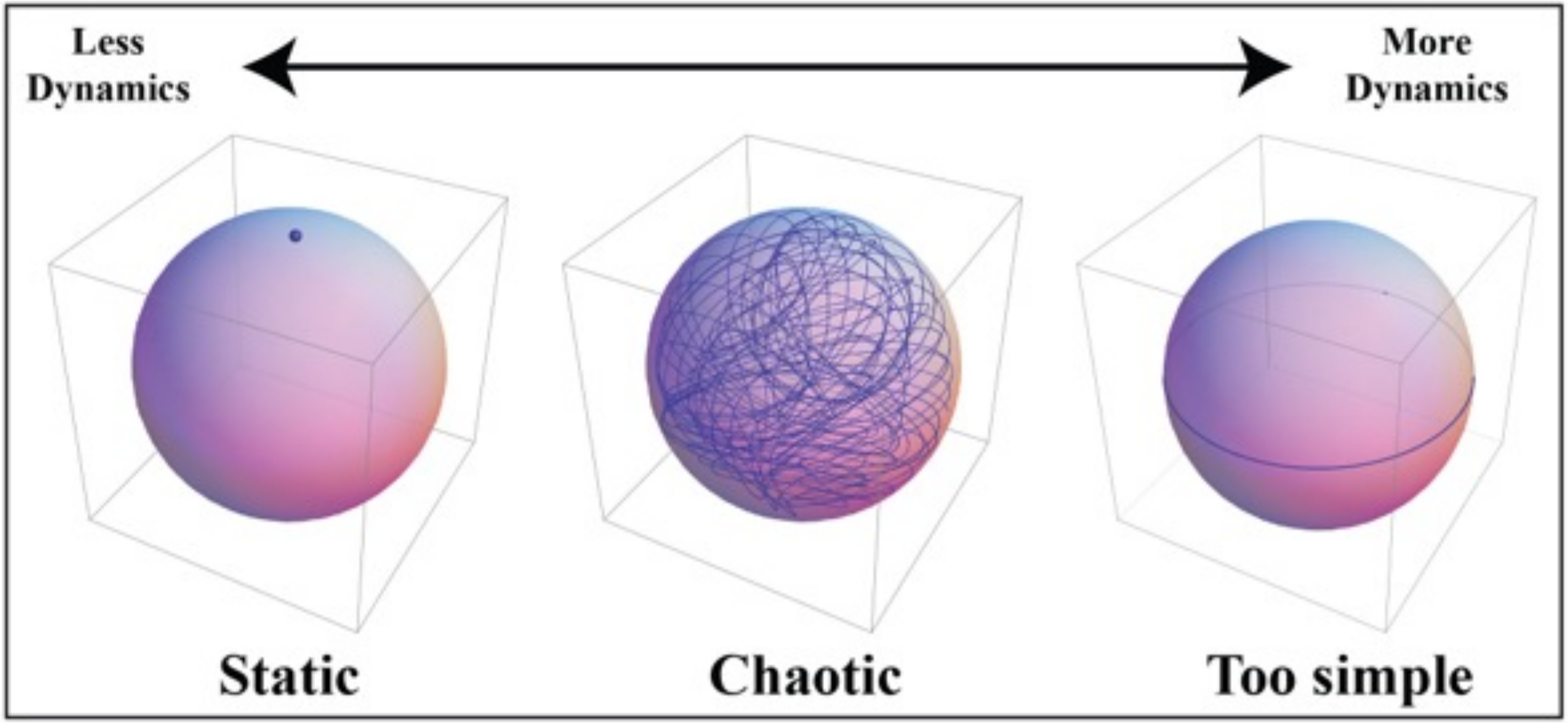




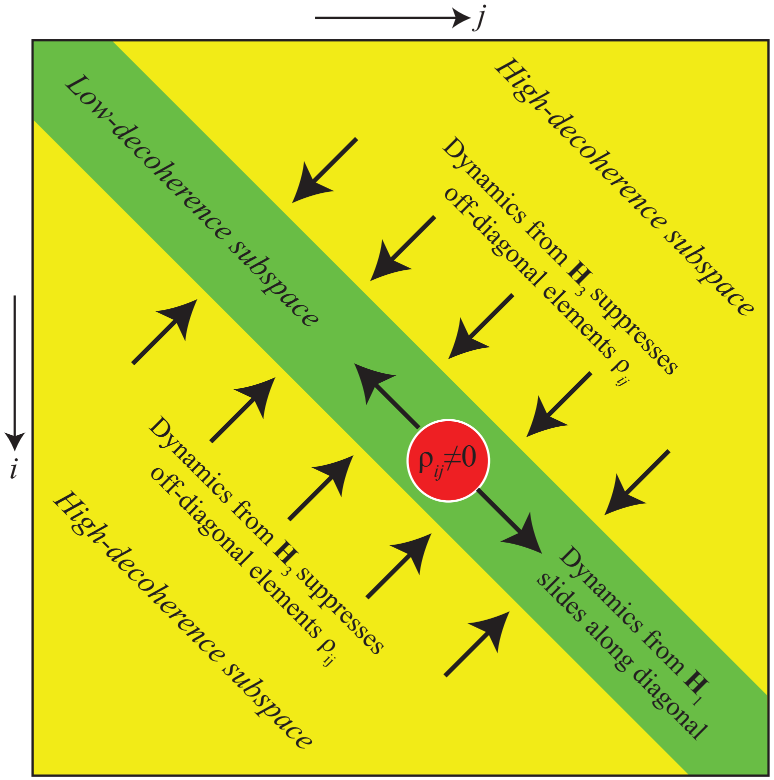
…the Quantum Zeno Paradox:

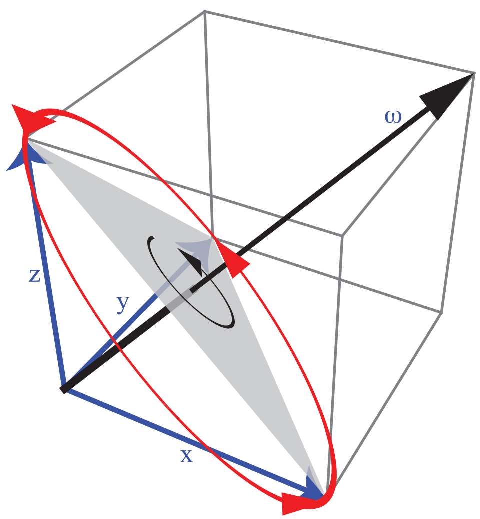
If we decompose our universe into maximally independent objects, then all change grinds to a halt.

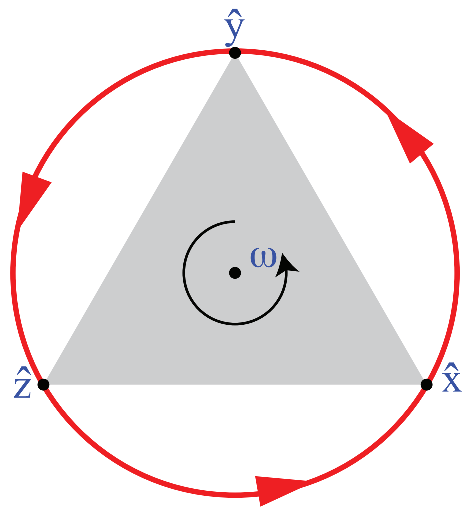


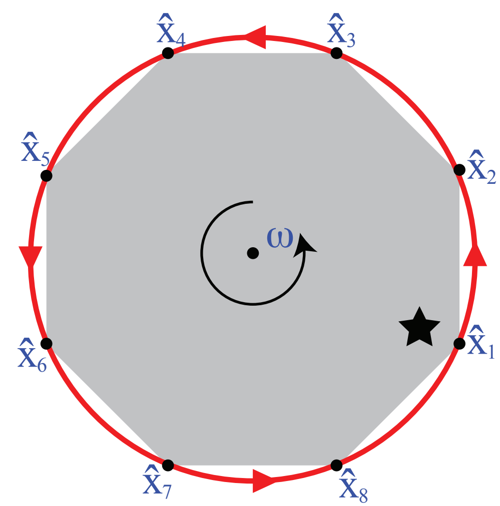

Less
Dynamics
More
Dynamics
Static
Chaotic
Too simple

Exponentially large autonomy can be attained by "sliding along the diagonal":



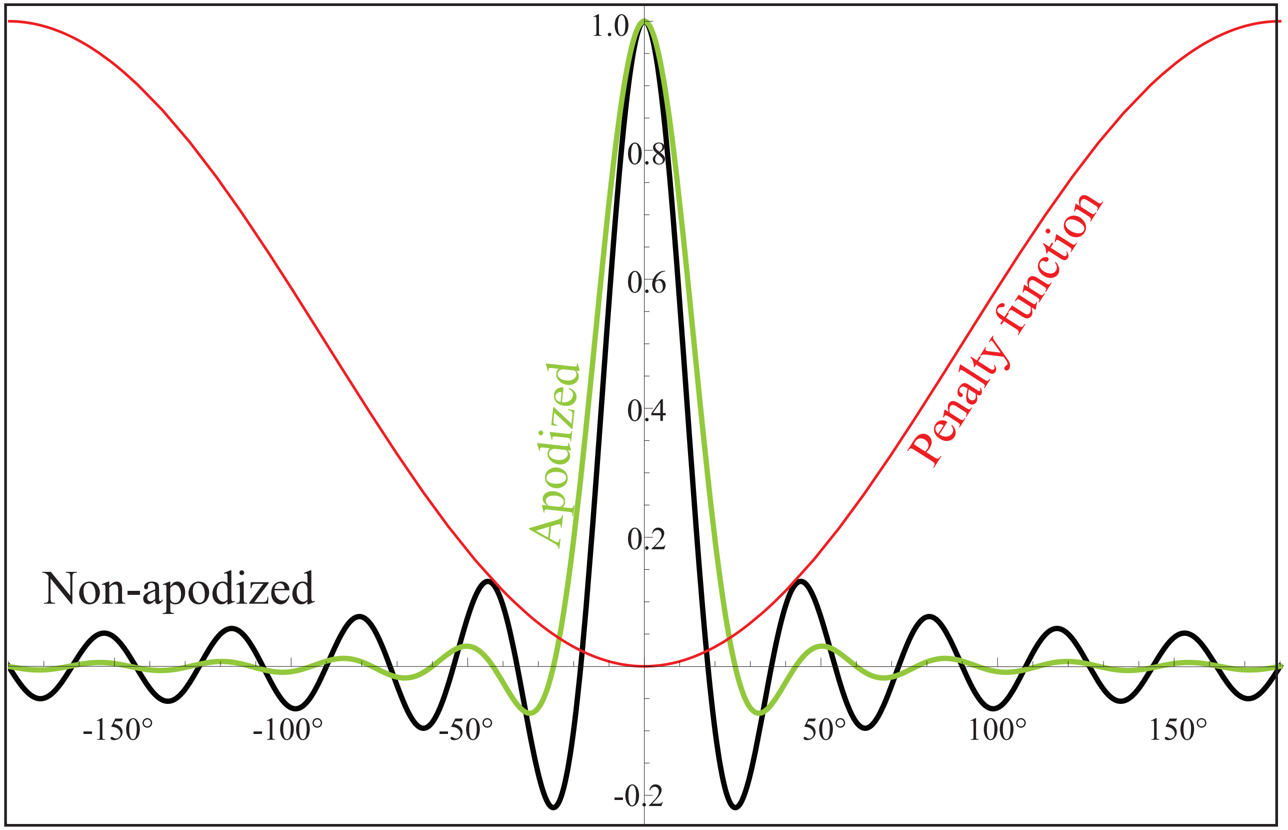




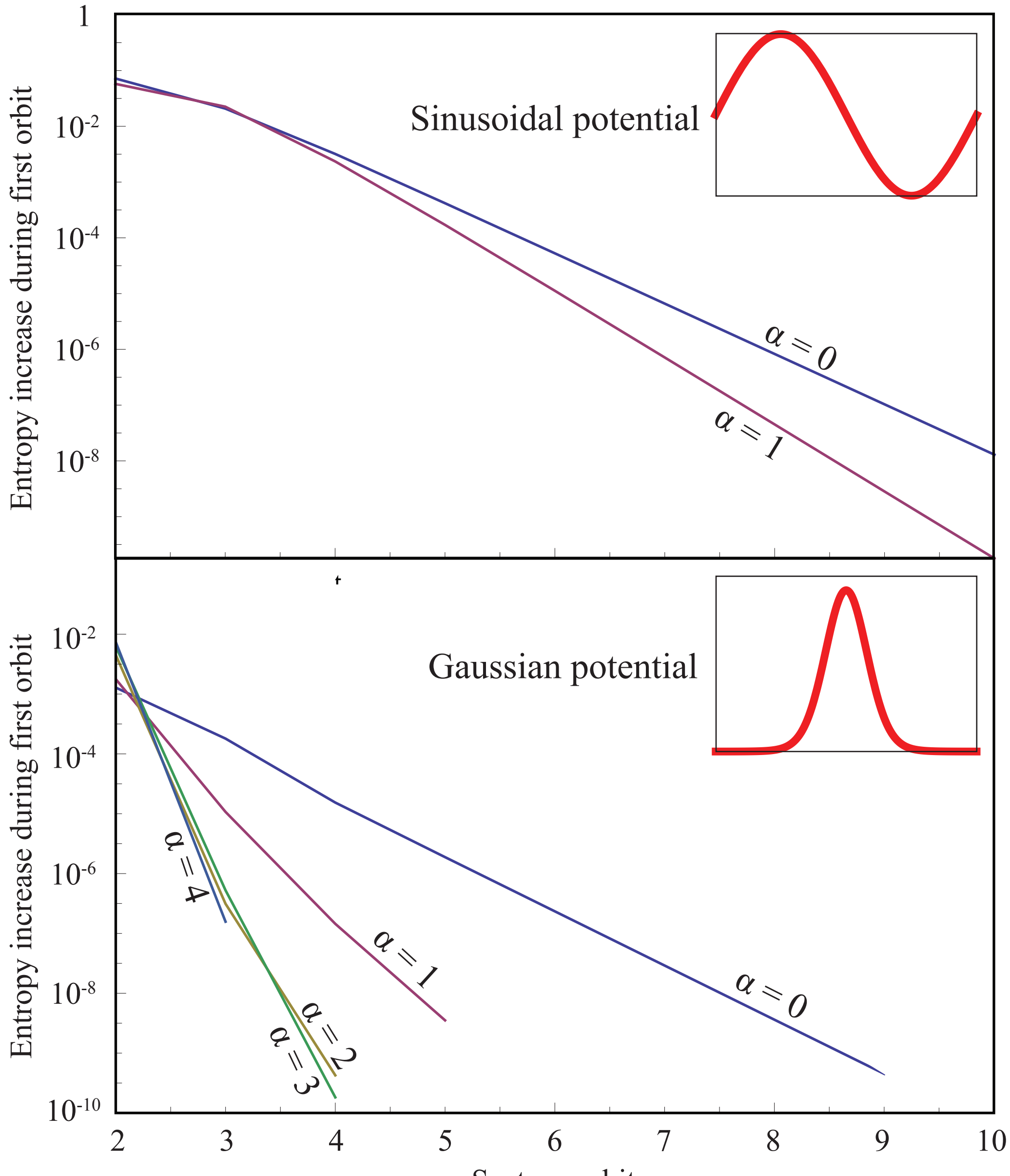


Summary

- I've explored the hypothesis that consciousness can be understood as a state of matter, "perceptronium", with distinctive information processing abilities
- We explore five basic principles that may distinguish conscious matter from other physical systems such as solids, liquids and gases: the information, integration, independence, dynamics and utility principles
- If such principles can identify conscious entities, then they can help solve the quantum factorization problem: why do conscious observers like us perceive the particular Hilbert space factorization corresponding to classical space (rather than Fourier space, say), and more generally, why do we perceive the world around us as a dynamic hierarchy of objects that are strongly integrated and relatively independent?
- Tensor factorization of matrices is found to play a central role, and our technical results include a theorem about Hamiltonian separability (defined using Hilbert-Schmidt superoperators) being maximized in the energy eigenbasis.
- The goal is to generalizes Giulio Tononi's integrated information framework for neural-network-based consciousness to arbitrary quantum systems
- It's hard! There are many fun open problems, and there are interesting links to error-correcting codes, condensed matter criticality, and the Quantum Darwinism program, as well as an interesting connection between the emergence of consciousness and the emergence of time.



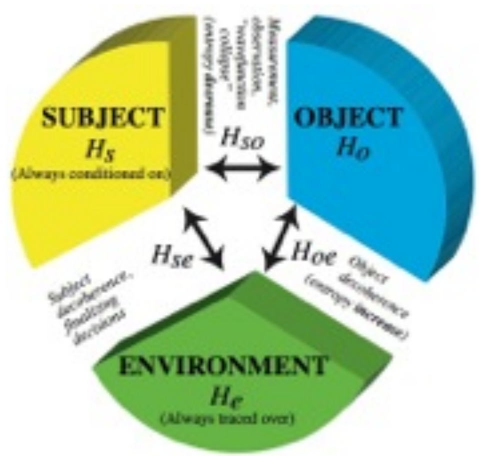
1.0
0.8
0.6
0.4
0.2
-0.2
-150°
-100°
-50°
50°
100°
150°
Non-apodized
Apodized
Penalty function



Sinusoidal potential
Gaussian potential
Entropy increase during first orbit
Entropy increase during first orbit
System qubits
1
10-2
10-4
10-6
10-8
10-10
2
3
4
5
6
7
8
9
10
α = 0
α = 1
α = 0
α = 1
α = 2
α = 3
α = 4

Quantum Thermodynamics

1) **A framework**

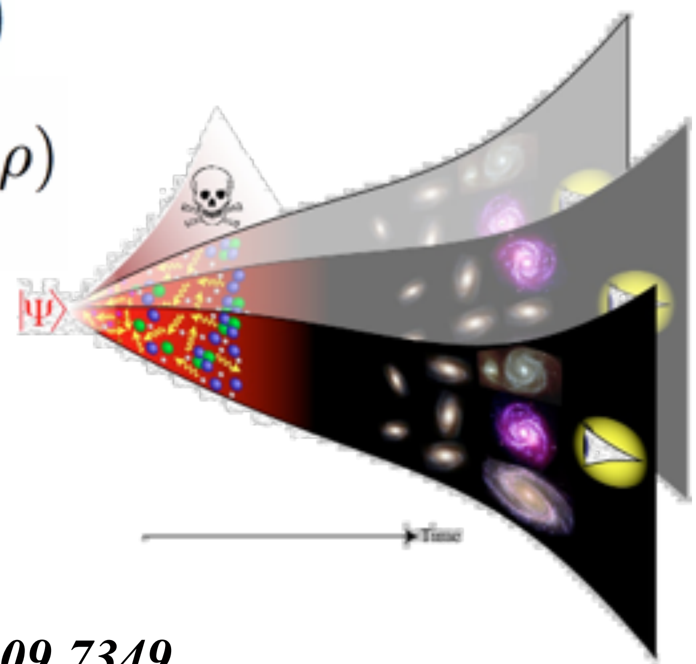


(Cosmology + QM)

2) **Two theorems**

$$S(\rho \circ \mathbf{E}) \geq S(\rho)$$

$$\sum_i p_i S\left(\rho^{(i)}\right) \leq S(\rho)$$



3) **Application to cosmology and inflation**

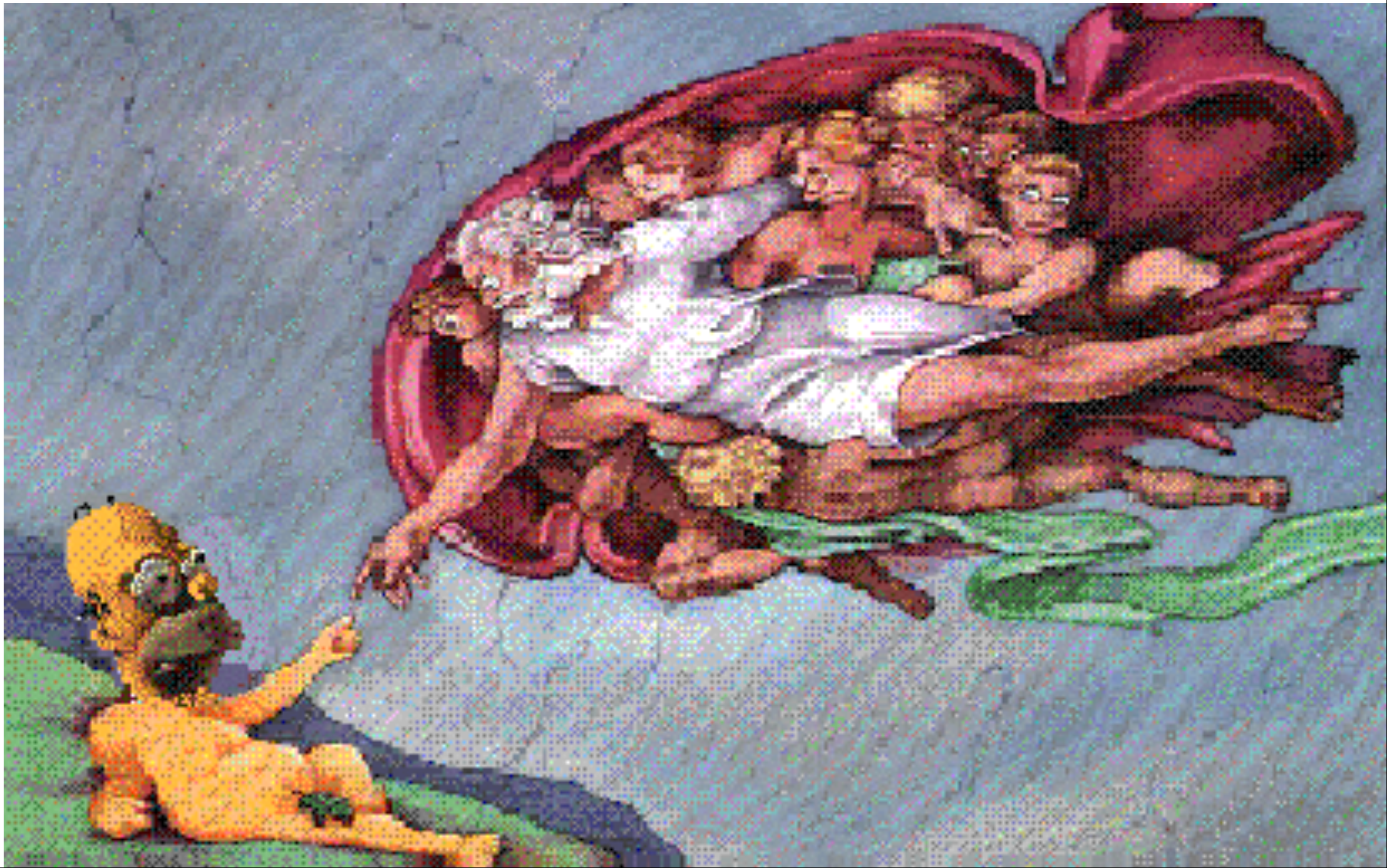
Based on:
MT: quant-ph/9907009
MT: arXiv:1108.3080
Hrant Gharibyian & MT: arXiv:1309.7349

Why should you care?

