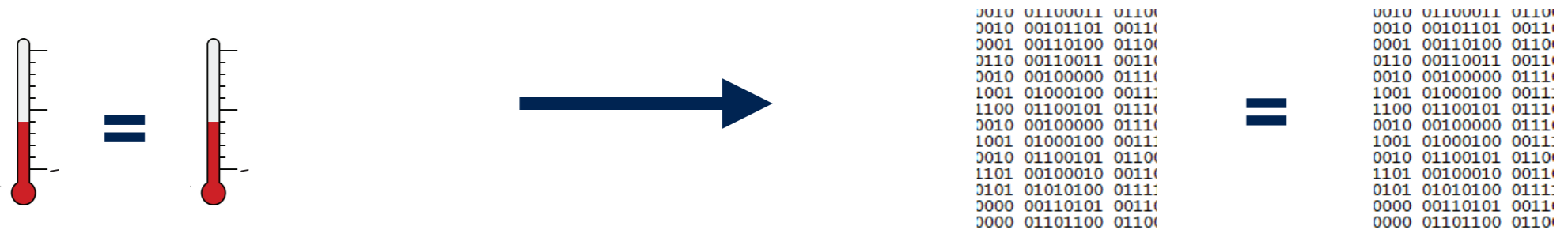


# Death and Resurrection of the Zero-th principle of thermodynamics

*Carlo Rovelli,*

*with Hal Haggard. Also: Eugenio Bianchi, Aldo Riello, Goffredo Chirco, FQXi postdoc*

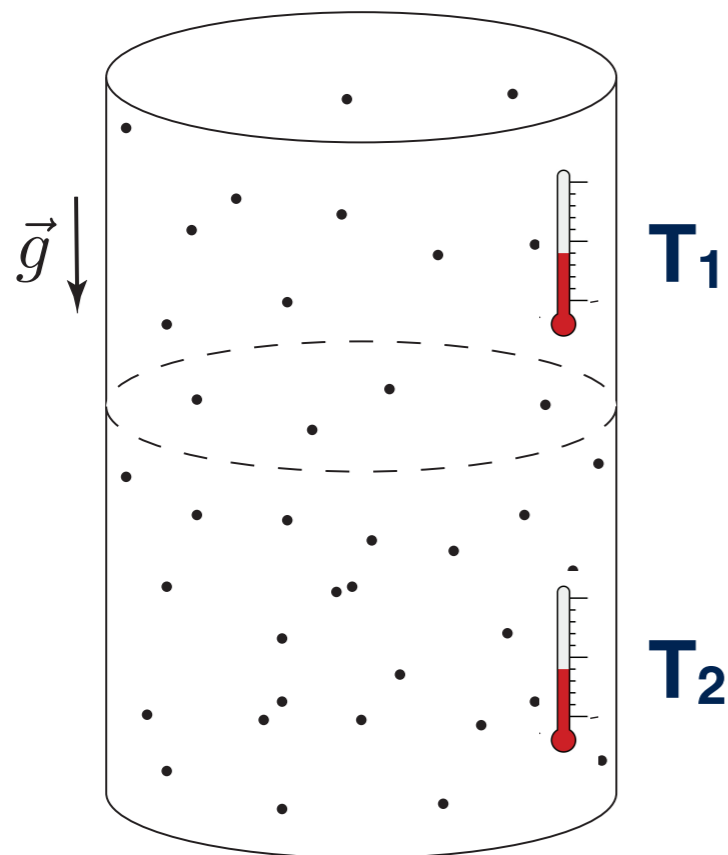


From equal temperature to equal information

0th : Temperature is constant at equilibrium

1st : Energy is constant in time

2nd : Entropy grows in time



$$T_1 = T_2$$

***False !***

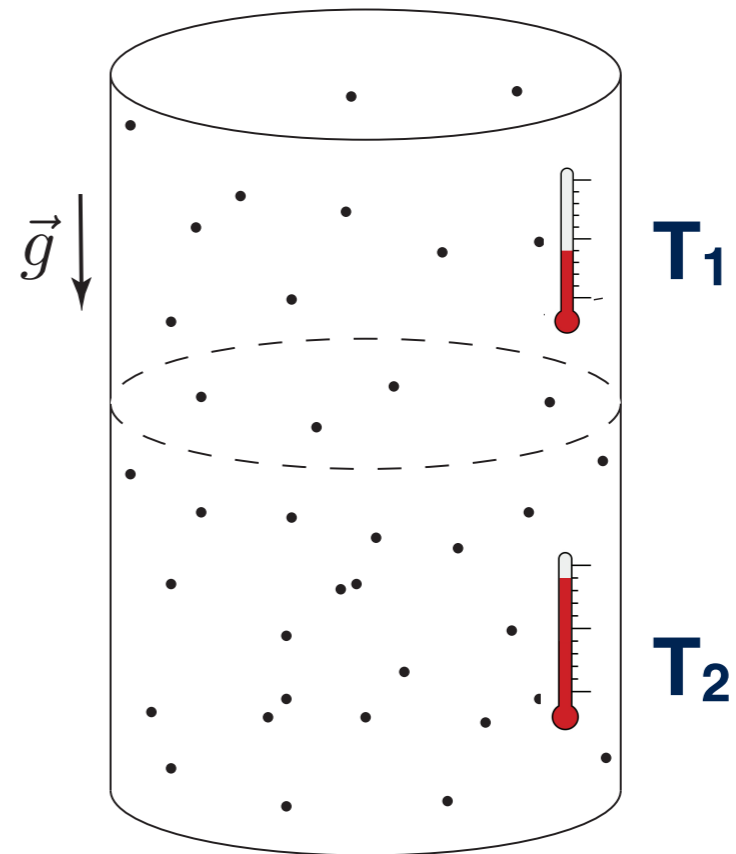
0th : Temperature is **not** constant at equilibrium in a gravitational field

$$T_2 = T_1 \left( 1 + \frac{gh}{c^2} \right)$$

Tolman-Ehrenfest effect (1930)