- Cosmological entropy problem
- Inflationary entropy problem
- Quantum measurement problem

} Same solution!

How did it all begin?



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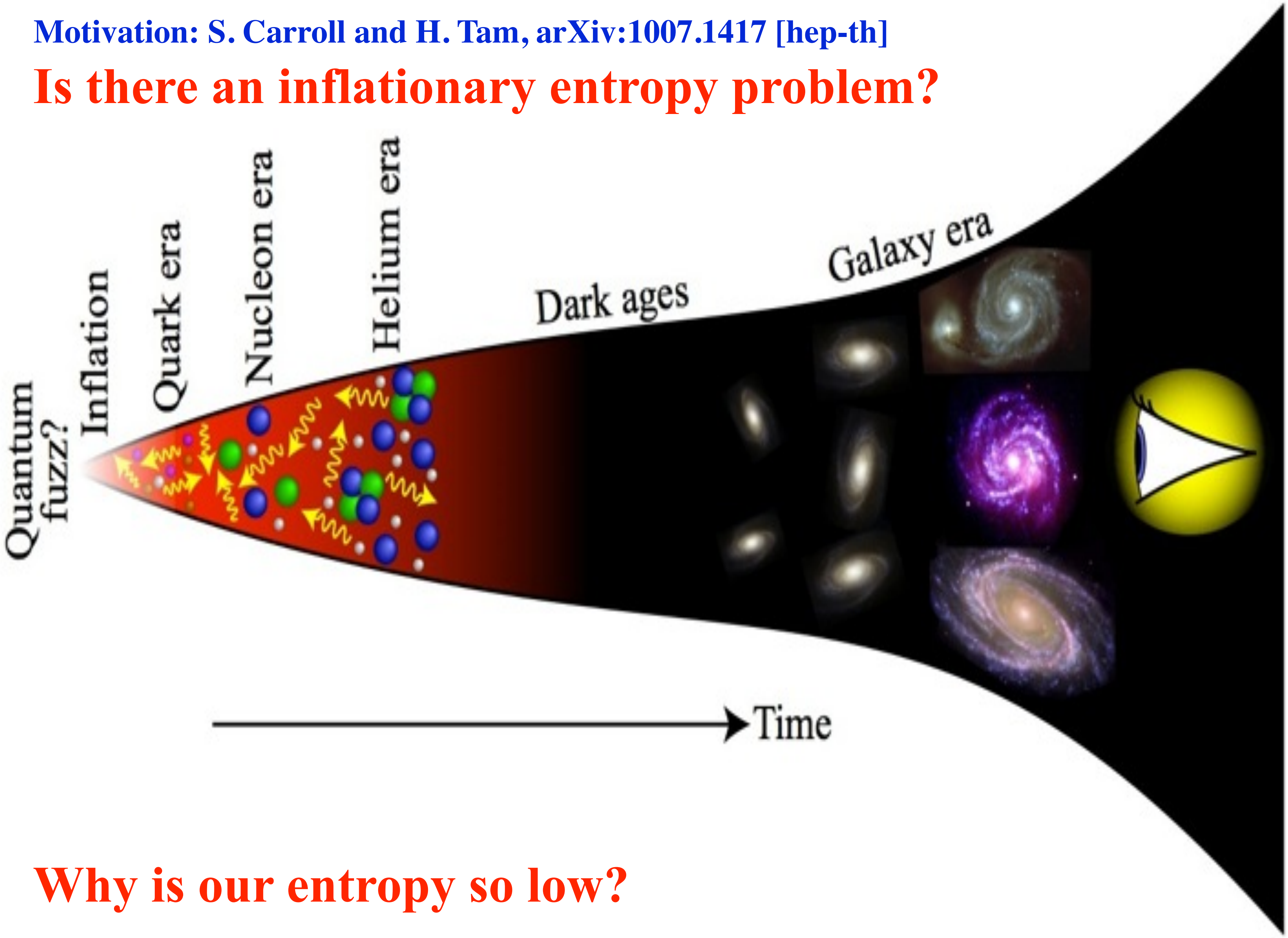
Motivation: S. Carroll and H. Tam, arXiv:1007.1417 [hep-th]

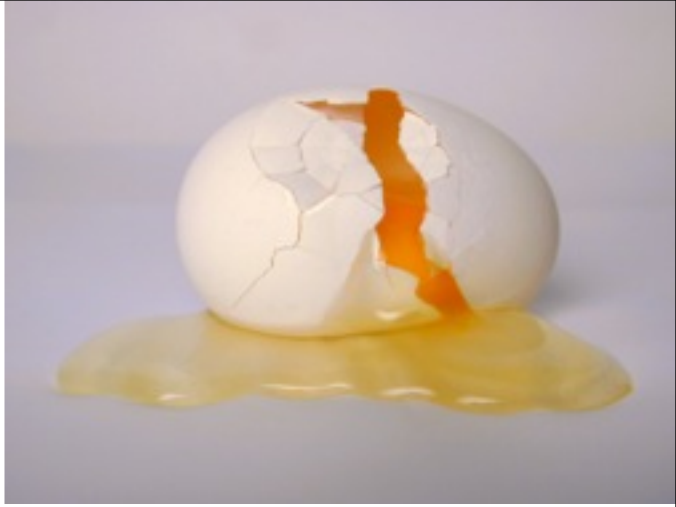
Is there an inflationary entropy problem?



Why is our entropy so low?

FRAMEWORK

The entropy controversy



Seth Lloyd 1988:

Theorem:

(Second law of thermodynamics for isolated systems.)

If the time evolution of an isolated classical or quantum mechanical system is known only inexactly, and if under such an evolution the entropy S goes to S', then $S' \geq S$.

Entropy is a measure of our ignorance

Boltzmann's viewpoint vs Gibbs' viewpoint:

"[Thinking] that entropy has anything to do with what we know about a system is a bad misleading mistake."

David Albert, Copenhagen, Aug. 28, 2011

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I disagree!

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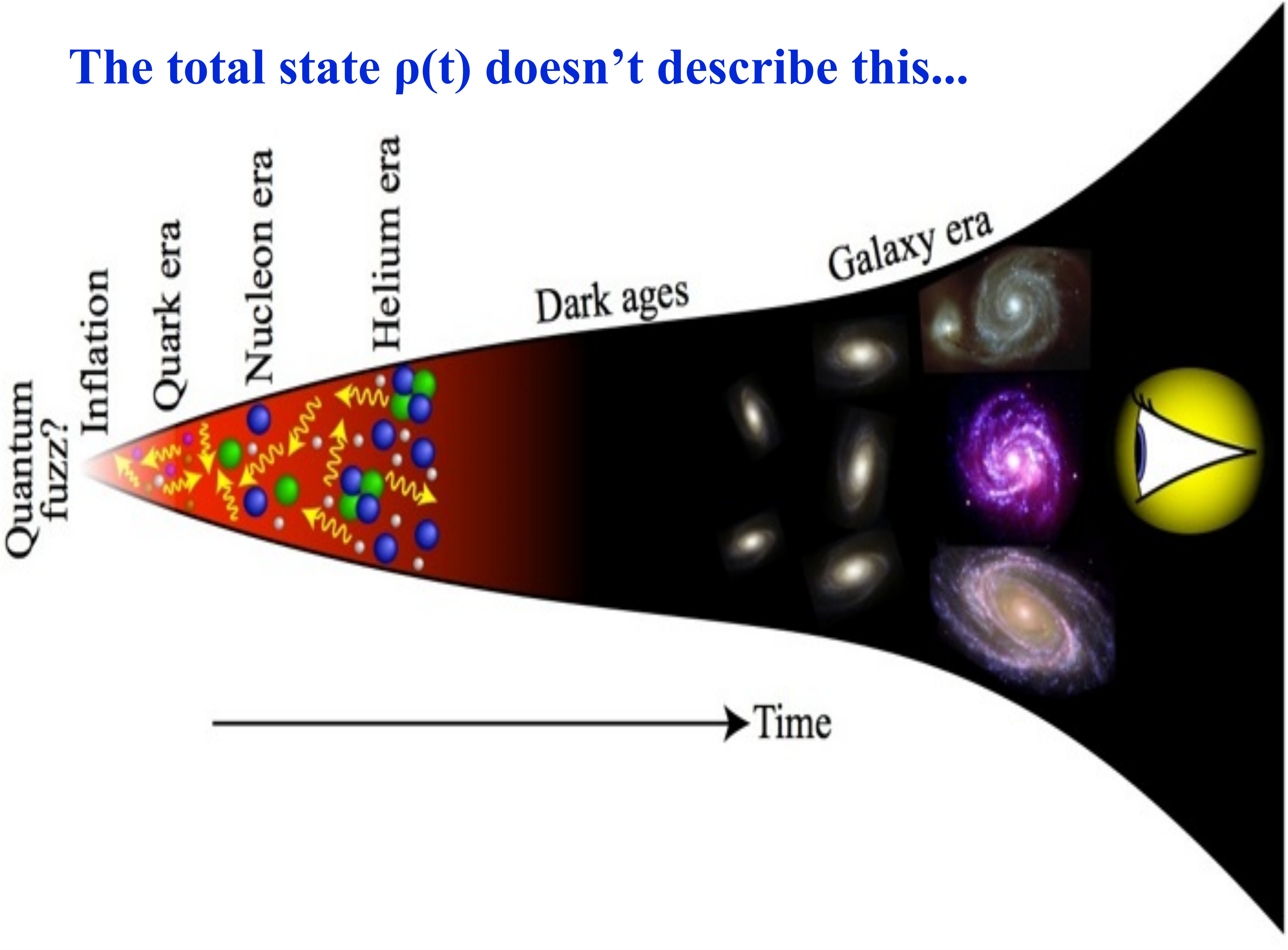
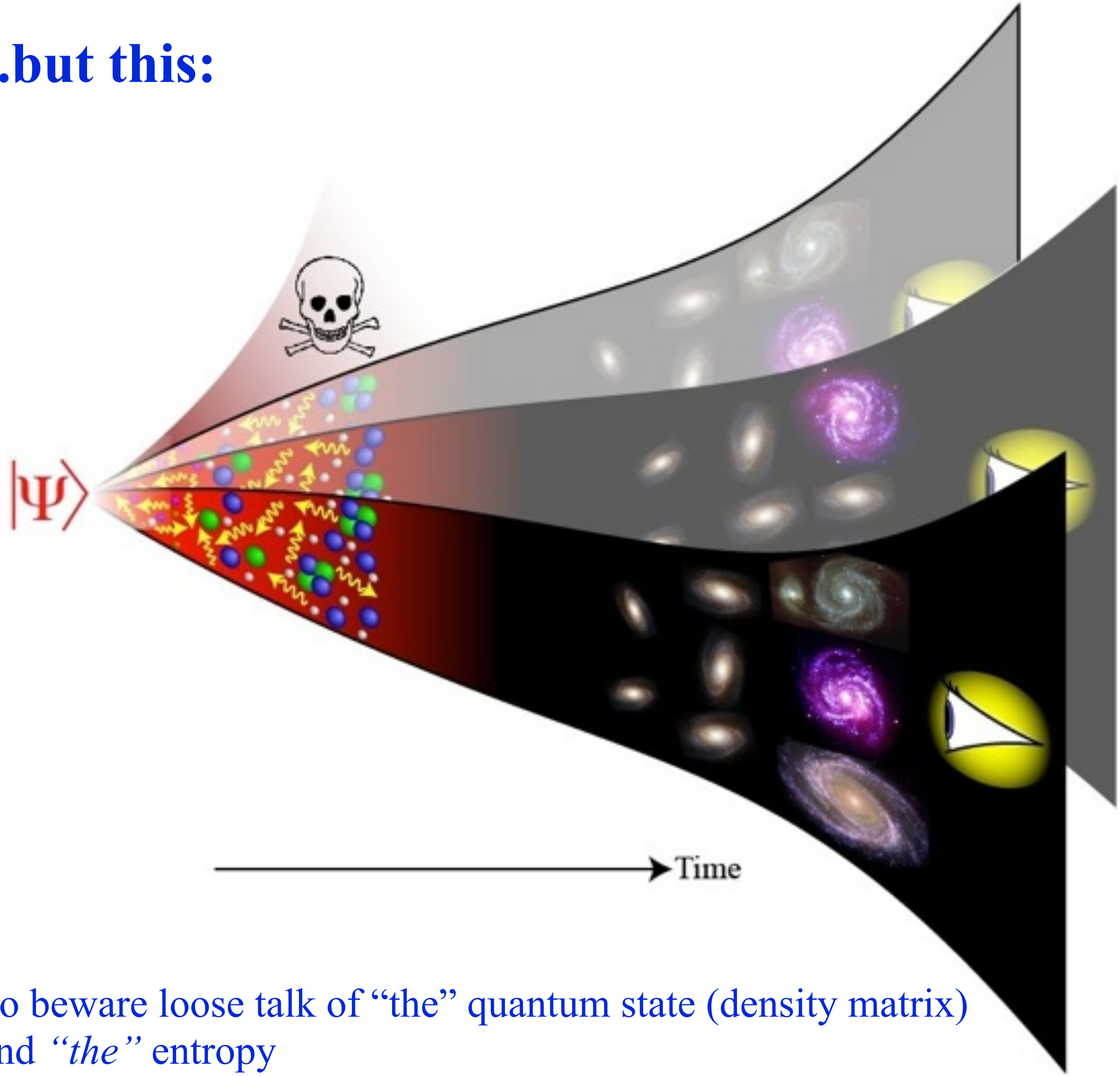
The total state $\rho(t)$ doesn't describe this...



...but this:



So beware loose talk of "the" quantum state (density matrix) and *"the"* entropy

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I disagree!

I'm interested in using physics to make predictions about my future. Knowing the full quantum state ρ of the whole multiverse

1) **Isn't enough:**

   I also need to take into account what I know about my location:
   - in 3D space
   - in Hilbert space

2) **Isn't necessary:**

   I only need to know the quantum state "nearby"
   - in 3D space
   - in Hilbert space

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1) **Isn't enough:** ← **Conditionalize! (QBT)**

I also need to take into account what I know about my location:
- in 3D space
- in Hilbert space

2) **Isn't necessary:** ← **Partial trace!**

I only need to know the quantum state "nearby"
- in 3D space
- in Hilbert space

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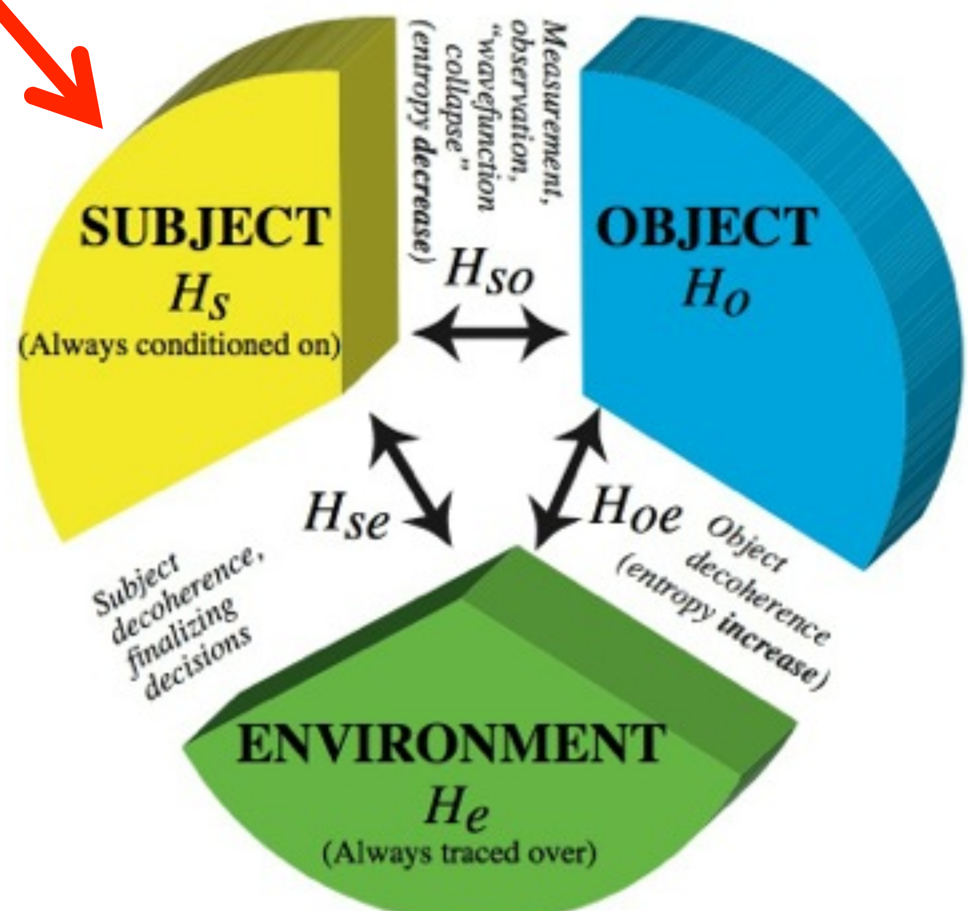
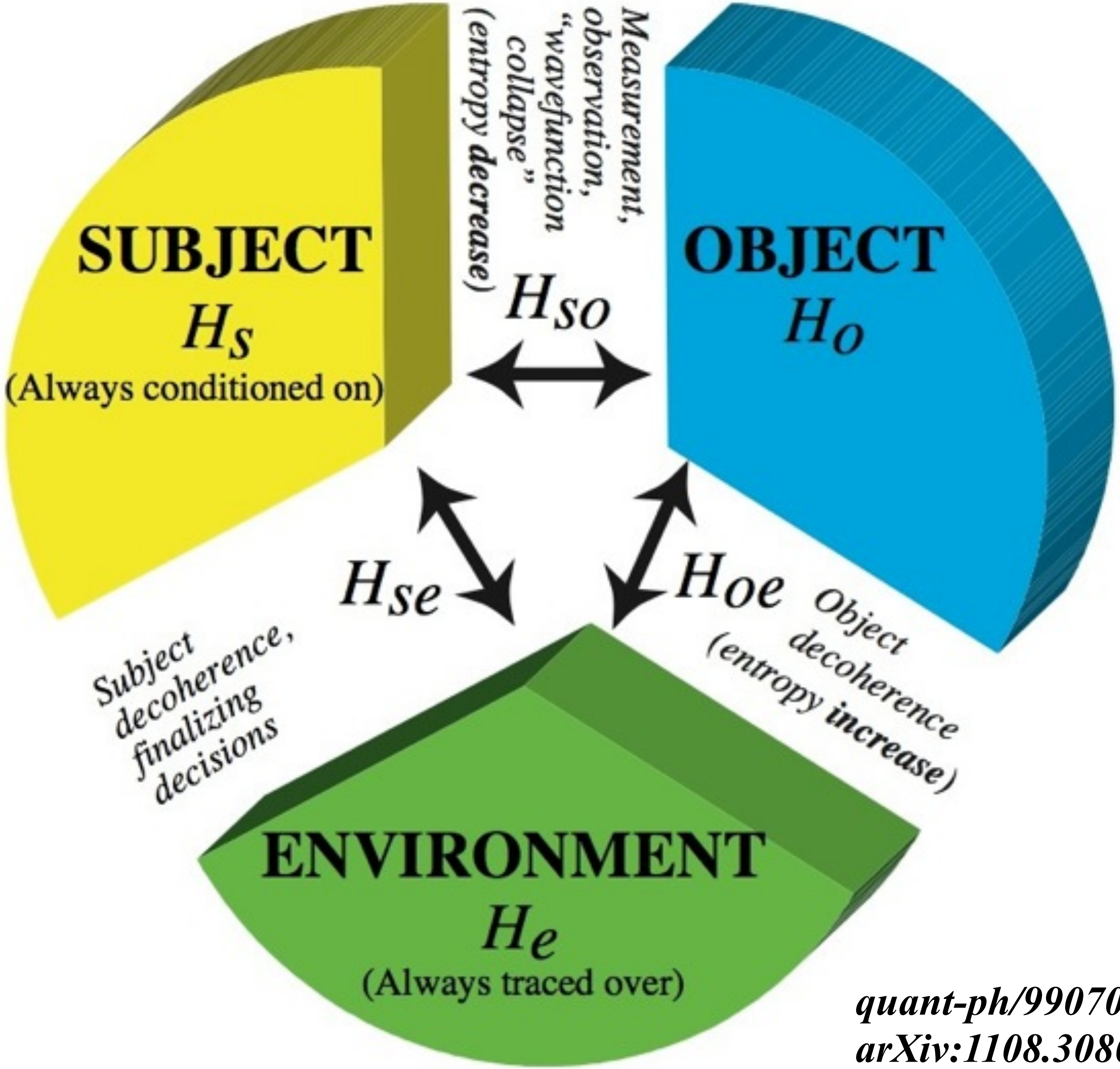
Basic theme: don't ignore the *subject!*

I think that consciousness is the way information feels when being processed in certain complex ways. No "secret sauce"!



Based on: **MT quant-ph/9907009, MT arXiv:1108.3080, Hrant Gharibyan & MT 2013**



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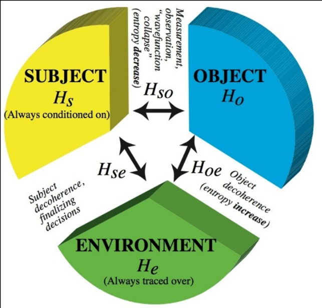
quant-ph/9907009
arXiv:1108.3080

THEOREMS



The second law of quantum thermodynamics:

<u>Decoherence</u>: *When an object is probed by the environment, its entropy increases*

<u>Observation</u>: *When an object is probed by the subject, its entropy on average decreases*

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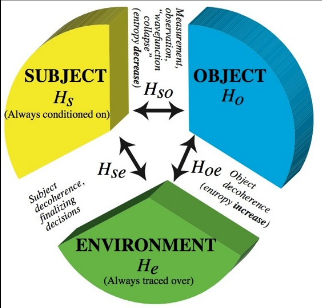

SUBJECT
H_S
(Always conditioned on)
OBJECT
H_O
ENVIRONMENT
H_E
(Always traced over)
H_{SO}
H_{SE}
H_{OE}
Measurement, observation, "wavefunction collapse" (entropy **decrease**)
Subject decoherence, finalizing decisions
Object decoherence (entropy **increase**)


Hrant Gharibyan

APPLICATIONS

Q: How solve inflationary entropy problem?

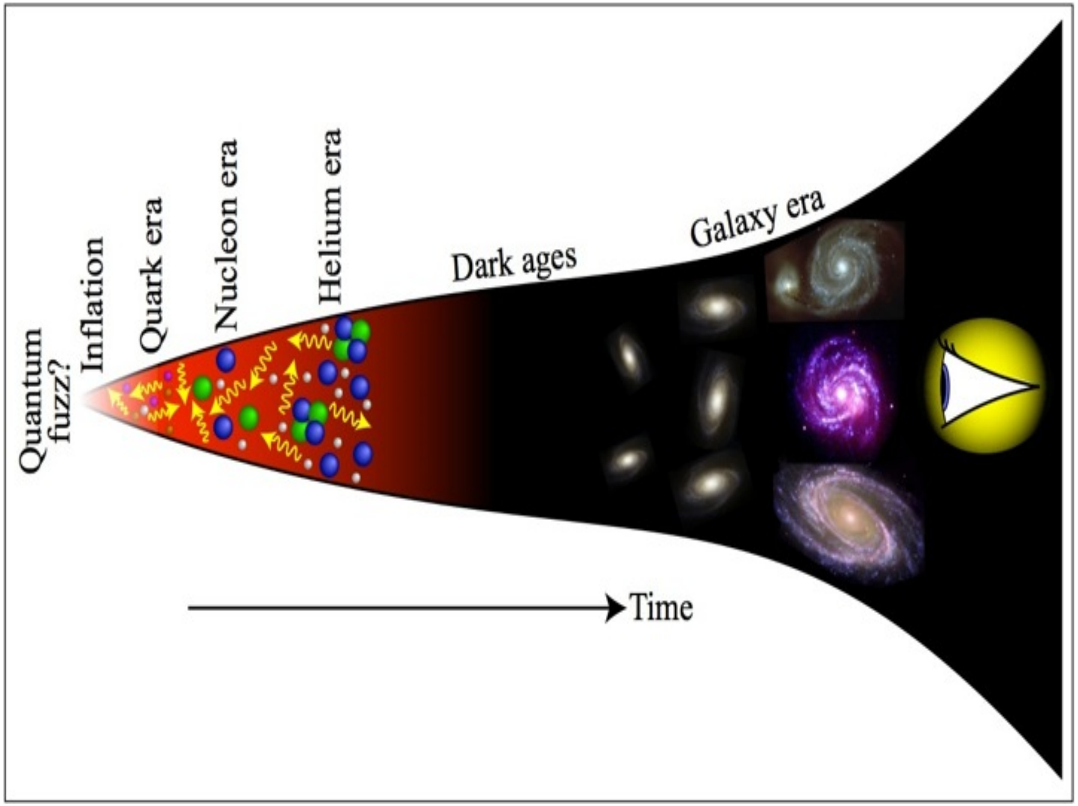
A: By unifying quantum mechanics with Alan Guth!

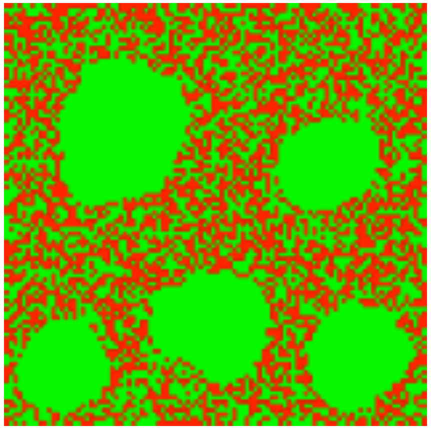
Summary:

- How lower entropy?

Basically, the entropy of something decreases when you look at it and increases when you don't (quant-ph/9907009, arXiv:1108.3080)

- How lower entropy by a lot? (inflate+observe!)





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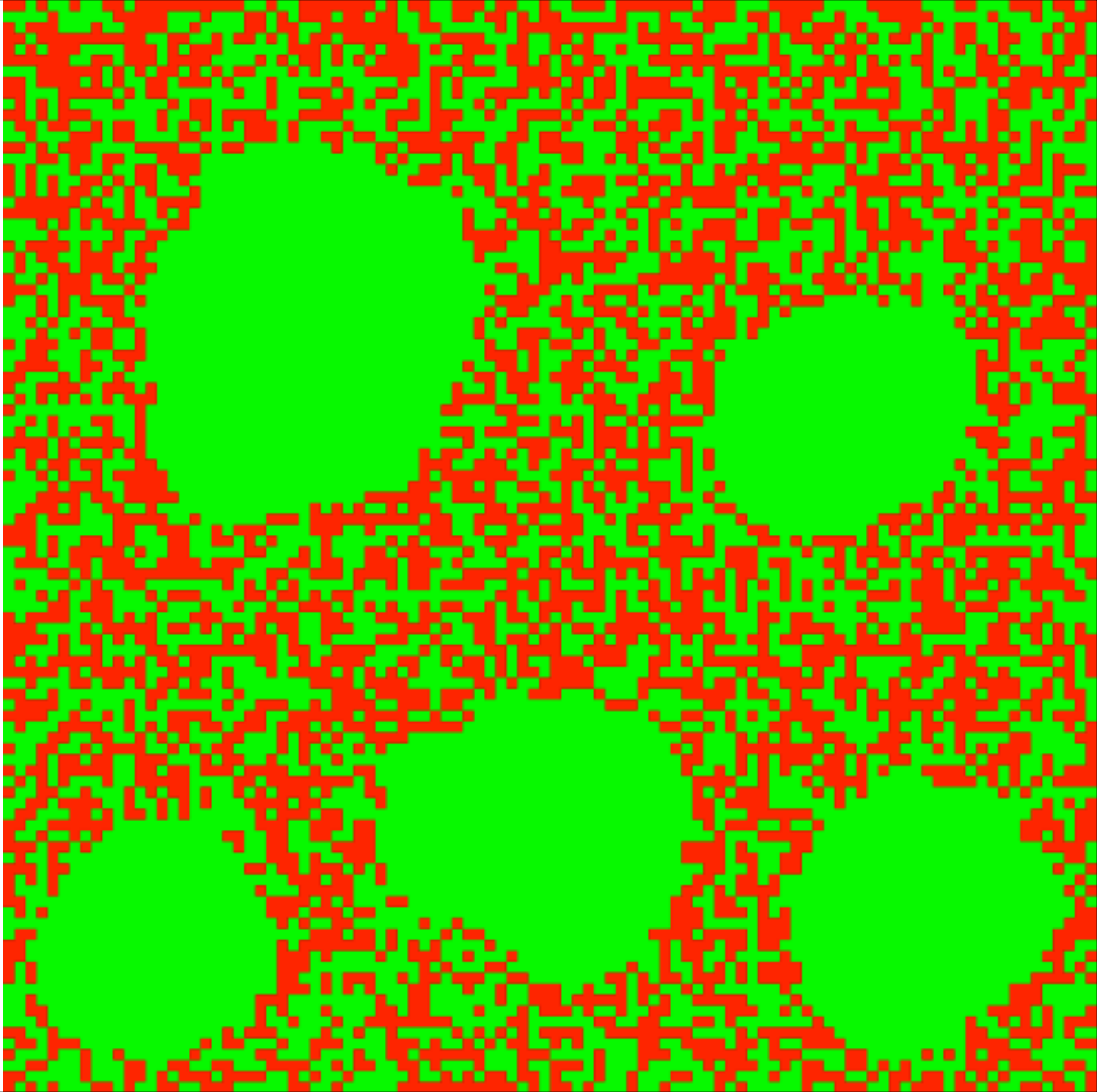
(Phase transitions do the trick too)



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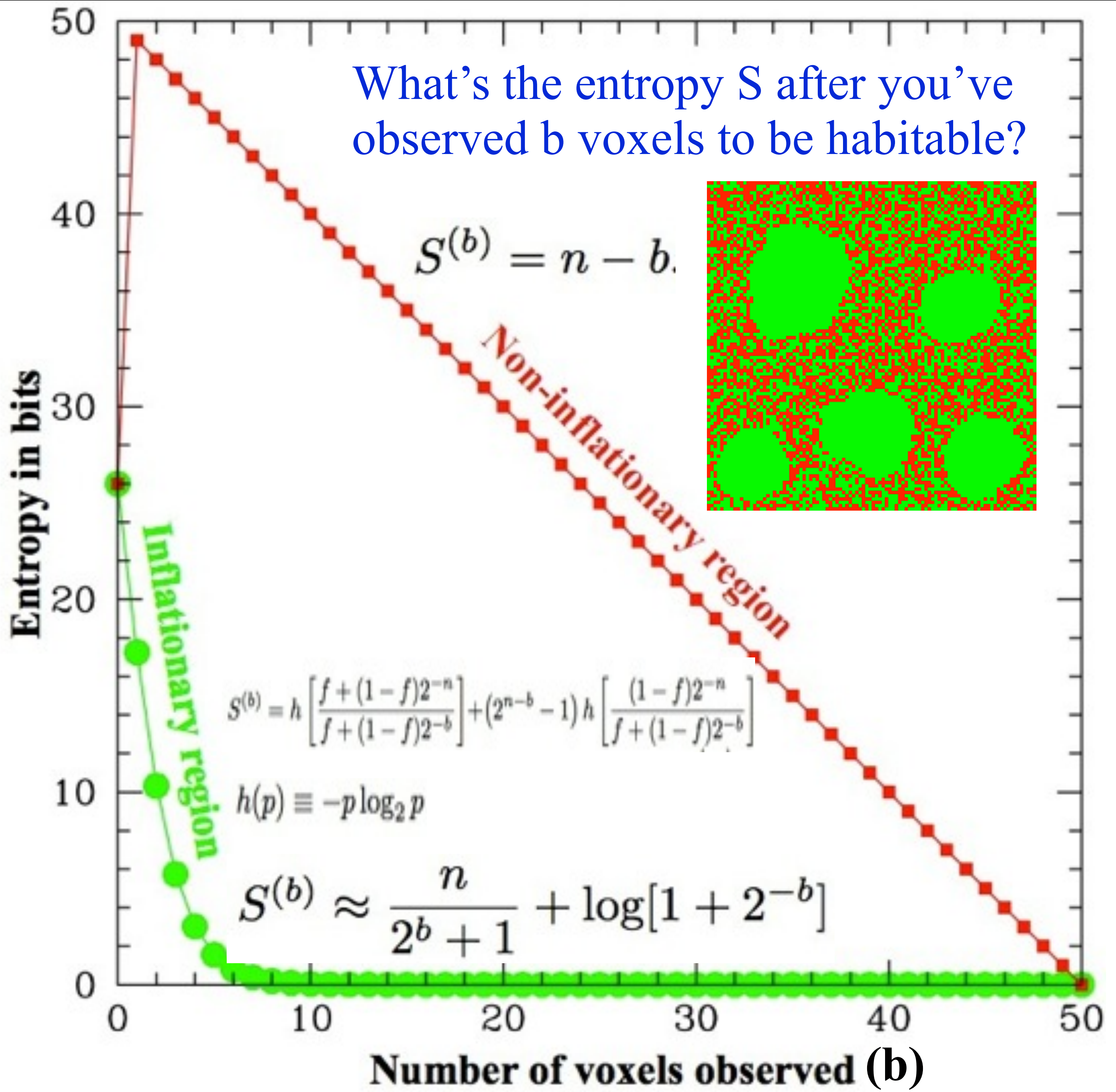
What's the entropy S after you've observed b voxels to be habitable?



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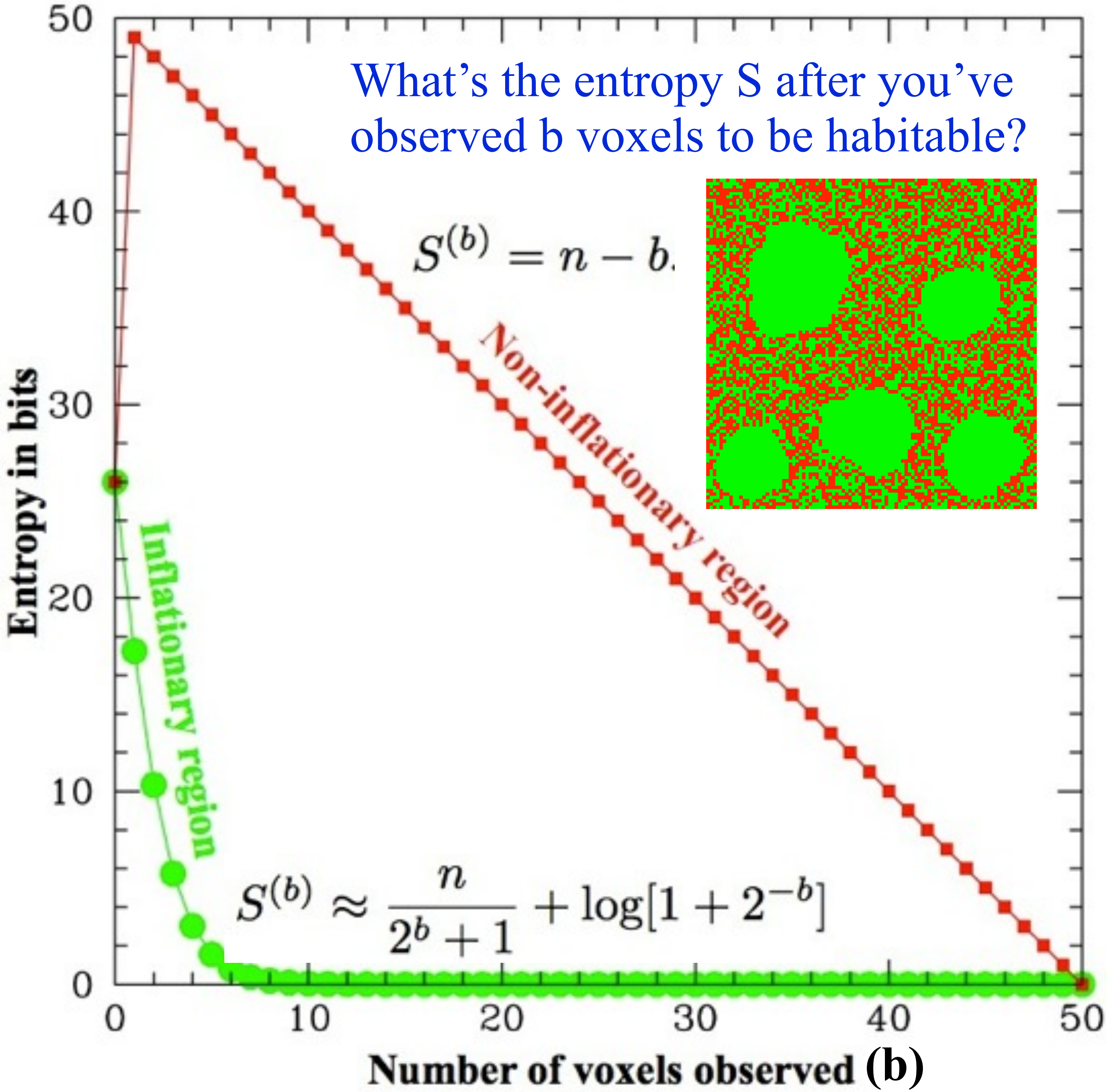

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$$G_{\mu\nu} \overset{?}{\approx} 8\pi G \langle T_{\mu\nu} \rangle$$

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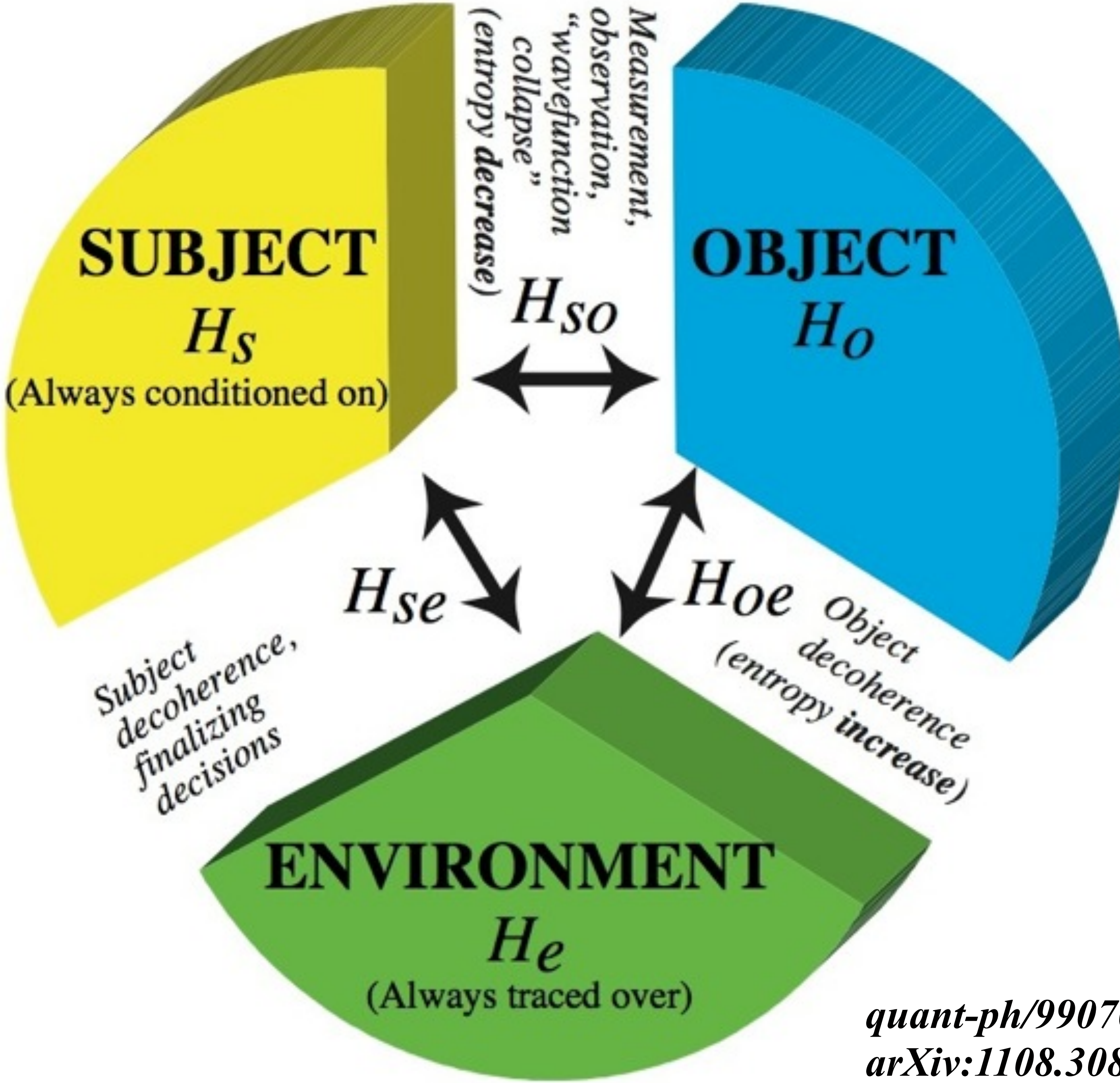