$$T|\xi| = \text{constant}$$

On Earth surface:  $\frac{\Delta T}{T} = 10^{-18} \text{cm}^{-1}$



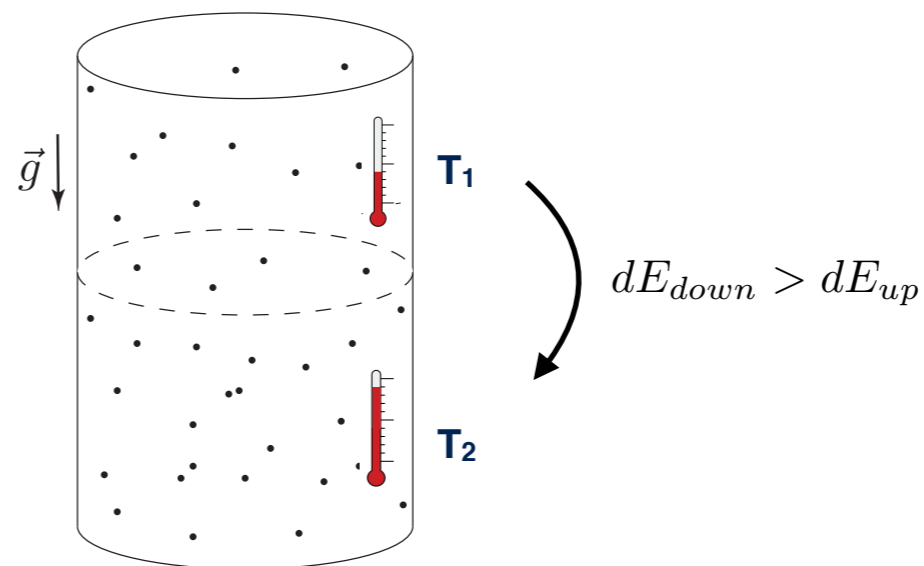
## How can this happen?

In micro-canonical:  $S(E) = k \ln N(E)$

$$\frac{1}{kT} = \frac{dS(E)}{dE}$$

Maximizing  $N = N_1 N_2$  under an energy transfer  $dE$  gives  $T_1 = T_2$ .

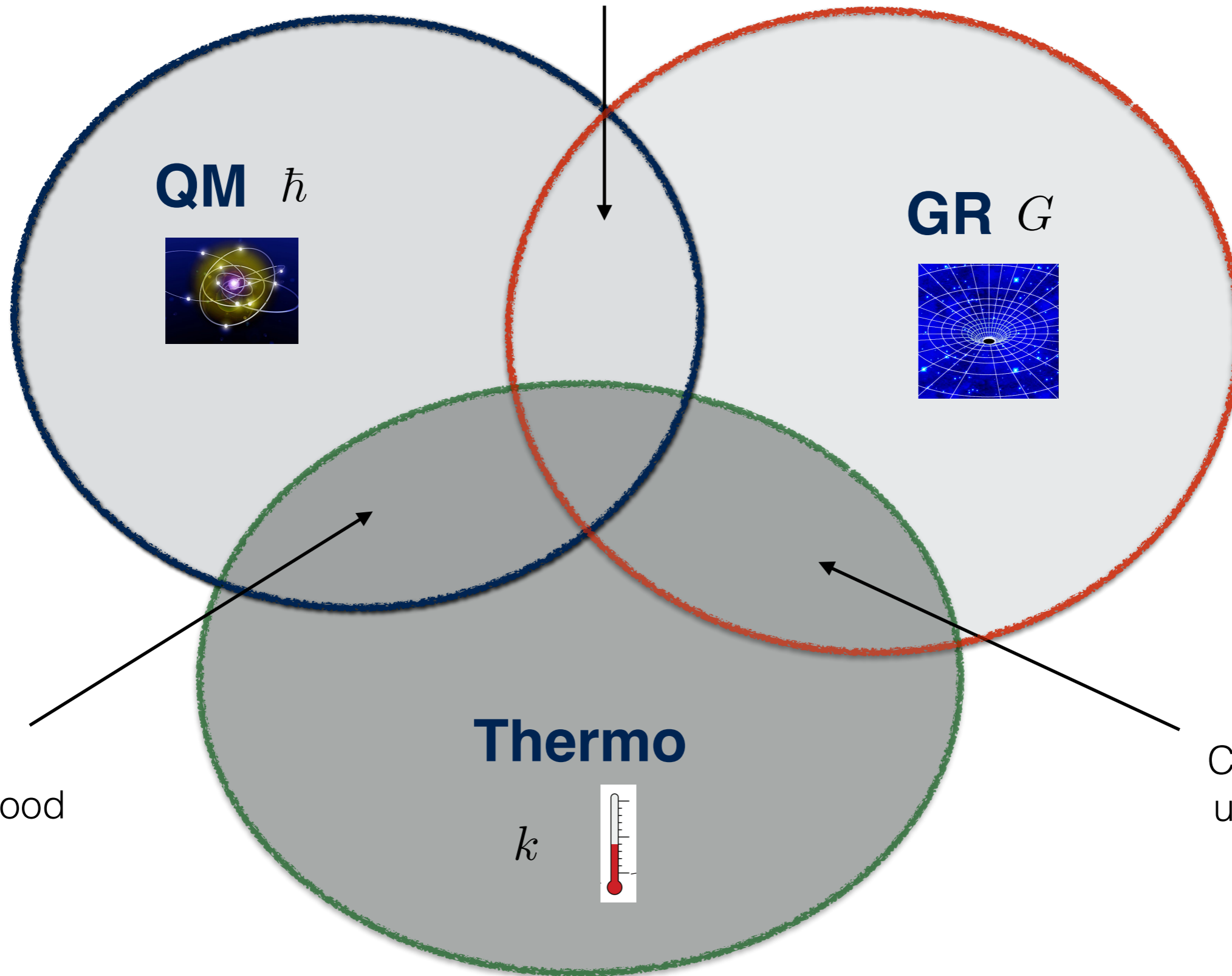
But conservation of Energy is tricky in GR:  $dE$  falls!