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quant-ph/9907009
arXiv:1108.3080

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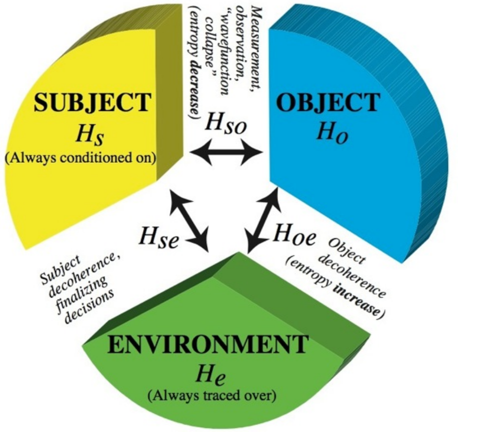
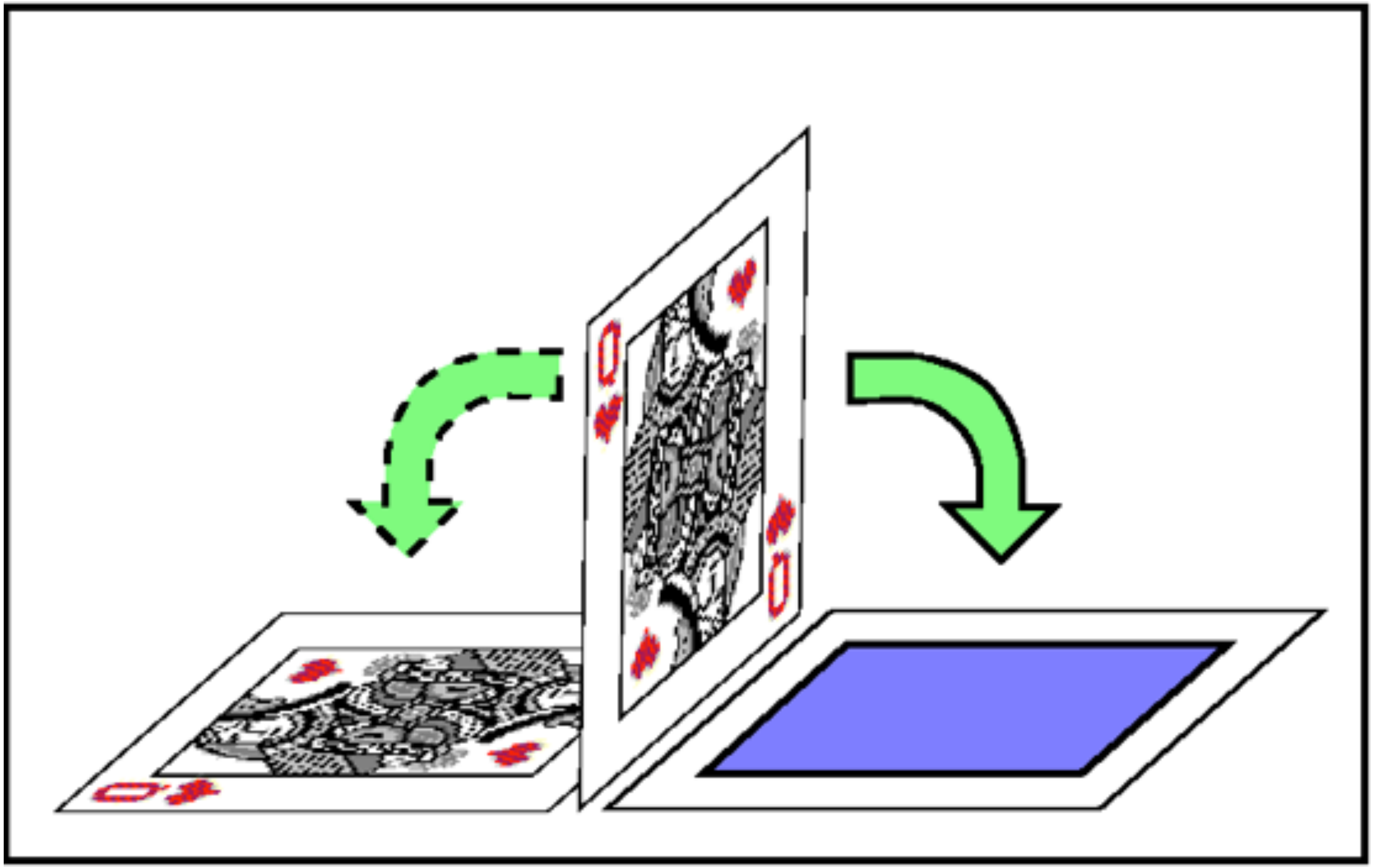
Unitary evolution

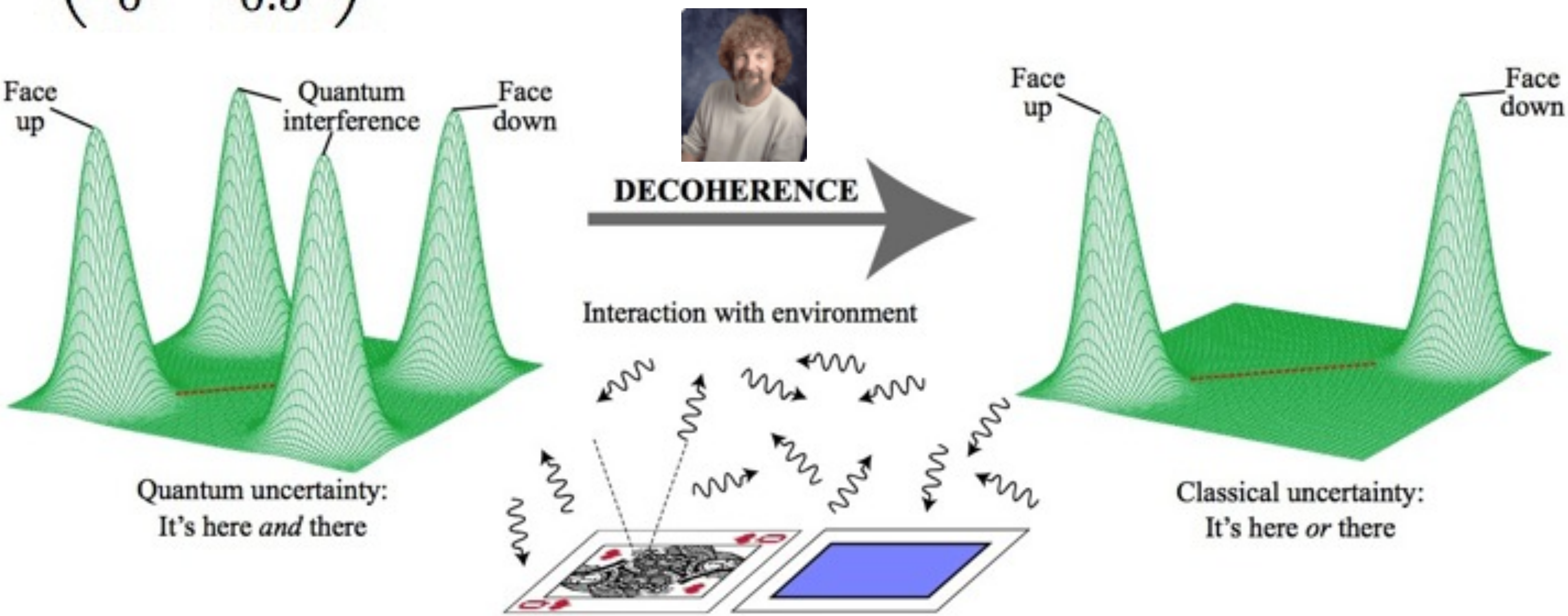




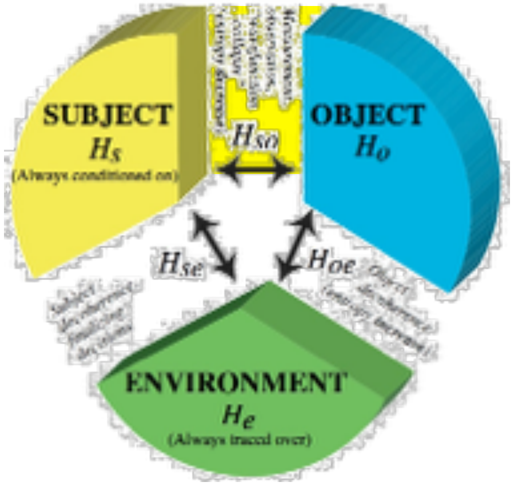
Decoherence

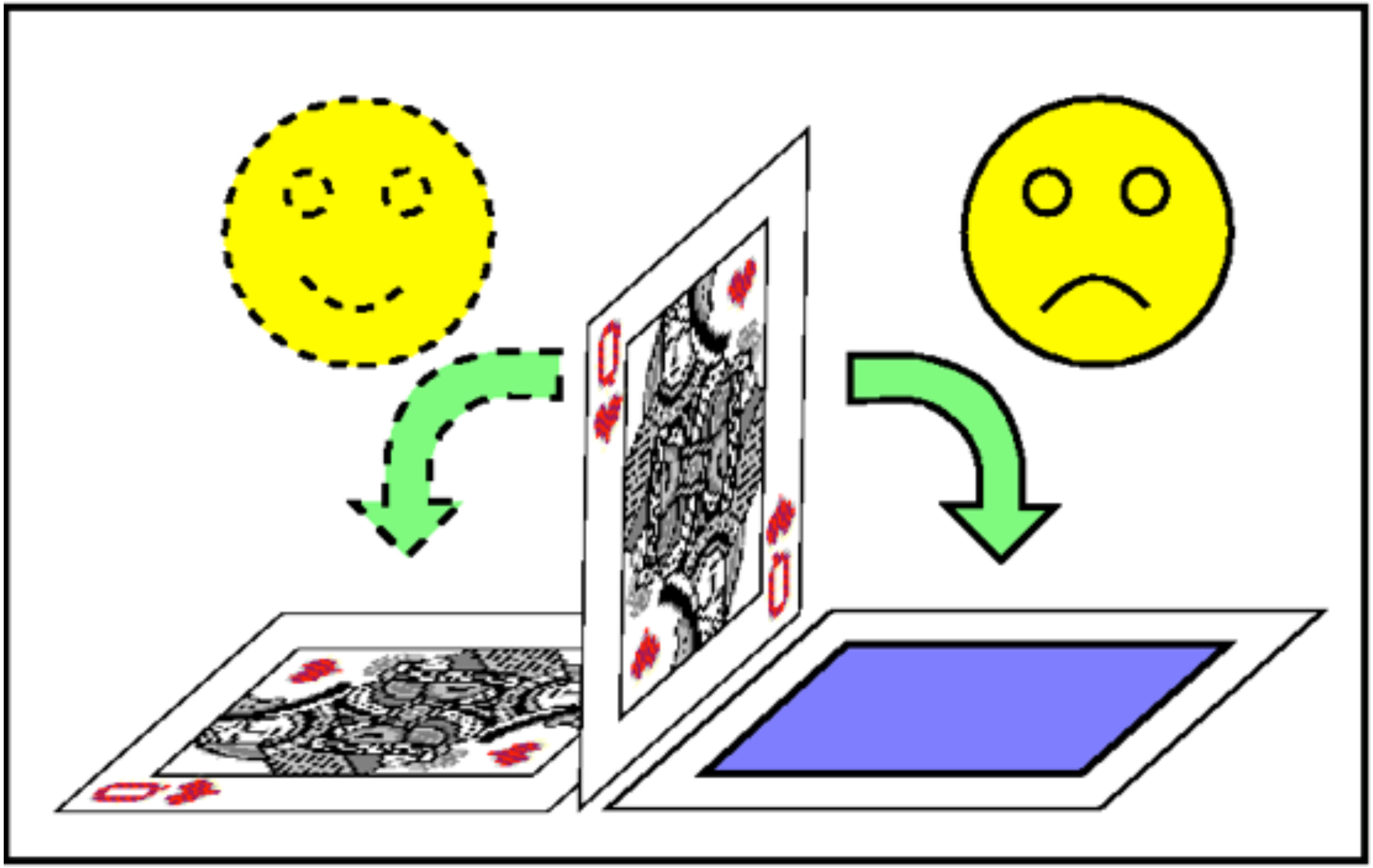
$$\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} = \text{"It's equally here } \textit{and} \text{ there at the same time"}$$

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} = \text{"It's here } \textit{or} \text{ there — I just don't know which"}$$



Observation





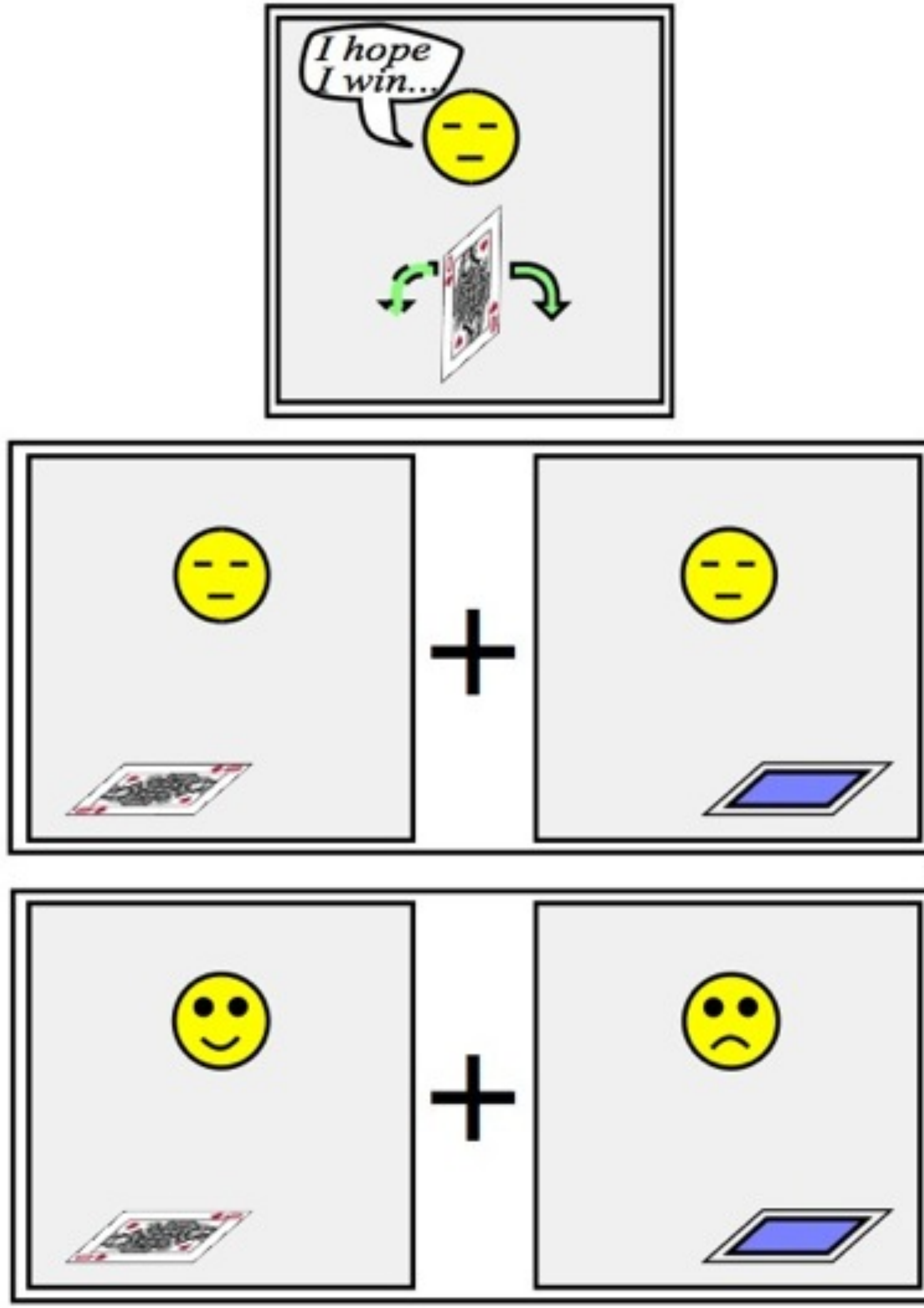
Wavefunction
at 10:00:00 AM:

Card Falls

Wavefunction
at 10:00:10 AM:

Eyes opened

Wavefunction
at 10:00:20 AM:



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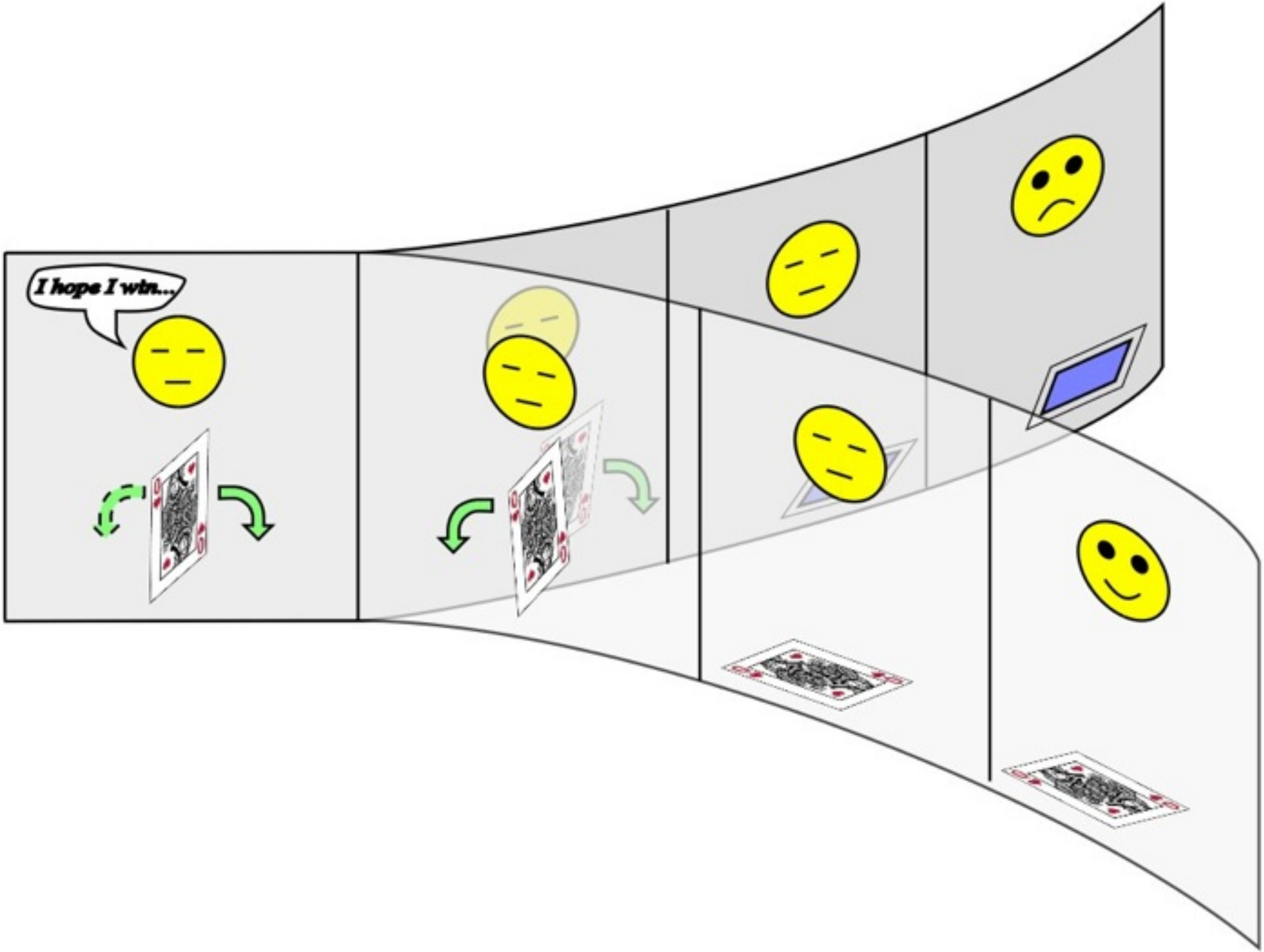
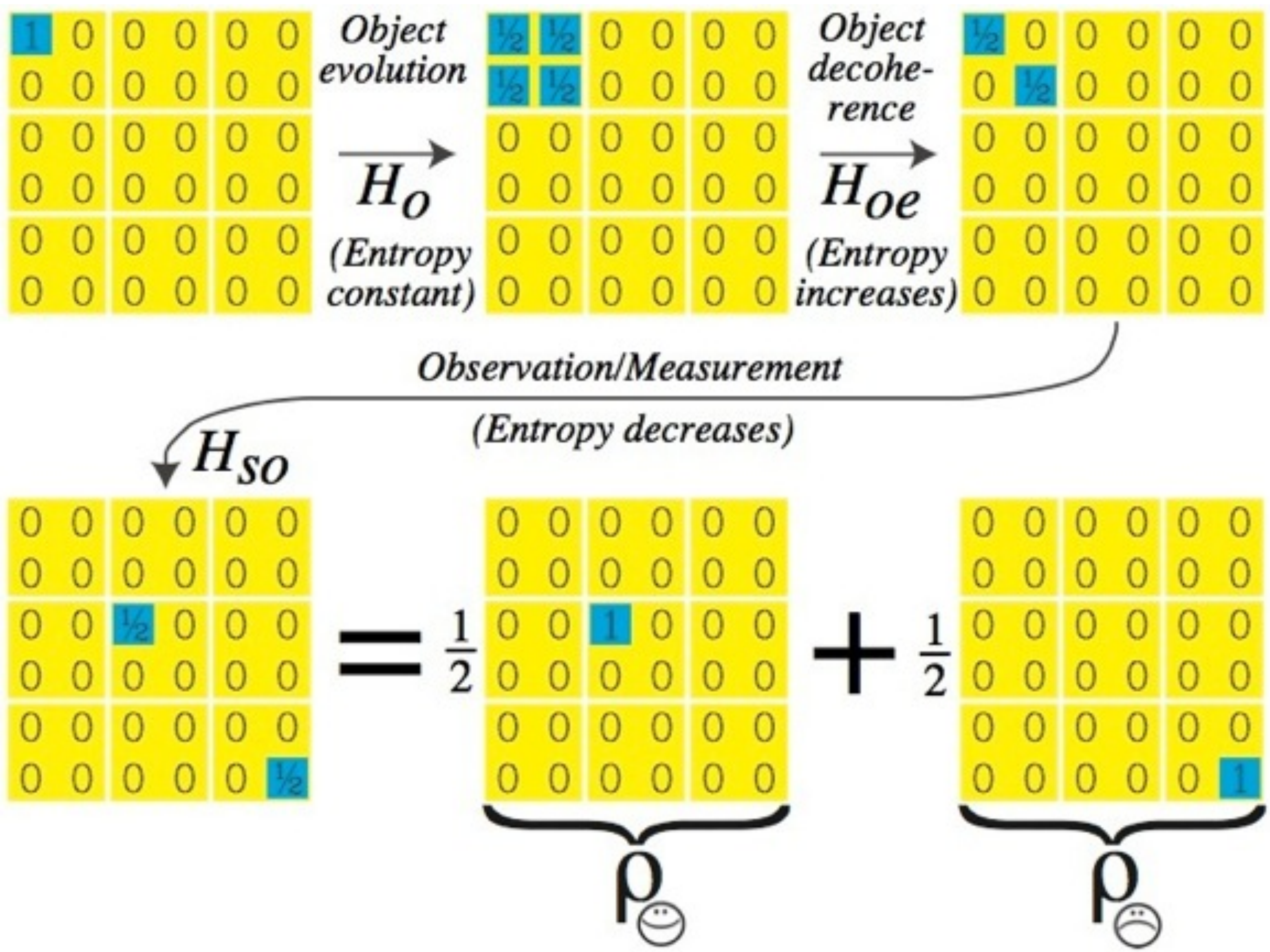

I hope I win...



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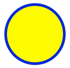
Q: What counts as an observer?

- A human?
- A mouse?
- A robot?
- A photon?

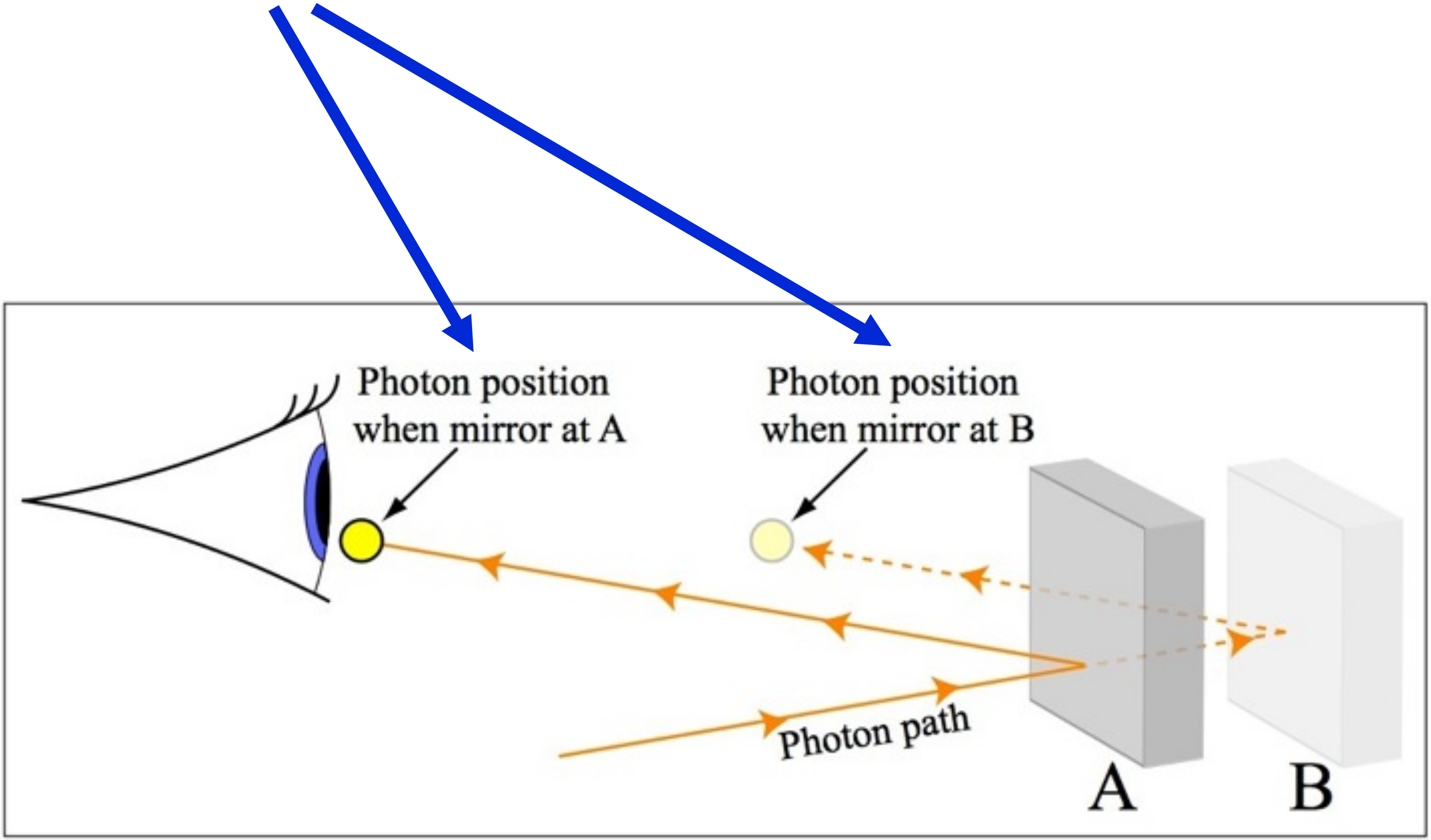


A: All of the above!

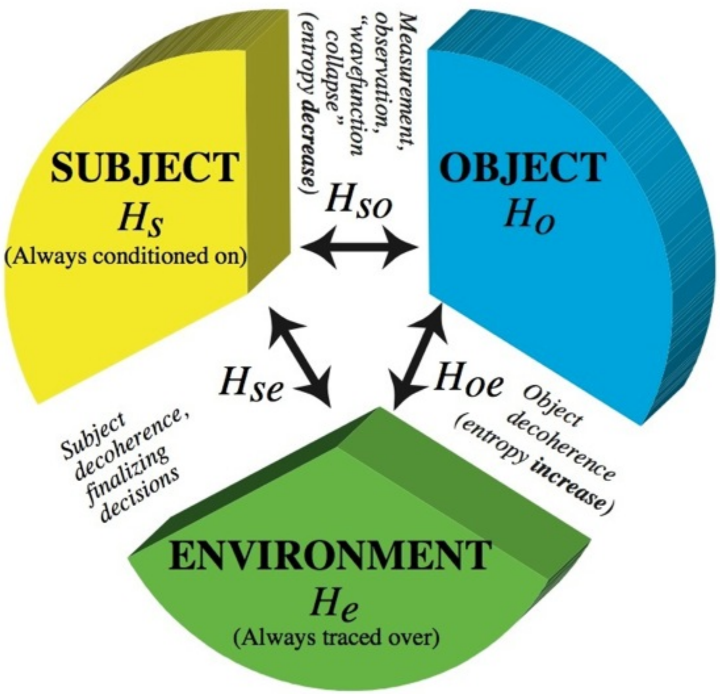
(It's simply information transfer that matters)

Photon as observer:

It's position encodes whether block is at A or B



Interaction	Dynamics	Example	Effect	Entropy
Object-object	$\rho \mapsto U\rho U^\dagger$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mapsto \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	Unitary evolution	Unchanged
Object-environment	$\rho \mapsto \sum_{ij} P_i \rho P_j \langle \epsilon_j \vert \epsilon_i \rangle$	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \mapsto \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$	Decoherence	Increases
Object-subject	$\rho \mapsto \frac{\Pi_i^\dagger \rho \Pi_i}{\mathrm{tr}\, \Pi_i \rho \Pi_i}, \quad \Pi_i = \sum_j \langle s_i \vert \sigma_j \rangle P_j$	$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	Observation	Decreases



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Decoherence

$$U|e_0\rangle|o_i\rangle = |\epsilon_i\rangle|o_i\rangle$$

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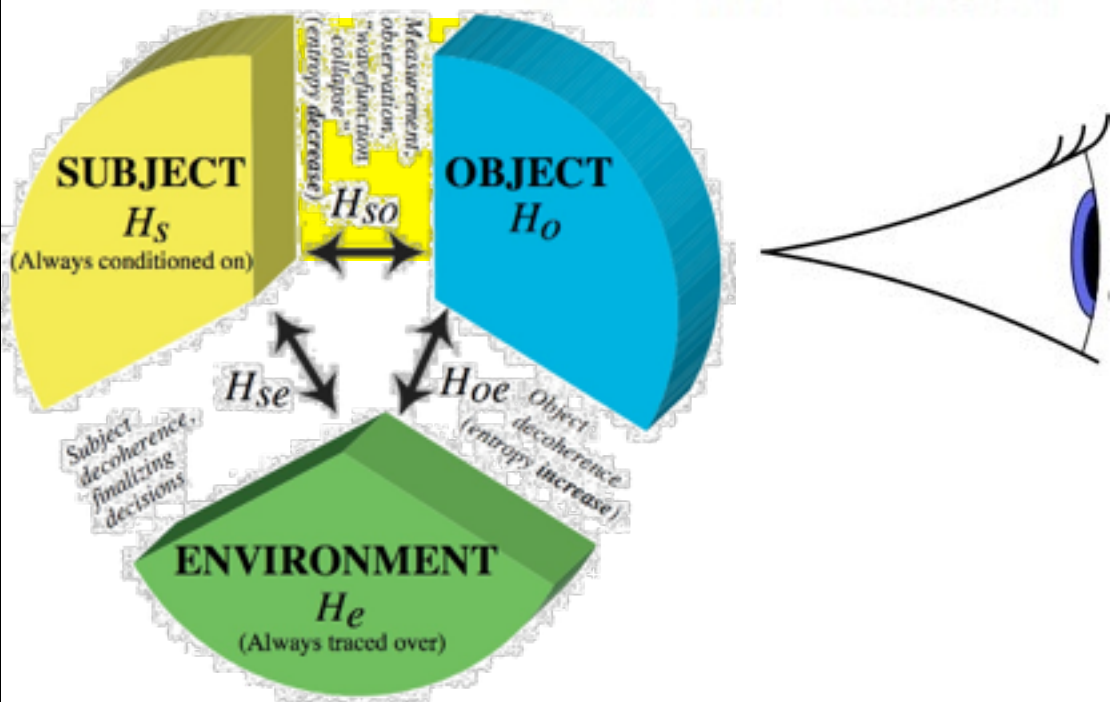
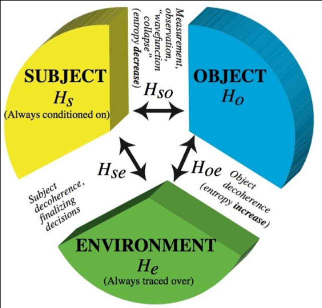
Observation

$$U|s_0\rangle|o_i\rangle = |\sigma_i\rangle|o_i\rangle$$



Quantum Bayes Theorem!



Another law of thermodynamics:
The object's entropy can't increase unless it interacts with the environment.

Zeh, Zurek & co: unitary object-environment

$$U|e_0\rangle|o_i\rangle = |\epsilon_i\rangle|o_i\rangle$$

changes the reduced density as

$$\rho \mapsto \rho \circ \mathbf{E}$$

Shur product

$$(\rho \circ \mathbf{E})_{ij} = \rho_{ij} E_{ij}$$

$$E_{ij} \equiv \langle \epsilon_j | \epsilon_i \rangle$$

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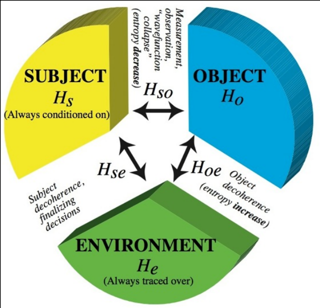

Another law of thermodynamics:
The object's entropy can't increase unless it interacts with the environment.

Theorem: $S(\rho \circ \mathbf{E}) \geq S(\rho)$

Shur product

$E_{ij} \equiv \langle \epsilon_j | \epsilon_i \rangle$

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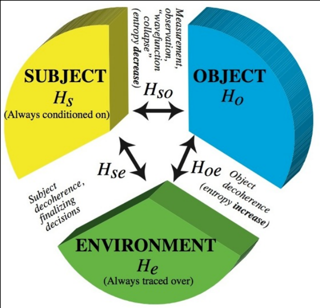

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Proof (MT, arXiv:1108.3080):

From a 1985 theorem of Bapat & Sunder, one can prove that the eigenvalues of ρ *majorize* those of $\rho \circ \mathbf{E}$.
This implies that decoherence will increase any sum of a concave function of these eigenvalues.
Chosing the concave functions -*p* log *p*, 1-p^α and ln *p* implies that decoherence increases the Shannon entropy, the Renyi entropy and the determinant, respectively.

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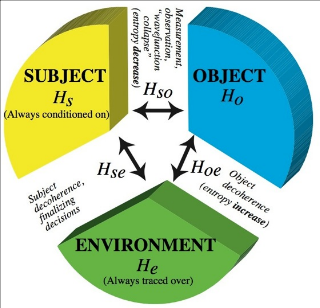

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Second law of thermodynamics:
The object's entropy can't decrease unless it interacts with the subject.

Another law of thermodynamics:
The object's entropy can't increase unless it interacts with the environment.

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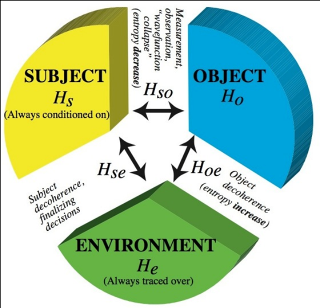

Second law of thermodynamics:
The object's entropy can't decrease unless it interacts with the subject.

Theorem: $$\sum_i p_i S\left(\rho^{(i)}\right) < S(\rho)$$

$$p_i = \sum_j \rho_{jj}|S_{ij}|^2 \qquad \rho^{(i)}_{jk} = \frac{\rho_{jk}S_{ij}S^*_{ik}}{p_i} \qquad S_{ij} \equiv \langle s_i|\sigma_j\rangle$$

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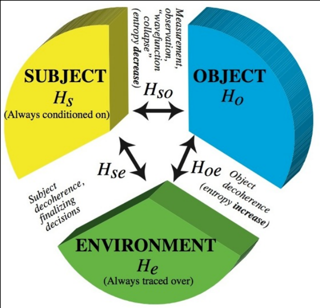

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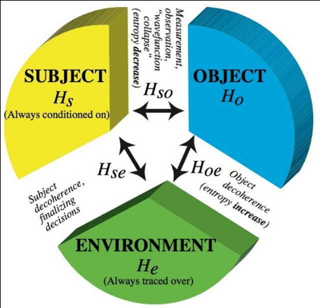

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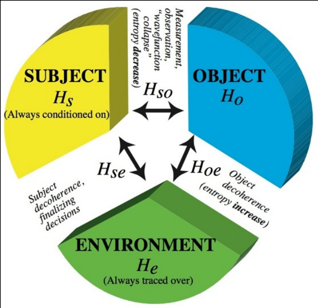
Hrant Gharibyan



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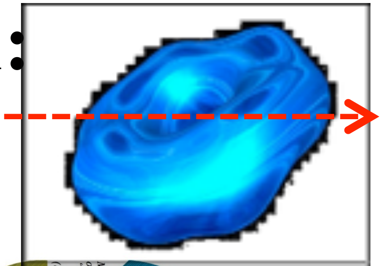
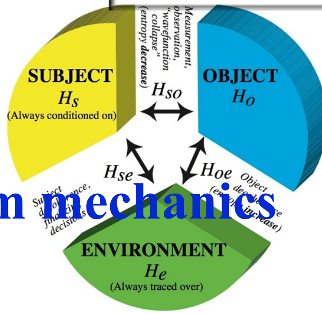
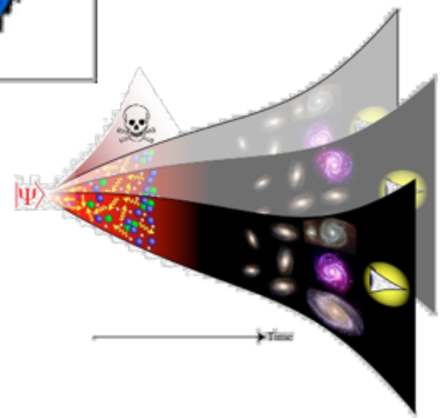
Today's menu:

1) How decoherence makes spacetime so classical



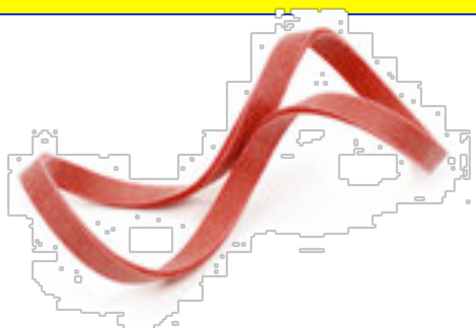
2) Decoherence, observation & the 2nd law(s)

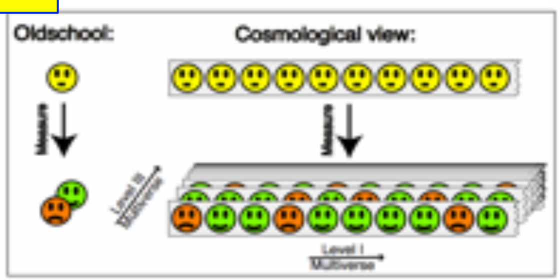




3) The cosmological interpretation of quantum mechanics

4) The big snap



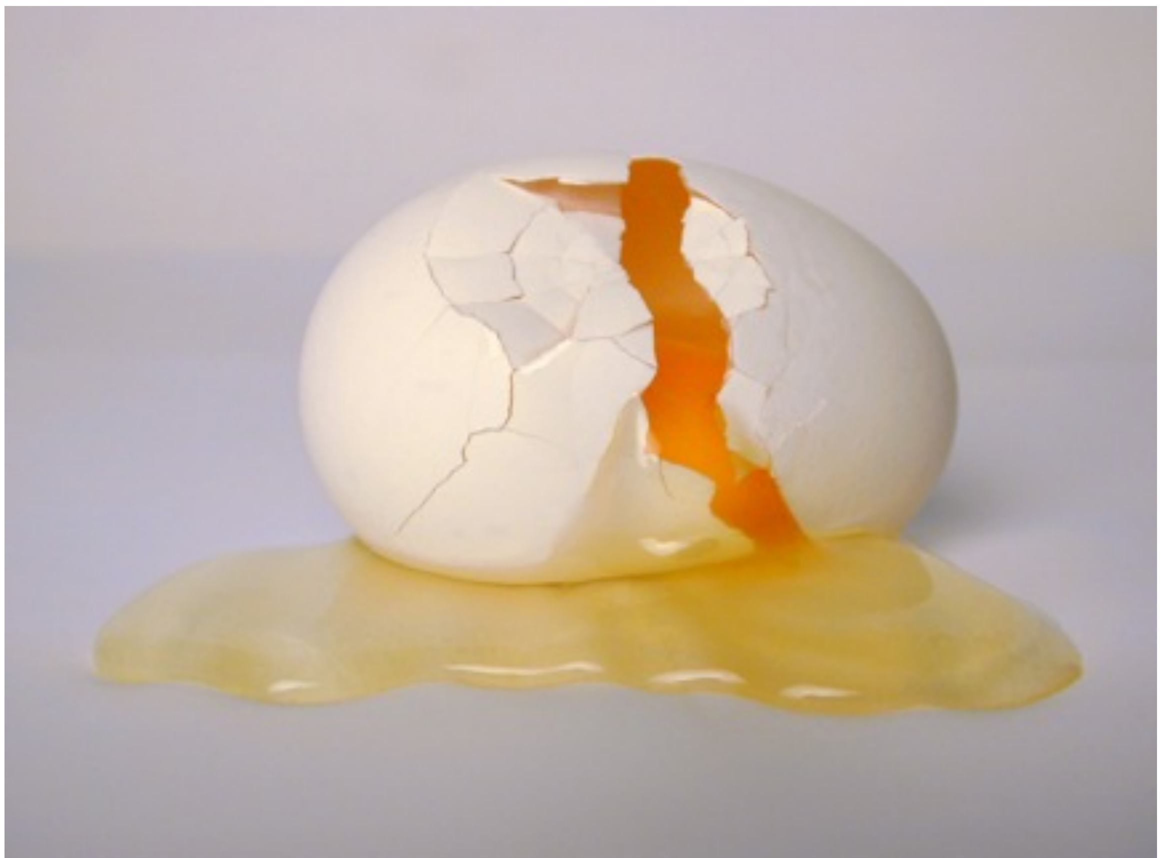


Based on:

MT: quant-ph/9907009
Anthony Aguirre & MT: arXiv:1008.1066
MT: arXiv:1108.3080
Hrant Gharibyian & MT 2013
Dan Roberts & MT 2013

Classical physics:
Entropy is a measure of our ignorance

Shannon



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Probability

1

Classical physics:

0.5

Shannon Entropy $S = -\sum p_i \log p_i = 3$ bits

Shannon

= ?