$T_1 = T_2$     *is dead !*

*What replaces it ?*

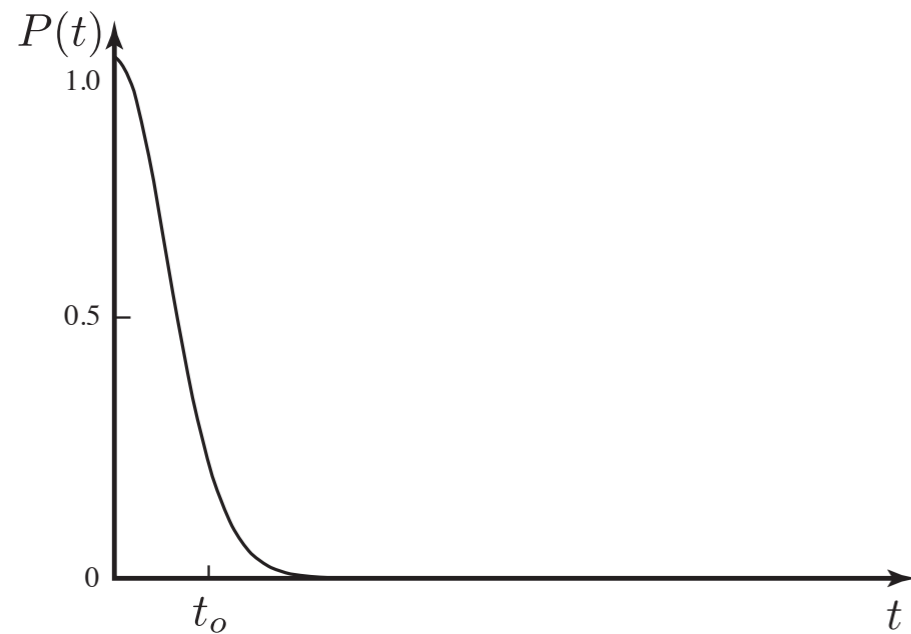
Better understood  
that commonly claimed



Well  
understood

Completely  
uncovered

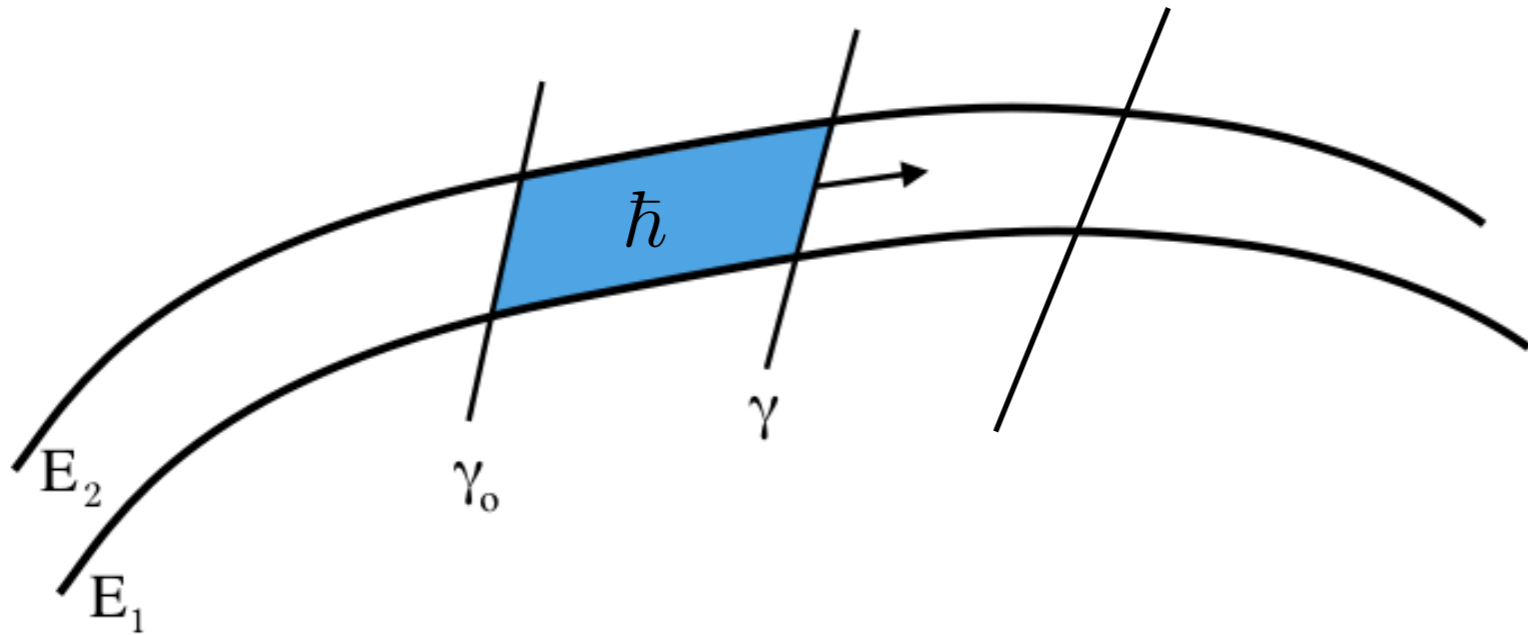
## Time needed by a state to move away from itself



$$\begin{aligned} P(t) &= |\langle \psi(0) | \psi(t) \rangle|^2 \\ &= |\langle \psi(0) | e^{-\frac{i}{\hbar} H t} | \psi(0) \rangle|^2 \end{aligned}$$

$$\frac{d^2 P}{dt^2} = \frac{1}{\hbar^2} (\langle H \rangle^2 - \langle H^2 \rangle) = -\frac{(\Delta E)^2}{\hbar^2}$$

$$t_0 = \frac{\hbar}{\Delta E}$$



$$\omega(X) = -dH$$

$$\begin{aligned} \frac{dV(t)}{dt} &= \frac{d}{dt} \int_{R(t)} \omega = \int_{\gamma} \omega(X) \\ &= - \int_{\gamma} dH = E_2 - E_1 \equiv \Delta E \end{aligned}$$

$$t_o = \frac{\hbar}{\Delta E}$$

Specialize to thermal states. Then  $\Delta E \sim kT$  and therefore

$$t_o = \frac{\hbar}{kT}$$

This is a **universal time step**.

At temperature  $T$ , any system moves from a state to an orthogonal state in a time  $t_o$ !