1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8

Card

M

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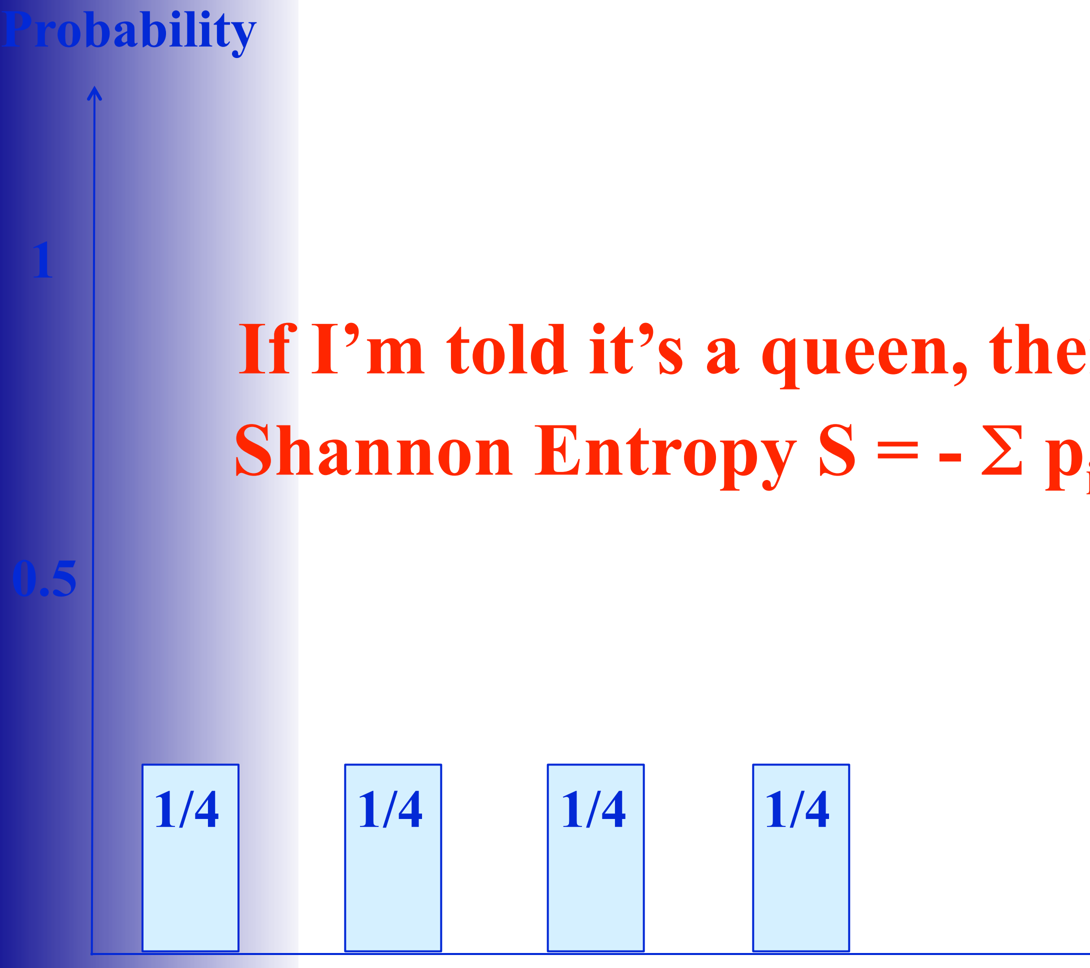
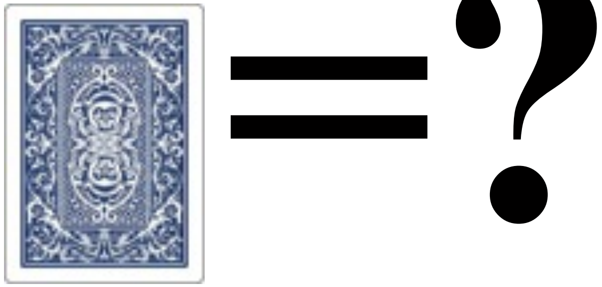
If I'm told it's a queen, then

Shannon Entropy $S = -\sum p_i \log p_i = 2$ bits





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Probability

1

0.5

If I'm told it's black, then

Shannon Entropy $S = -\Sigma\, p_i \log p_i = 1$ bit

Shannon

= ?

1/2

1/2

Card

Probability

If I'm told it's the queen of spades, then

Shannon Entropy $S = -\sum p_i \log p_i = 0$ bits

1

0.5

1

= ?

Card

Quantum mechanics:

- replace probability distribution by density matrix
- replace Shannon entropy by von Neumann entropy $-\mathrm{tr}\,\rho \log \rho$ (Shannon entropy of eigenvalues)

von Neumann

S = 0 bits

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} = \text{“It’s here \textit{and} there at the same time”}$$

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} = \text{“It’s here \textit{or} there — I just don’t know which”}$$

S = 1 bit

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Quantum mechanics:

- replace probability distribution by density matrix
- replace Shannon entropy by von Neumann entropy –tr ρ log ρ (Shannon entropy of eigenvalues)

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mapsto \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

S = 0 bits

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} = \text{“It’s here } \textit{and} \text{ there at the same time”}$$

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} = \text{“It’s here } \textit{or} \text{ there — I just don’t know which”}$$

S = 1 bit

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How will it all end?

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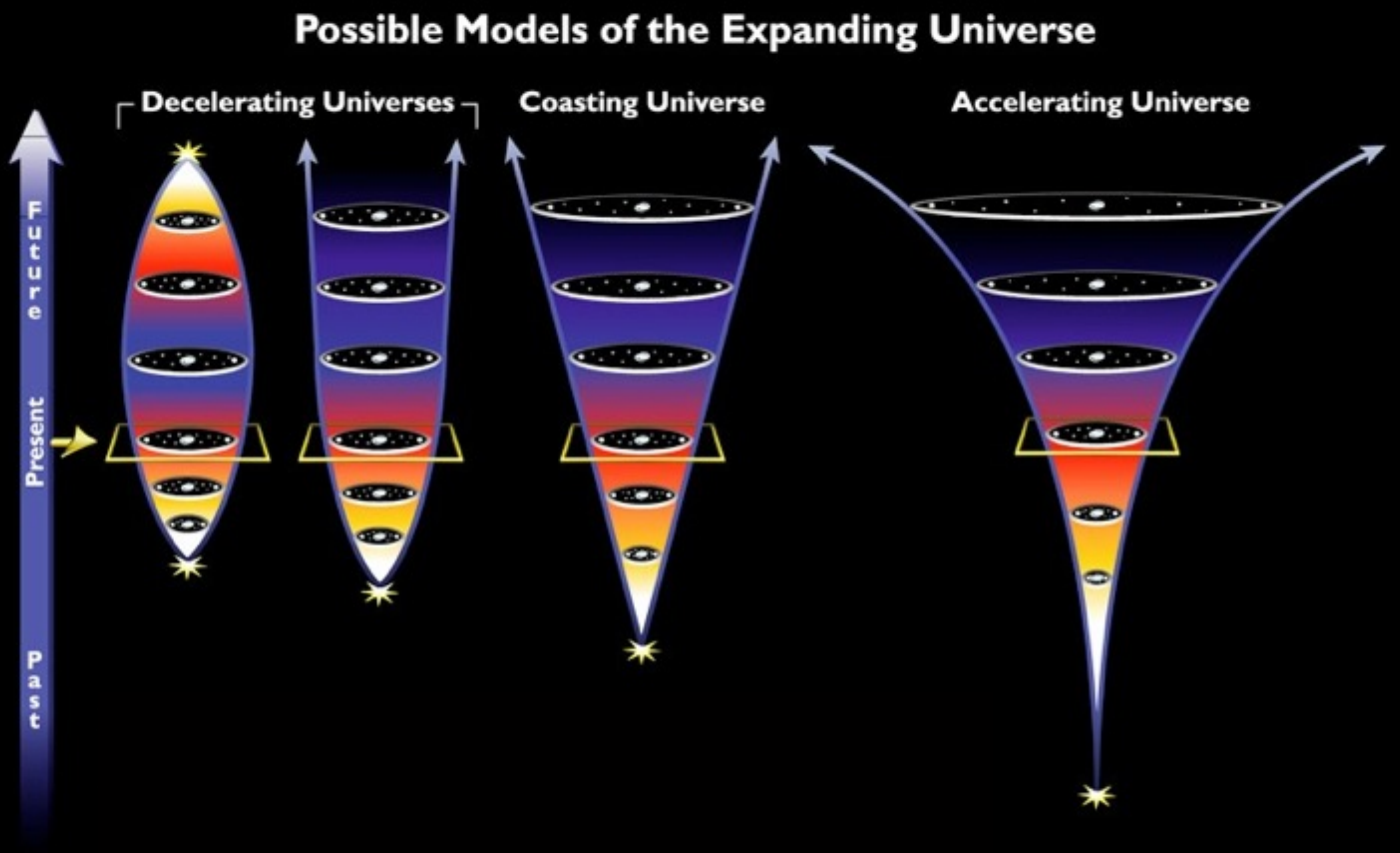
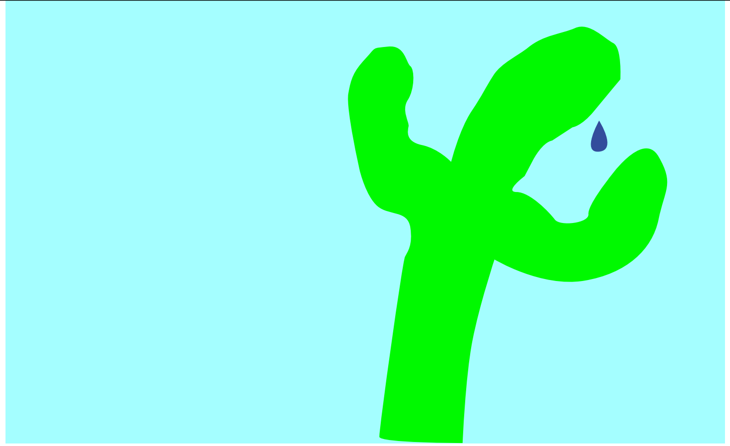


Figure from STScI



The cosmological interpretation of quantum mechanics

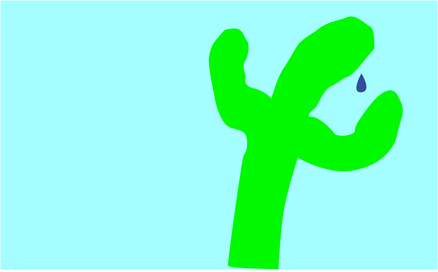
Co-conspirator: Anthony Aguirre

arXiv:1008.1066

Textbook quantum mechanics is either logically inconsistent or incomplete (unable to make predictions for certain experiments):

1. All isolated systems evolve according to the Schrödinger equation

2. When a system is observed, its wavefunction collapses in a random way



Textbook quantum mechanics is either logically <u>inconsistent</u> or <u>incomplete</u> (unable to make predictions for certain experiments):

1. All isolated systems evolve according to the Schrödinger equation

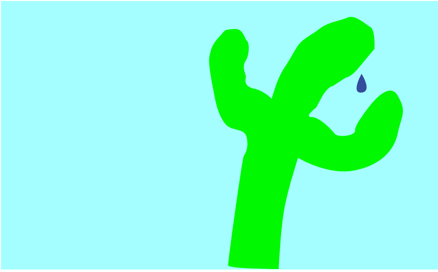
2. When a[...]erved, its wav[...]tion collaps[...]in a ran[...]m

Everett's idea

(People are just systems of atoms; no secret sauce)

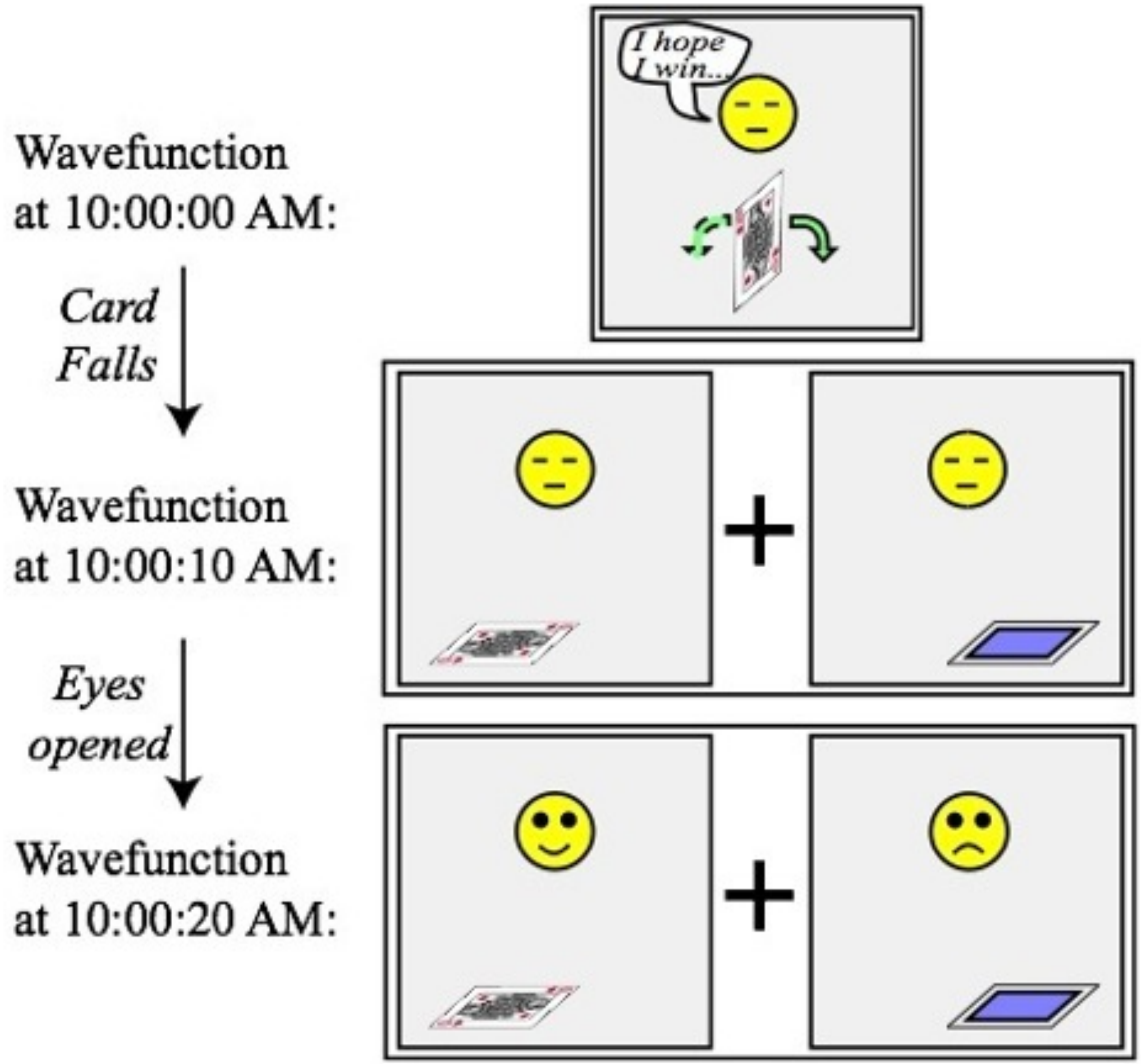
Textbook quantum mechanics is either logically inconsistent or unable to make predictions for certain experiments.

Interpretation	Maryland 1997	Harvard 2010
Copenhagen	13	0
Everett	8	16
Bohm	4	0
Consistent histories	4	2
Modified dynamics	1	1
None of the above/undecided	18	16
Total votes	46	35



The quantum weirdness isn't confined to the Microworld:

1. We're made of particles
2. Microsuperpositions can be amplified into macrosuperpositions:



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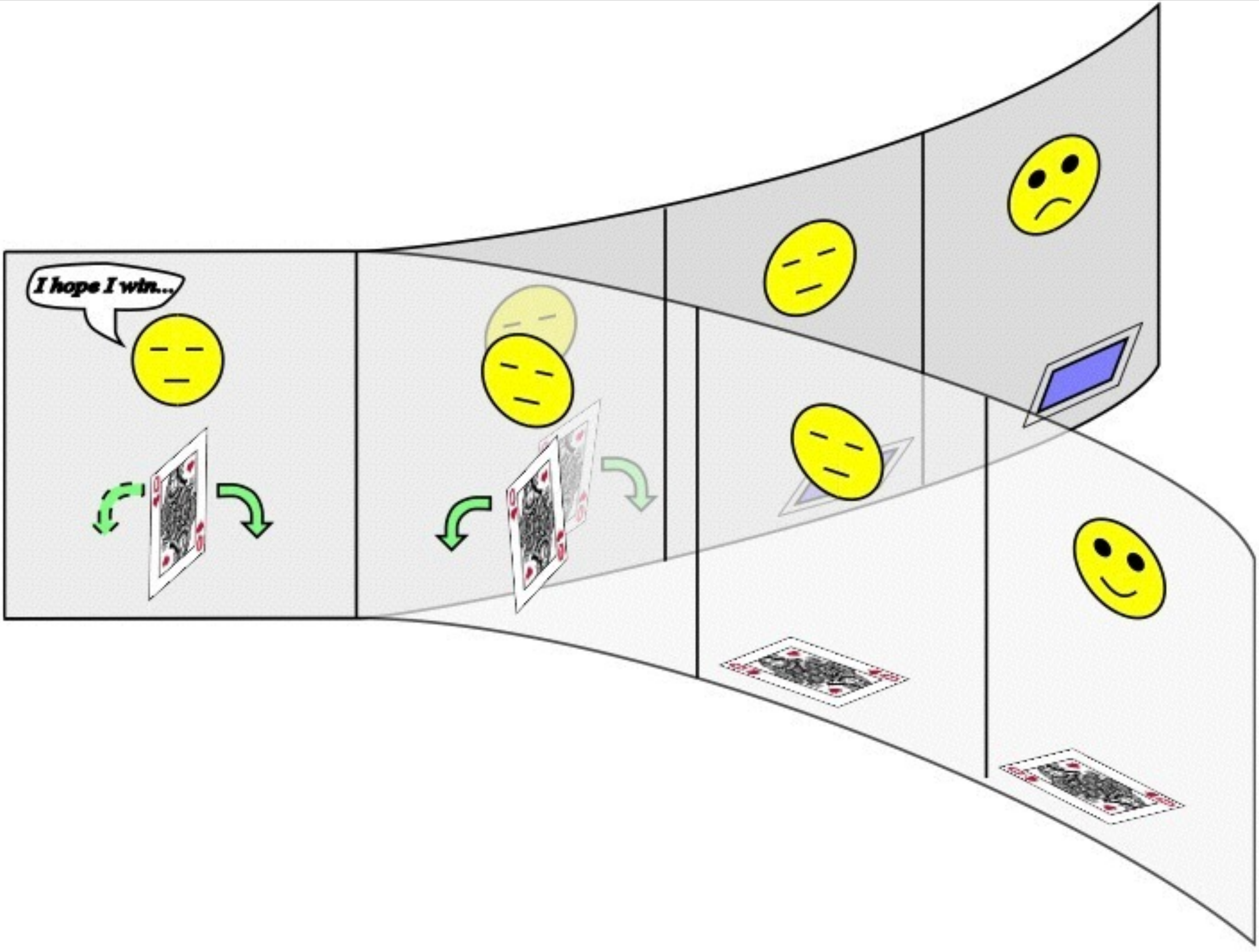
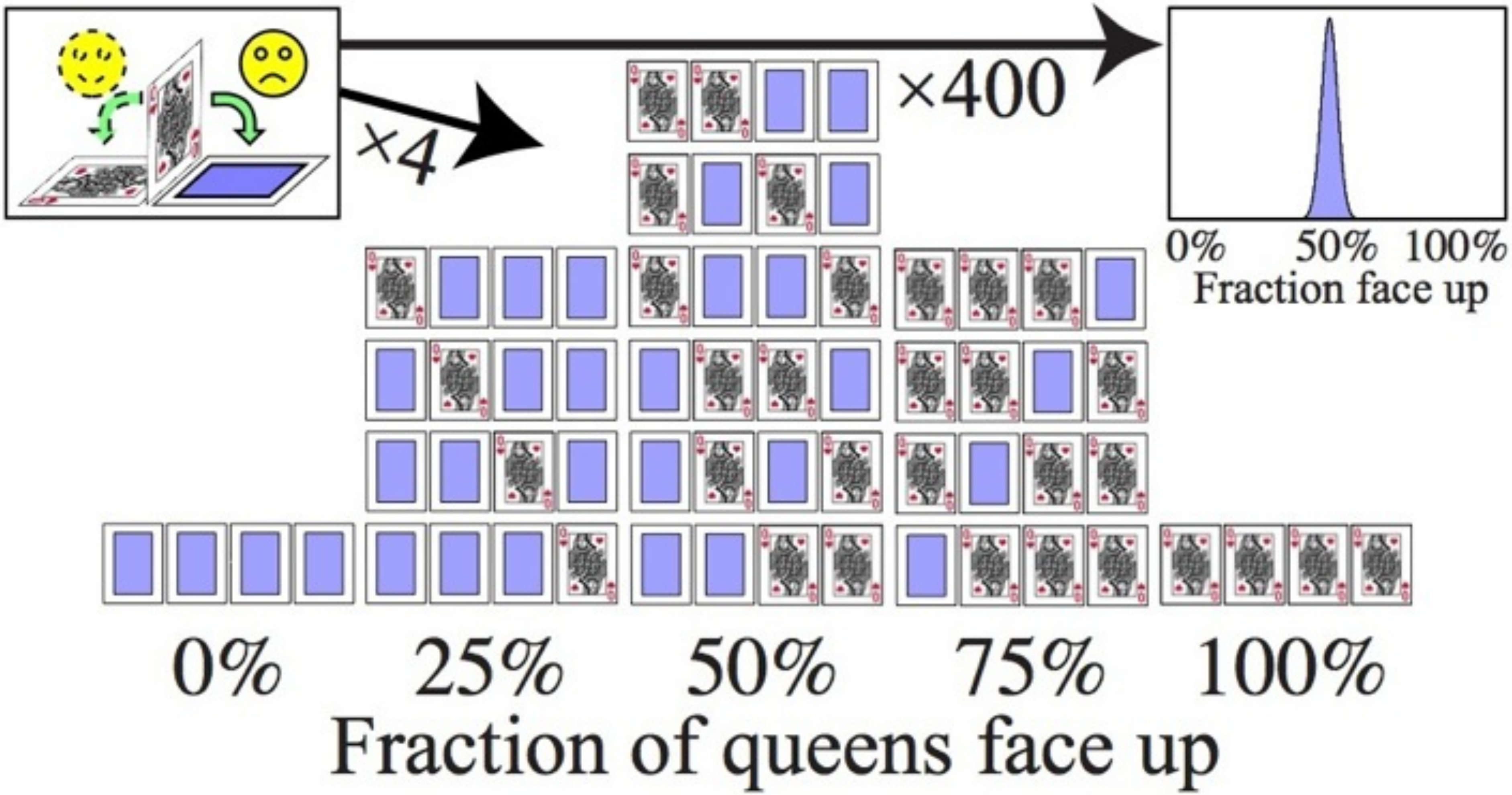
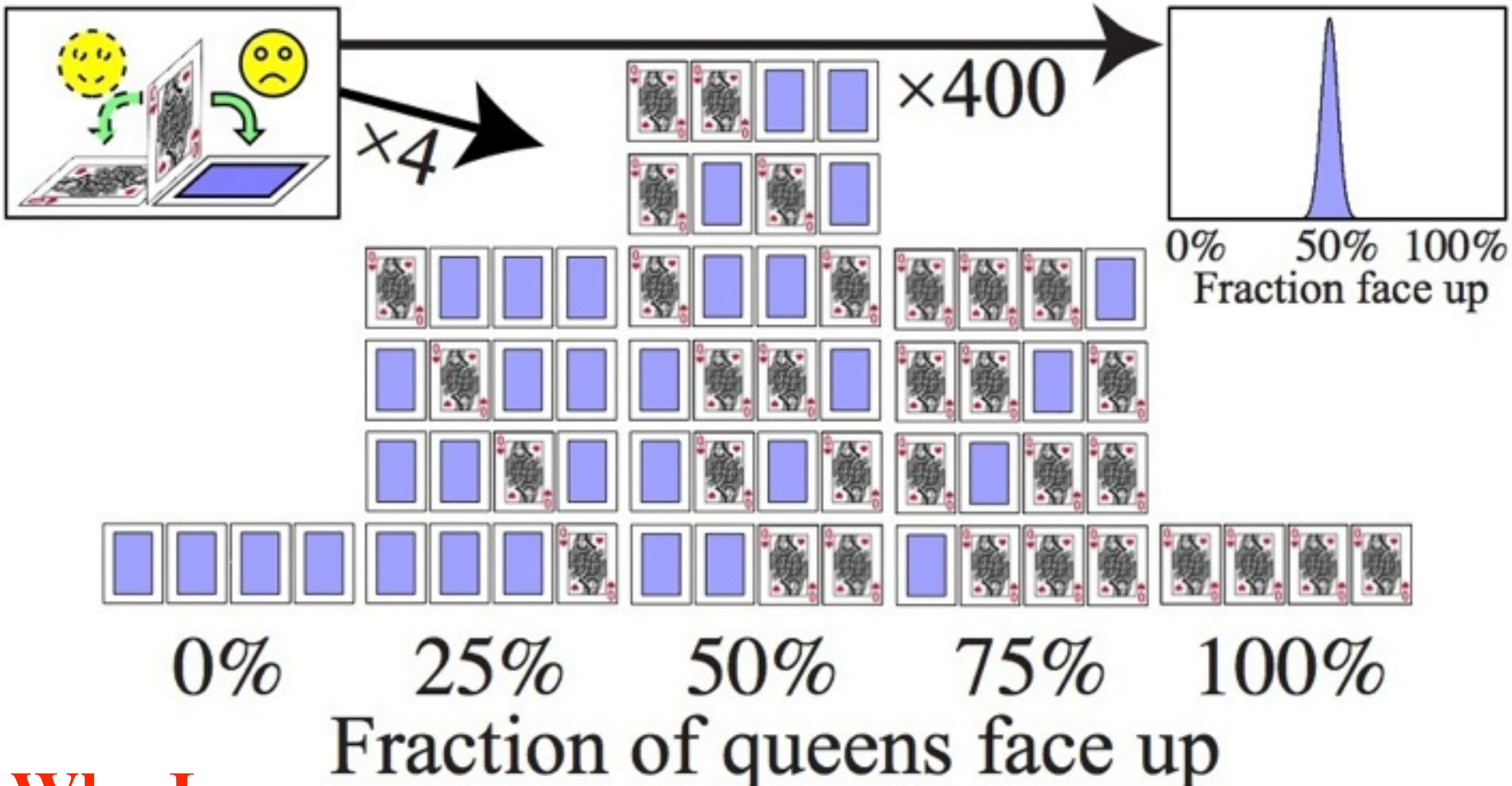

I hope I win...


×4
×400
0%
50%
100%
Fraction face up
0%
25%
50%
75%
100%
Fraction of queens face up



Why I care → ***But are some worlds more equal than others?***

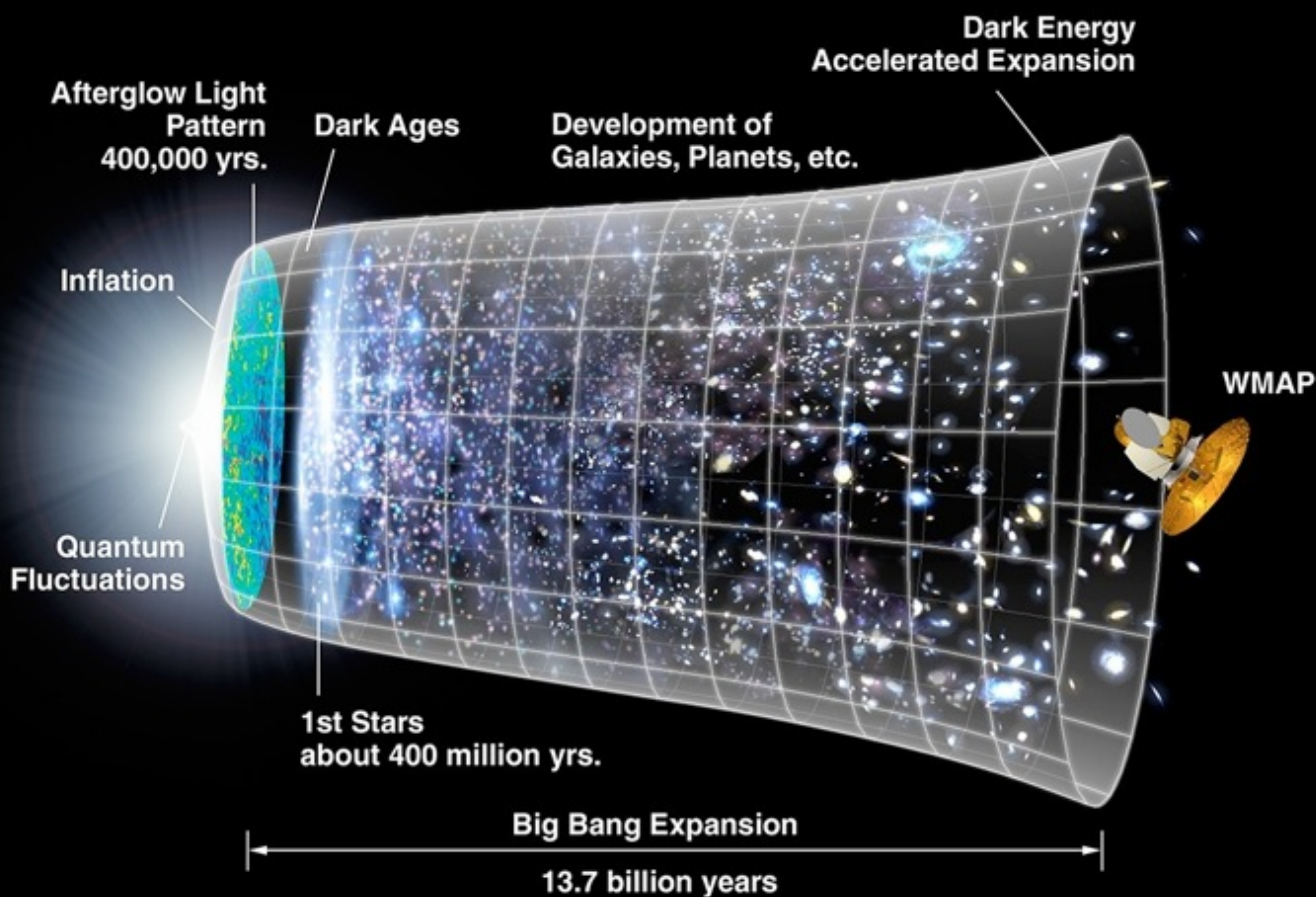
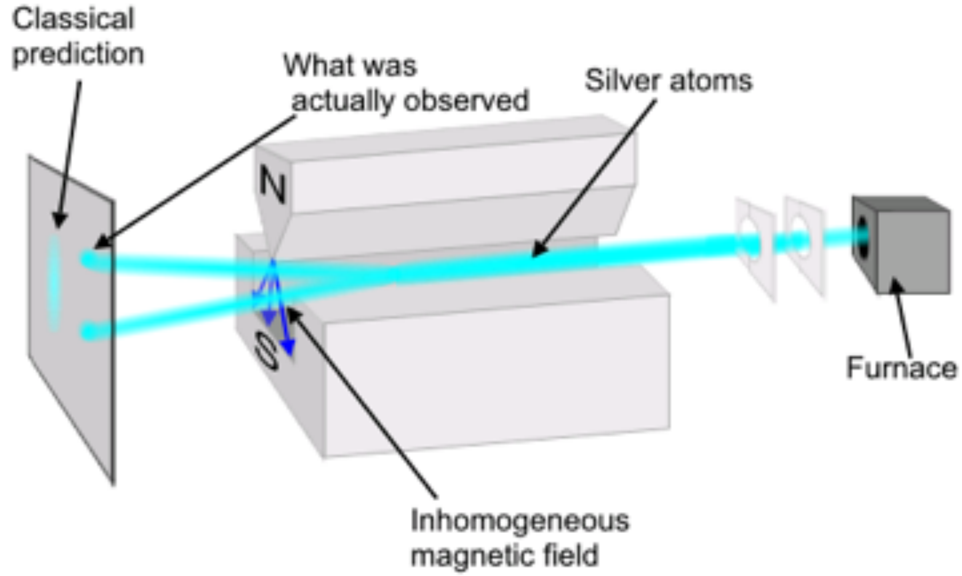


Figure from WMAP team

Eternal inflation: “Our space isn’t just huge, it’s infinite.”

$$|\psi\rangle = (\alpha|\downarrow\rangle + \beta|\uparrow\rangle)$$

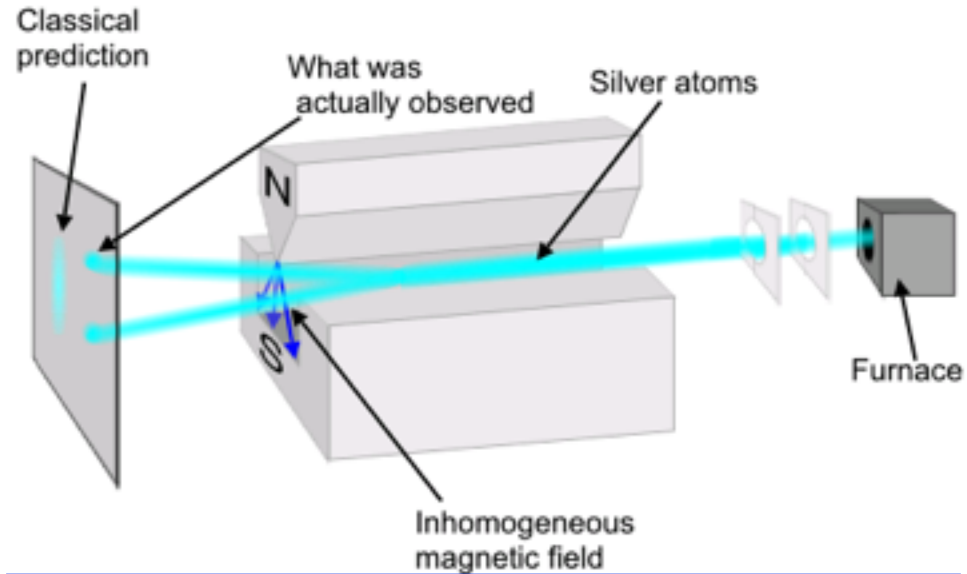


Born and his rule:

Probability of observing spin up is $p = |\beta|^2$



$$|\psi\rangle = (\alpha|\downarrow\rangle + \beta|\uparrow\rangle) \otimes (\alpha|\downarrow\rangle + \beta|\uparrow\rangle) \otimes (\alpha|\downarrow\rangle + \beta|\uparrow\rangle)$$
$$= \alpha^3|\downarrow\downarrow\downarrow\rangle + \alpha^2\beta|\downarrow\downarrow\uparrow\rangle + \ldots + \beta^3|\uparrow\uparrow\uparrow\rangle.$$



$$\begin{pmatrix} \langle\downarrow\downarrow\downarrow|\psi\rangle \\ \langle\downarrow\downarrow\uparrow|\psi\rangle \\ \langle\downarrow\uparrow\downarrow|\psi\rangle \\ \langle\uparrow\downarrow\downarrow|\psi\rangle \\ \langle\uparrow\uparrow\uparrow|\psi\rangle \\ \langle\uparrow\uparrow\downarrow|\psi\rangle \\ \langle\uparrow\downarrow\uparrow|\psi\rangle \\ \langle\uparrow\uparrow\uparrow|\psi\rangle \end{pmatrix} = \begin{pmatrix} \alpha^3 \\ \alpha^2\beta \\ \alpha^2\beta \\ \alpha^2\beta \\ \alpha\beta^2 \\ \alpha\beta^2 \\ \alpha\beta^2 \\ \beta^3 \end{pmatrix}$$

Don Page, arXiv:0903:4888:
"This isn't the square modulus of a quantum amplitude"

$$P_\uparrow = \sum_{n=0}^{N} \binom{N}{n} (\beta^*\beta)^n (\alpha^*\alpha)^{(N-n)} \frac{n}{N} = \beta^*\beta = p$$

Quantum probability

Classical probability

$$\begin{aligned}|\psi\rangle &= (\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\otimes(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\otimes(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\\ &= \alpha^3|\downarrow\downarrow\downarrow\rangle + \alpha^2\beta|\downarrow\downarrow\uparrow\rangle + \ldots + \beta^3|\uparrow\uparrow\uparrow\rangle.\end{aligned}$$

$$\begin{pmatrix}\langle\downarrow\downarrow\downarrow|\psi\rangle\\ \langle\downarrow\downarrow\uparrow|\psi\rangle\\ \langle\downarrow\uparrow\downarrow|\psi\rangle\\ \langle\uparrow\downarrow\downarrow|\psi\rangle\\ \langle\uparrow\uparrow\uparrow|\psi\rangle\\ \langle\uparrow\uparrow\downarrow|\psi\rangle\\ \langle\uparrow\downarrow\uparrow|\psi\rangle\\ \langle\uparrow\uparrow\uparrow|\psi\rangle\end{pmatrix} = \begin{pmatrix}\alpha^3\\ \alpha^2\beta\\ \alpha^2\beta\\ \alpha^2\beta\\ \alpha\beta^2\\ \alpha\beta^2\\ \alpha\beta^2\\ \beta^3\end{pmatrix}$$

Frequency operator:

$$\hat{F} = \frac{1}{3}\begin{pmatrix}0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&2&0&0&0\\0&0&0&0&0&2&0&0\\0&0&0&0&0&0&2&0\\0&0&0&0&0&0&0&3\end{pmatrix}$$

(Hartle, Finkelstein, Farhi, Goldstone, Gutman, Srednicki)

Confusion operator:

$$\widehat{\frownie} = \begin{pmatrix}1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix} = \theta\left(|\hat{F}-p|-\epsilon\right)$$

Theorem:

$$\begin{aligned}\|\widehat{\frownie}|\psi\rangle\|^2 &= \langle\psi|\widehat{\frownie}|\psi\rangle =\\ &= \sum_{n=0}^{N}\binom{N}{n}(1-p)^n p^{N-n}\theta\left(\left|\frac{n}{N}-p\right|-\epsilon\right)\\ &= \sum_{|n-Np|>N\epsilon}\binom{N}{n}(1-p)^n p^{N-n}\\ &\le 2e^{-2\epsilon^2 N}, \qquad (8)\end{aligned}$$

Use Hoeffding's inequality here

$$|\psi\rangle = (\alpha|\downarrow\rangle + \beta|\uparrow\rangle) \otimes (\alpha|\downarrow\rangle + \beta|\uparrow\rangle) \otimes (\alpha|\downarrow\rangle + \beta|\uparrow\rangle)$$
$$= \alpha^3|\downarrow\downarrow\downarrow\rangle + \alpha^2\beta|\downarrow\downarrow\uparrow\rangle + ... + \beta^3|\uparrow\uparrow\uparrow\rangle.$$

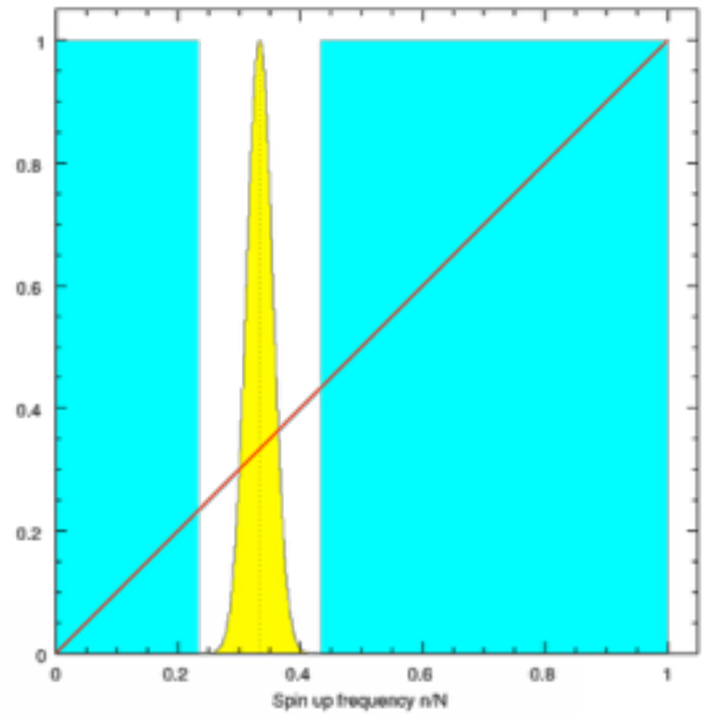
$$\begin{pmatrix} \langle\downarrow\downarrow\downarrow|\psi\rangle \\ \langle\downarrow\downarrow\uparrow|\psi\rangle \\ \langle\downarrow\uparrow\downarrow|\psi\rangle \\ \langle\uparrow\downarrow\downarrow|\psi\rangle \\ \langle\uparrow\uparrow\uparrow|\psi\rangle \\ \langle\uparrow\uparrow\downarrow|\psi\rangle \\ \langle\uparrow\downarrow\uparrow|\psi\rangle \\ \langle\uparrow\uparrow\uparrow|\psi\rangle \end{pmatrix} = \begin{pmatrix} \alpha^3 \\ \alpha^2\beta \\ \alpha^2\beta \\ \alpha^2\beta \\ \alpha\beta^2 \\ \alpha\beta^2 \\ \alpha\beta^2 \\ \beta^3 \end{pmatrix}$$