Introduce a “thermal time”

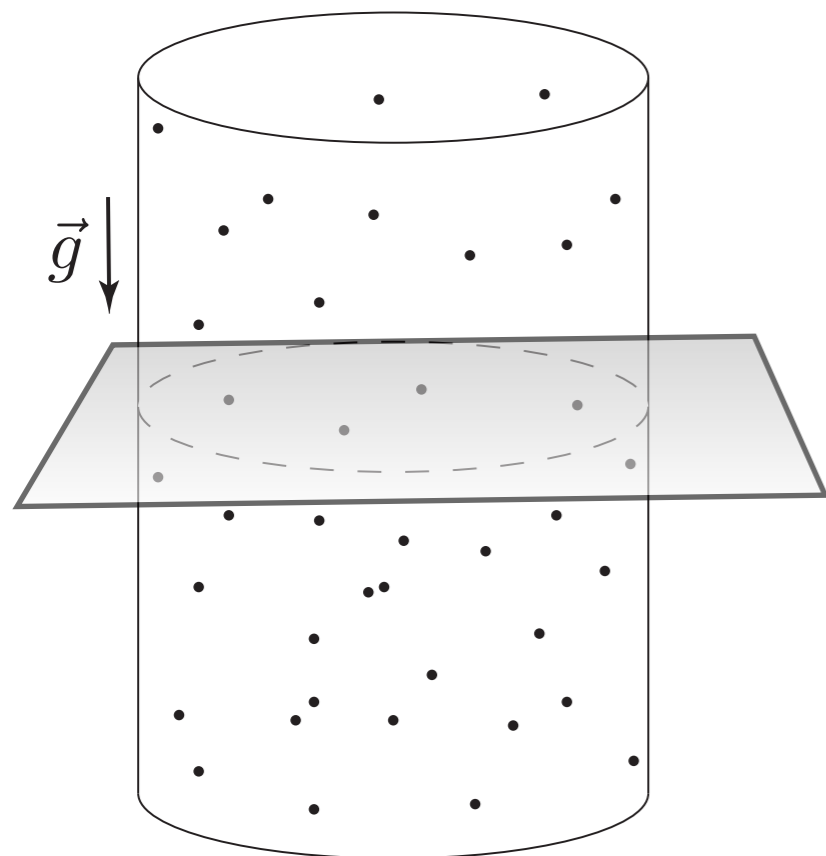
$$\tau = \frac{t}{t_o} = \frac{kT}{\hbar} t$$

This is precisely the thermal time (Connes-CR): the parameter of the Tomita flow in the algebra of the observables in a generally covariant QFT.

Physical meaning of Temperature (  $\hbar = k = 1$  ):

**Temperature is number of states transited per unit of time**

(room temperature  $\sim 3 \times 10^{12} s^{-1}$  ).



$N_1$



$N_2$

Amount of information about 1 that 2 has access to:  $N_1$

$$I_1 = \ln N_1$$

Amount of information about 2 that 1 has access to:  $N_2$

$$I_2 = \ln N_2$$

**Postulate: equilibrium is**  $\delta I = I_2 - I_1 = 0$

$$\delta I = 0$$

$$\tau = \frac{t}{t_0} = \frac{kT}{\hbar} t$$

Non relativistic physics

$$t_1 = t_2$$

$$T_1 = T_2$$

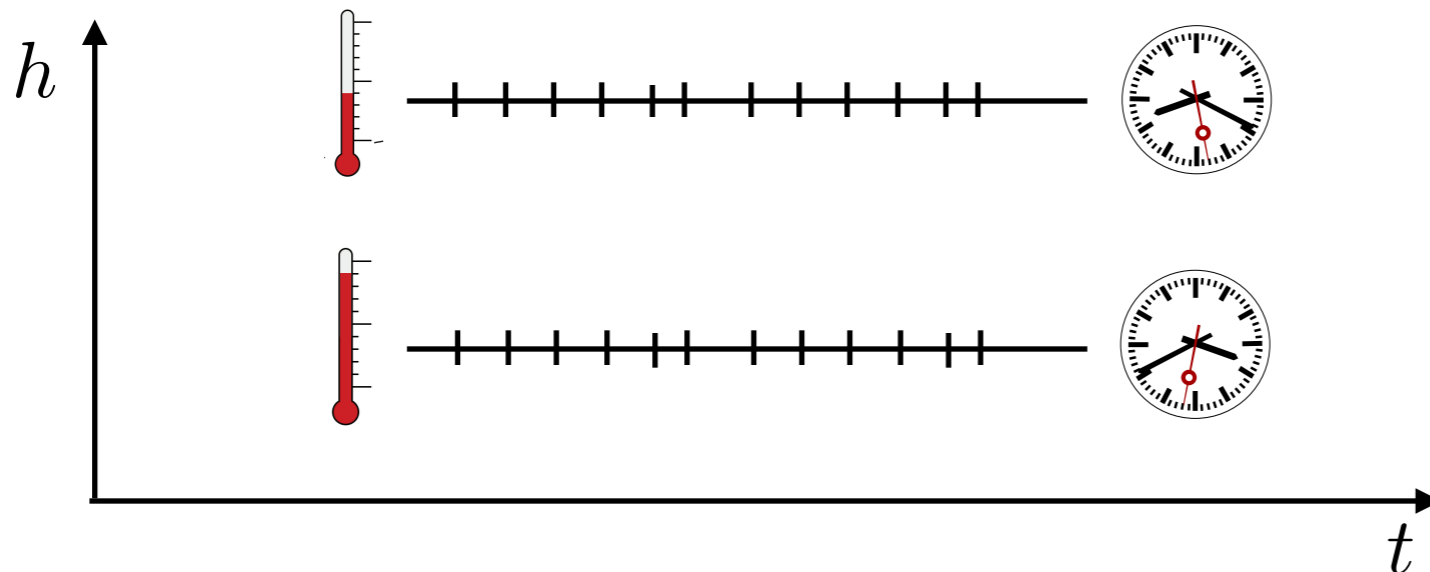
On curved spacetime

$$d\tau = \frac{kT}{\hbar} ds$$

Proper time along an orbit:

$$ds = |\xi| dt$$

$$T|\xi| = \text{constant}$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(1 - \frac{gh}{c^2}\right) dt^2 - d\vec{x}^2$$

- *Resurrected 0th principle of thermodynamics:*

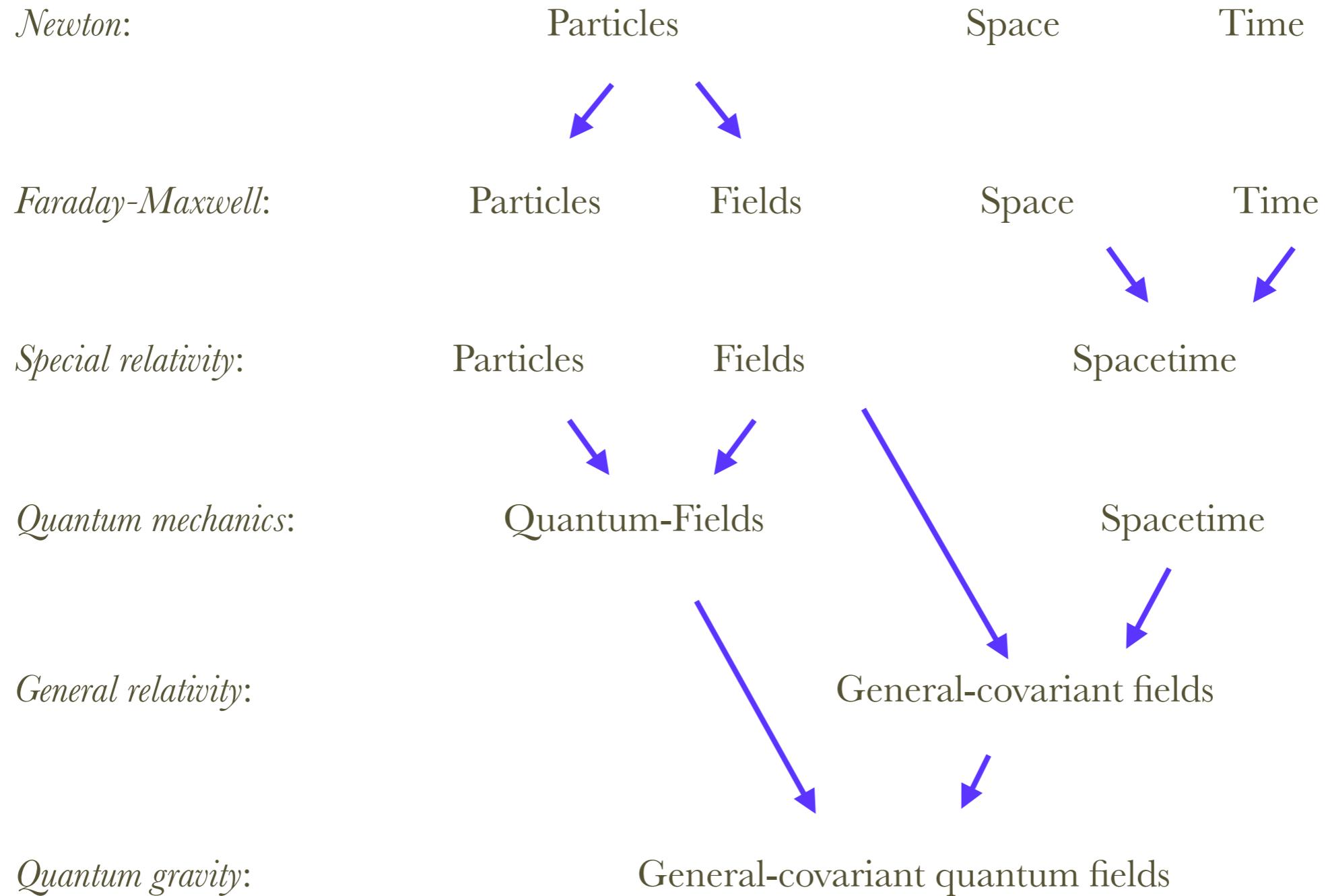
$$I_1 = I_2$$

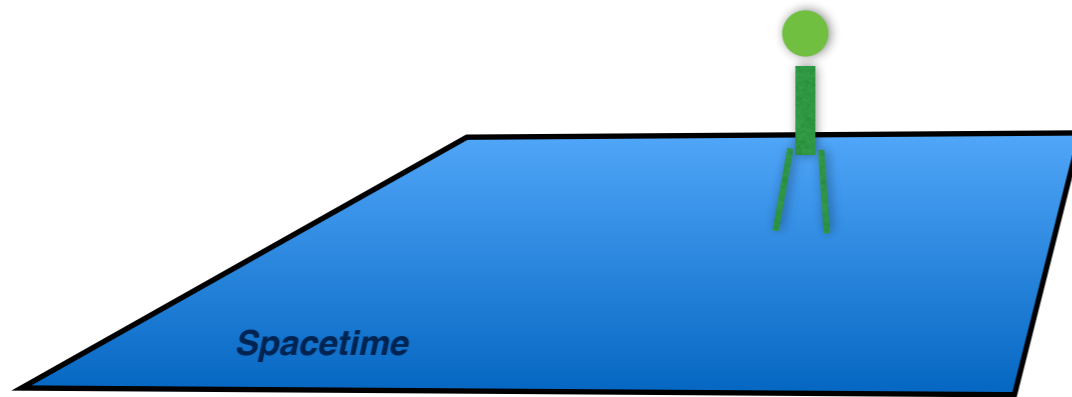
Two histories are in equilibrium if the net **information** flow between them is zero: if they transit the same number of states during the interaction period.