Frequency operator:

$$\widehat{F} = \frac{1}{3}\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&2&0&0&0 \\ 0&0&0&0&0&2&0&0 \\ 0&0&0&0&0&0&2&0 \\ 0&0&0&0&0&0&0&3 \end{pmatrix}$$

Confusion operator:

$$\widehat{☹} = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$



Theorem:

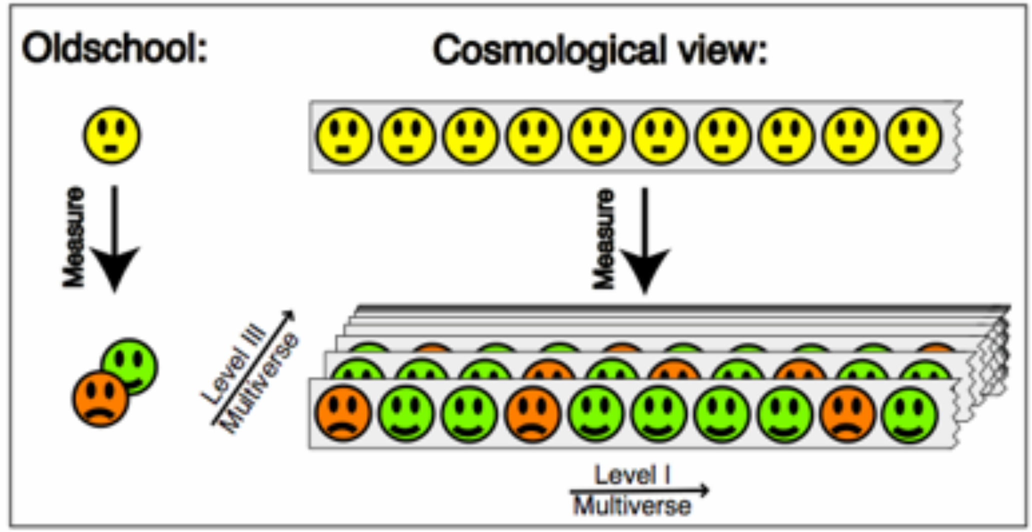
$$\|\widehat{☹}|\psi\rangle\|^2 = \langle\psi|\widehat{☹}|\psi\rangle =$$
$$= \sum_{n=0}^{N}\binom{N}{n}(1-p)^n p^{N-n}\theta\left(\left|\frac{n}{N}-p\right|-\epsilon\right)$$
$$= \sum_{|n-Np|>N\epsilon}\binom{N}{n}(1-p)^n p^{N-n}$$
$$\leq 2e^{-2\epsilon^2 N}, \quad (8)$$

Use Hoeffding's inequality here

Theorem:

When performing a quantum meaurement in an infinite uniform space and neglecting a wavefunction component of Hilbert space measure zero, one obtains a superposition of indistinguishable states, and each of which describes a spatial ensemble of outcomes with frequencies matching the Born rule.

EVERETT (NO WAVEFUNCTION COLLAPSE)



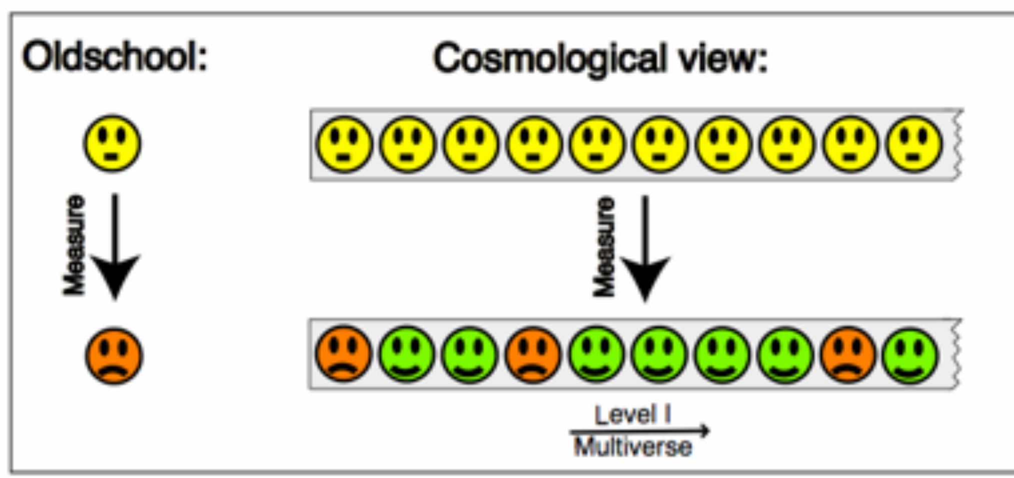
Proof:

Define confusion operator as in arXiv:1008:1066, use Hoeffding's inequality to show that

$$\|\widehat{☺}|\psi\rangle\|^2 \leq 2e^{-2\epsilon^2 N}$$

(that the total Hilbert space norm for states not matching the Born rule drops exponentially with spatial volume).

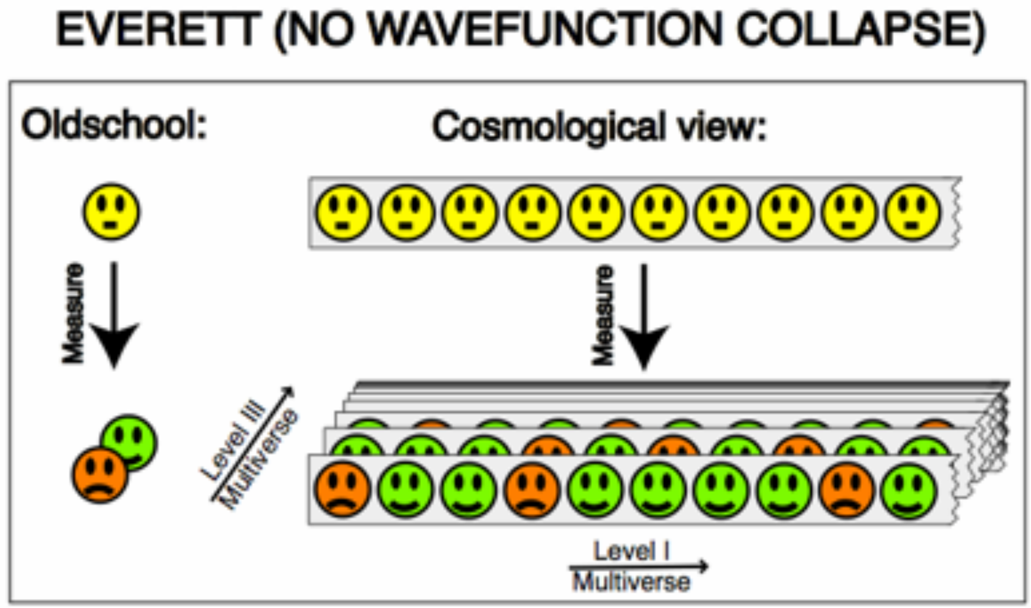
COPENHAGEN (WAVEFUNCTION COLLAPSES)

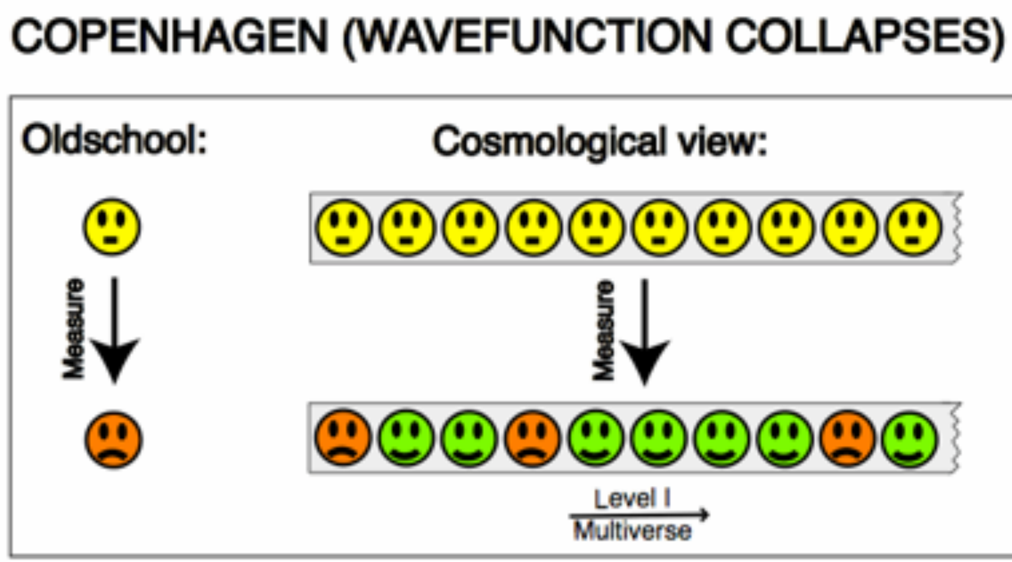


In the cosmological picture:

- You end up with parallel words with both outcomes even in the Copenhagen interpretation, which aimed to get rid of parallel universes.
- In the Everett interpretation, you can safely ignore the Level III (quantum) parallel universes, because they are all indistinguishable.
- Regardless of whether Everett or Copehagen is right, 2/3 of the planets have spin up (it doesn't matter whether the wavefunction collapses)
- Our cosmological interpretation brings Everett's many worlds home to our good old three-dimensional space
- The quantum uncertainty about whether you measure spin up or down reflects your uncertainty about which planet you're on
- Infinite space perhaps not necessary:

$$\|\widehat{\frown}|\psi\rangle\|^2 \leq 2e^{-2\epsilon^2 N} = 10^{-10^{800}}, \text{ say}$$





The cosmological interpretation of quantum mechanics:

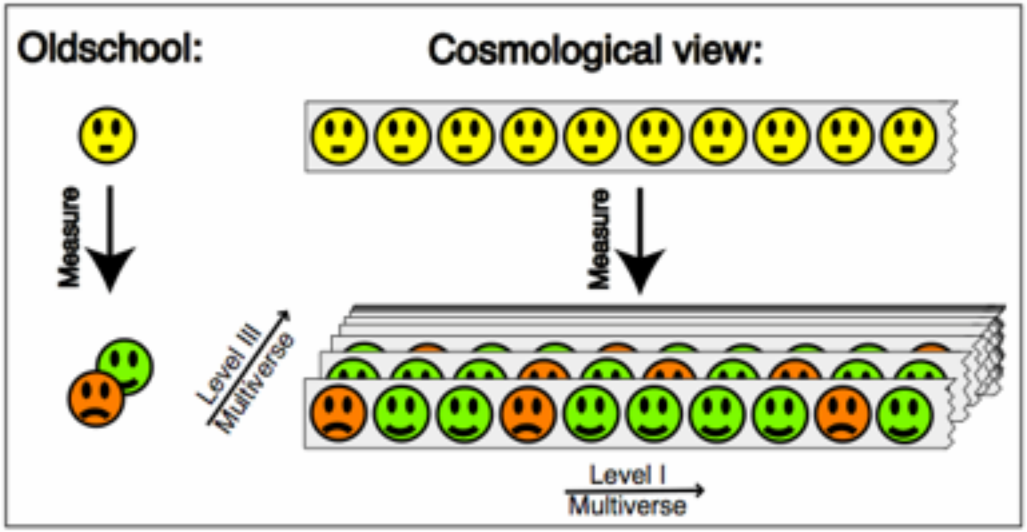
What does ψ describe?

The actual spatial collection of identical quantum systems

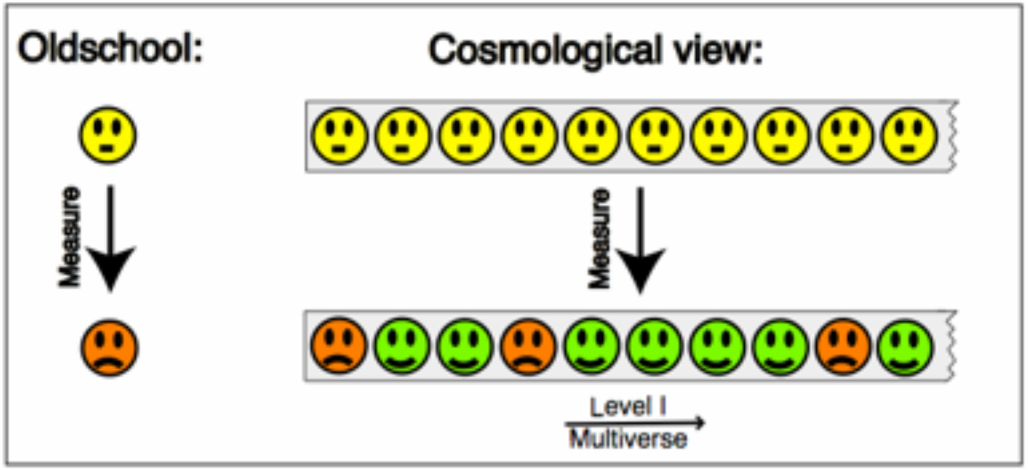
What's the cause of quantum uncertainty?

The observer's inability to self-locate in this collection

EVERETT (NO WAVEFUNCTION COLLAPSE)



COPENHAGEN (WAVEFUNCTION COLLAPSES)



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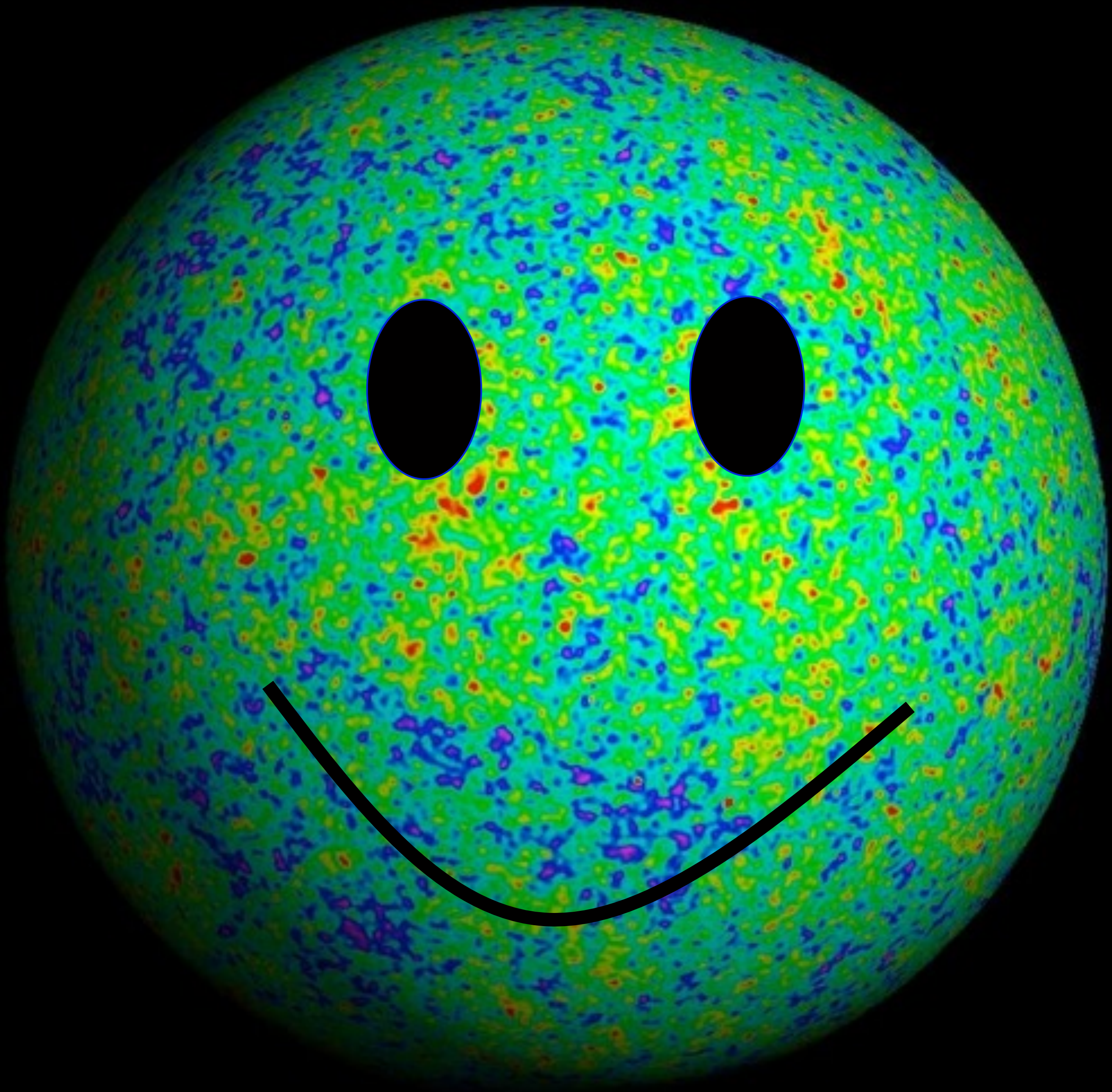
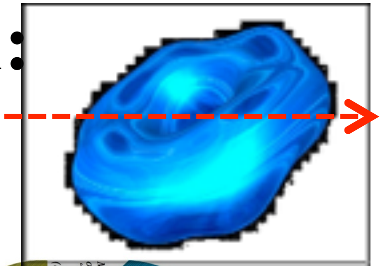
November 22, 2013

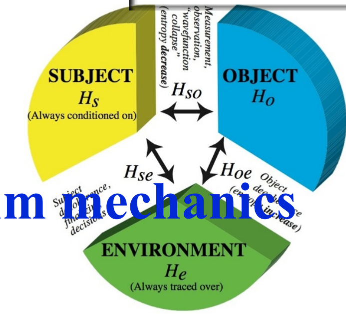


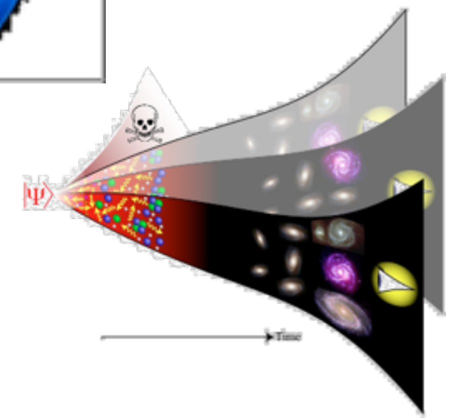
Today's menu:

1) How decoherence makes spacetime so classical



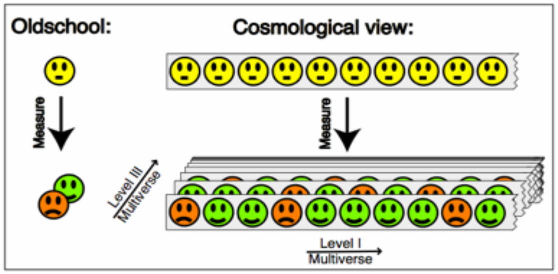
2) Decoherence, observation & the 2nd law(s)





3) The cosmological interpretation of quantum mechanics

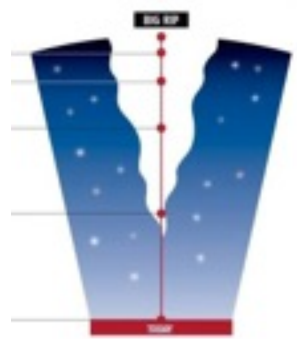
4) The big snap



Based on:
MT: arXiv:1108.3080

In related spirit:
Susskind/Bousso
holographic principle

BIG CHILL | BIG CRUNCH | BIG RIP | BIG SNAP | DEATH BUBBLES



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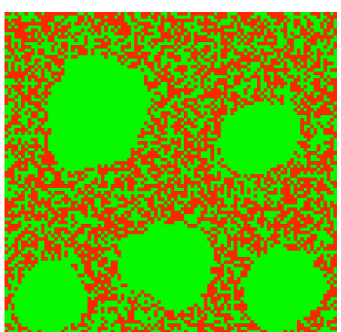
Summary:

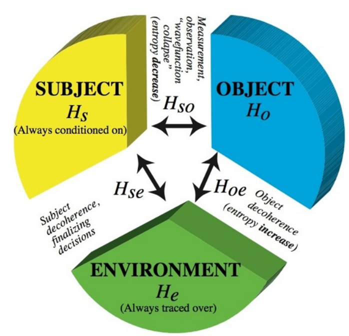
21 cm cosmology is lots of fun:

- Potentially much cheaper than we thought
- Potentially even better precision than CMB to help Alex
  (50-fold improvement on Ω_{tot}, can detect running of tilt, etc.)

Cosmology helps us understand quantum mechanics:

- **We can blame Born rule on Alex Vilenkin**
- **We can blame low entropy on Alex Vilenkin:** inflation+observation lowers entropy exponentially.
-**2nd law generalized: "***The system's entropy can't decrease unless it interacts with the observer, and it can't increase until it interacts with the environment."*
- **Our future:** the Big Snap? We're missing something!





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