- *Temperature is the rate at which a system moves from state to state.*

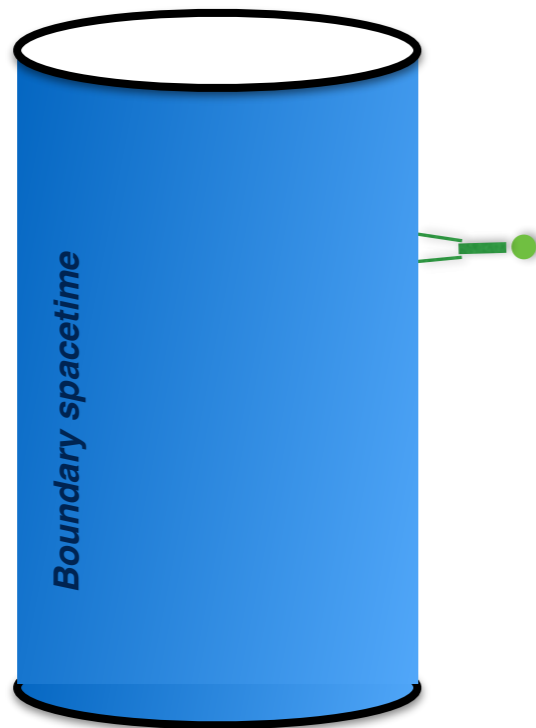
- *Thermal time:  $\tau = \frac{kT}{\hbar} t$  dimensionless time, in unit of universal step.*

# ■ The components of the world in fundamental physics





Conventional quantum field theorist



Post-Maldacena string theorist



Genuine quantum-gravity physicist

Quantum theory:

**information** exchanged  
in interaction between  
two systems

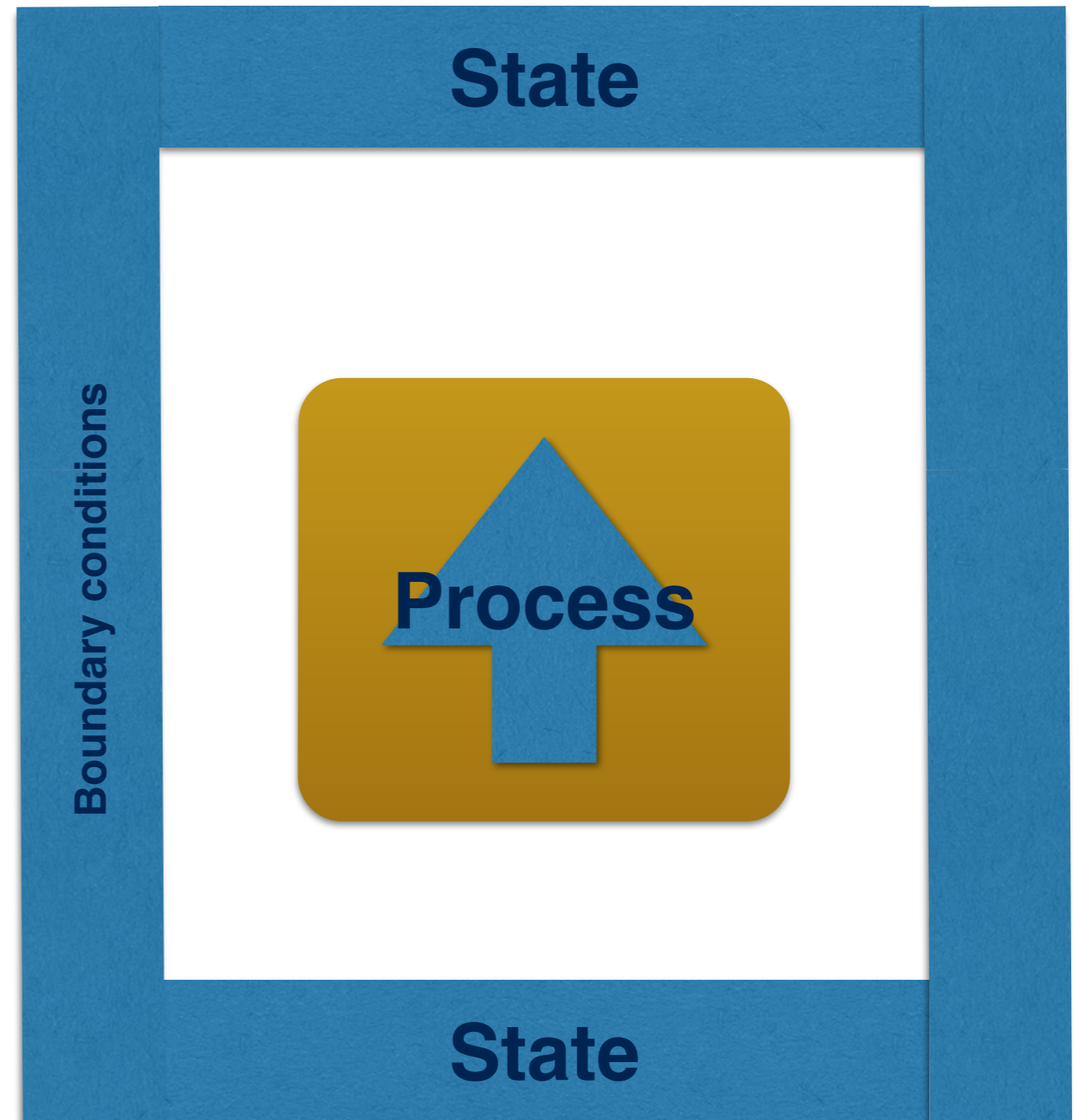
1. There is a finite amount of information in a physical system.
2. It is always possible to acquire new relevant information on a physical system.

State

Interaction

State

# Quantum field theory



Thermodynamics:  
**information** exchanged  
in interaction between  
two systems

**System**

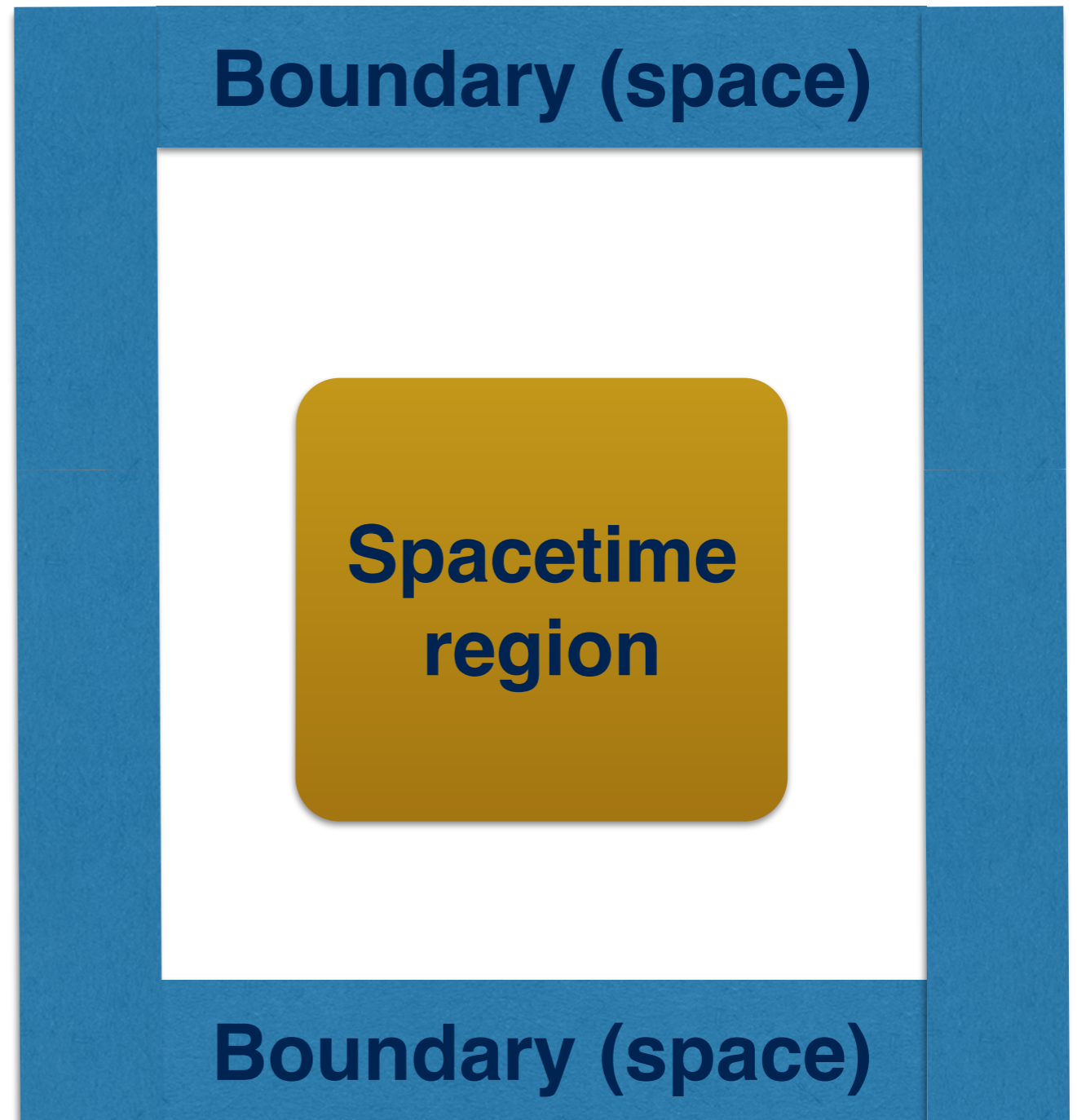


**Macro  
interaction**

**System**

General relativity:

**localization** defined relationally between physical entities



Quantum theory:

**Relations**  
between systems  
via **interactions**

General Relativity:

**Relations**  
between regions  
by **adjacency**

**Locality:**  
*Interactions are only  
among adjacent things*

System

Space region

Process

Spacetime region



**Locality**

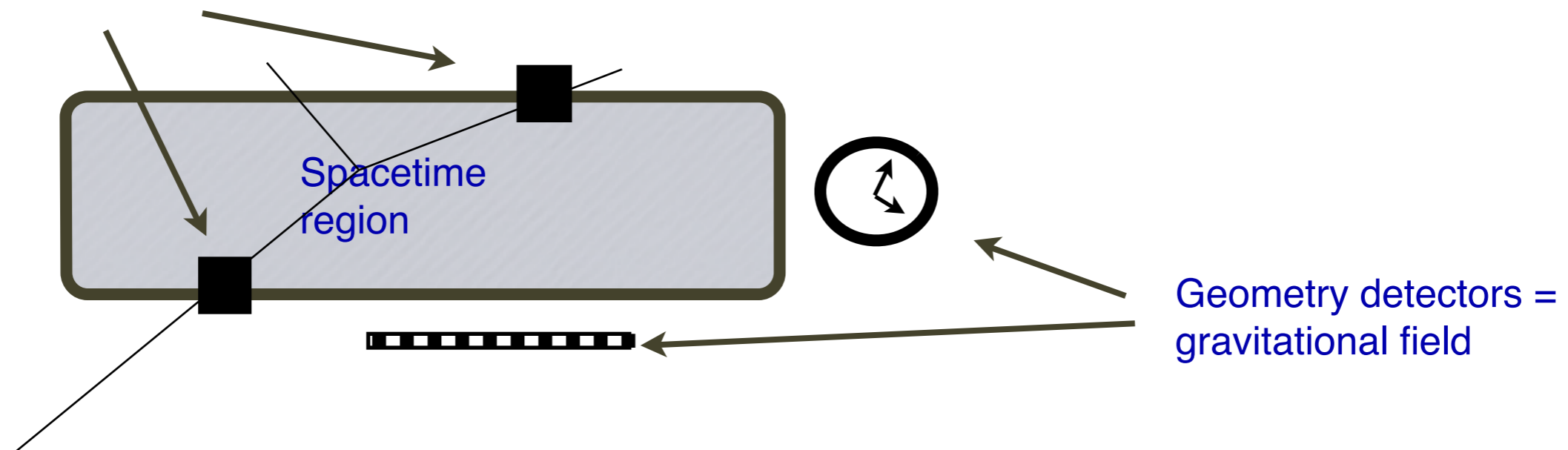


**Boundary (space)=states**

**Spacetime  
dynamics=  
Process**

- Boundary values of the gravitational field
- = geometry of box surface
- = distance and time separation of measurements

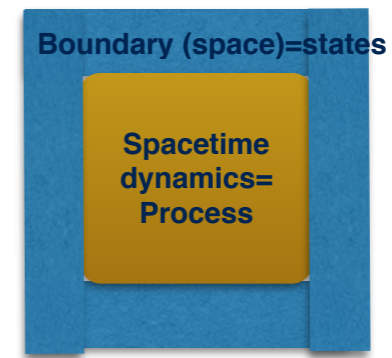
Particle detectors = fields



In GR, distance and time measurements are field measurements like others: they are part of the boundary data of the problem.

**Boundary (space)=states**

**Spacetime  
dynamics=  
Process**



# Loop Quantum Gravity

Kinematics

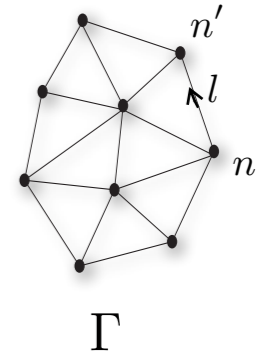


State space

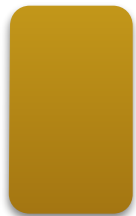
$$\mathcal{H}_\Gamma = L^2[SU(2)^L / SU(2)^N]$$

Operators:

$$\vec{L}_l = \{L_l^i\}, i = 1, 2, 3 \text{ where } L^i \psi(h) \equiv \left. \frac{d}{dt} \psi(h e^{t\tau_i}) \right|_{t=0}$$



Dynamics



Transition amplitudes

$$W_C(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

$$h_f = \prod_v h_{vf}$$

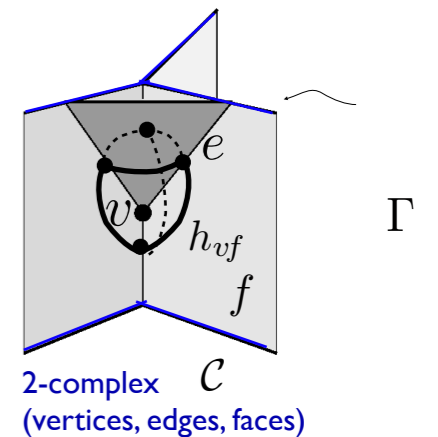
Vertex amplitude

$$A(h_f) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f + 1) \text{Tr}_j [h_f Y_\gamma^\dagger g_e g_{e'}^{-1} Y_\gamma]$$

Simplicity map

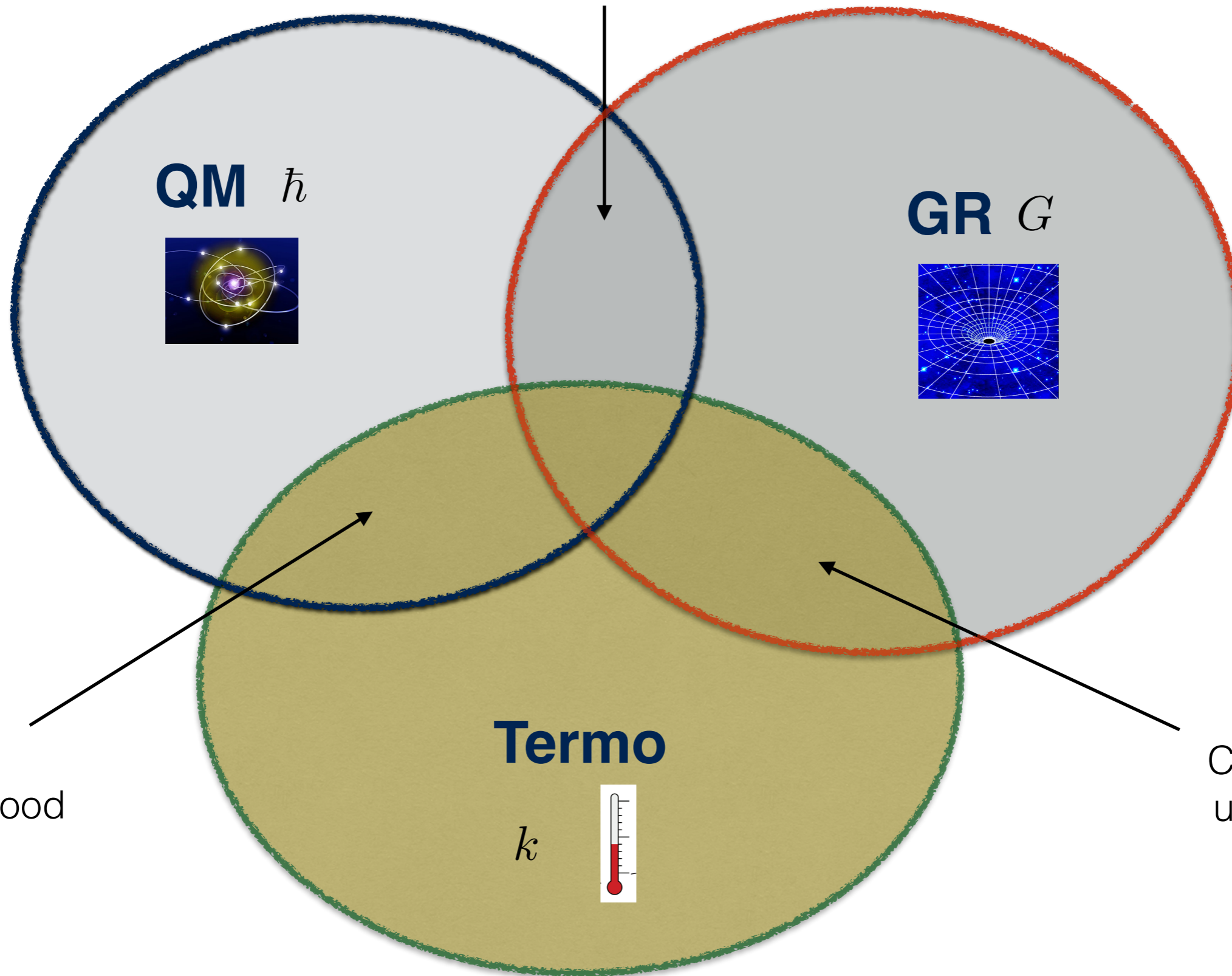
$$Y_\gamma : \mathcal{H}_j \rightarrow \mathcal{H}_{j,\gamma j}$$

$$|j; m\rangle \mapsto |j, \gamma(j+1); j, m\rangle$$



With a cosmological constant  $\Lambda > 0$ :  
Amplitude:  $SL(2,\mathbb{C}) \rightarrow SL(2,\mathbb{C})_q$  network evaluation.

Better understood  
that commonly claimed



Well  
understood

Completely  
uncovered