Infinite Phase Space and the Two-Headed Arrow of Time

Alan Guth
Massachusetts Institute of Technology
FQXi: The Physics of Information, January 7, 2013

Infinite Phase Space and the Two-Headed Arrow of Time

1) Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

Infinite Phase Space and the Two-Headed Arrow of Time

1) Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

YES: if there is an infinite available phase space.

Infinite Phase Space and the Two-Headed Arrow of Time

1) Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

   YES: if there is an infinite available phase space.

2) Can fine-tuning be explained in the context of Hamiltonian evolution?

Infinite Phase Space and the Two-Headed Arrow of Time

1) Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

   YES: if there is an infinite available phase space.

2) Can fine-tuning be explained in the context of Hamiltonian evolution?

   YES: if there is an infinite available phase space.

Infinite Phase Space and the Two-Headed Arrow of Time

1) Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

   YES: if there is an infinite available phase space.

2) Can fine-tuning be explained in the context of Hamiltonian evolution?

   YES: if there is an infinite available phase space.

3) If we assume that the universe has a finite available phase space, do we get into trouble?

Infinite Phase Space and the Two-Headed Arrow of Time

1) Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

   YES: if there is an infinite available phase space.

2) Can fine-tuning be explained in the context of Hamiltonian evolution?

   YES: if there is an infinite available phase space.

3) If we assume that the universe has a finite available phase space, do we get into trouble?

   YES: We would predict that the world we see should be described by state-counting statistics, and not by an historical picture.

- ☆ Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

- ☆ Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?
- ☆ It seems to be commonly believed that the answer is NO.

- ☆ Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?
- ☆ It seems to be commonly believed that the answer is NO.

> *No scenario based on CPT-invariant dynamical laws can explain the time asymmetry apart from postulating or explaining these special initial conditions, as Penrose has emphasized.*
>
> — Don Page (1983)

Proposal: Spontaneous Two-Headed Arrow of Time

Original source: Sean Carroll and Jennifer Chen, "Spontaneous inflation and the origin of the arrow of time," hep-th/0410270.

Recent related work: Bousso, Susskind, with collaborators.

Upcoming paper: Carroll, Chien-Yao Tseng, and me.

- ☆ **KEY IDEA:** If the maximum possible entropy is infinite, then any entropy is a low entropy. The entropy can increase from any given starting point!
- ☆ This talk will differ from previous work by giving a precise description of a simple toy model, rather dealing with the complexities of a "realistic" one.

Toy Model: A Gas Without a Box

- ☆ Purpose of this toy model: to show that it is possible for an arrow of time (monotonic change in coarse-grained entropy with time) to develop from time-symmetric laws of physics and time-symmetric initial conditions.

- ☆ The model: Consider a gas of N (large number) non-interacting particles, moving in empty space. Choose the initial conditions by making up a probability distribution for positions and velocities, and use a random number generator with these probabilities to choose the initial positions and velocities for the N particles.

- ☆ Insist that the probabilities be normalizable — probabilities must add up to one. This rules out ill-defined options, such as a uniform probability to be anywhere.

Why is a Uniform Probability Distribution on the Real Line Impossible?

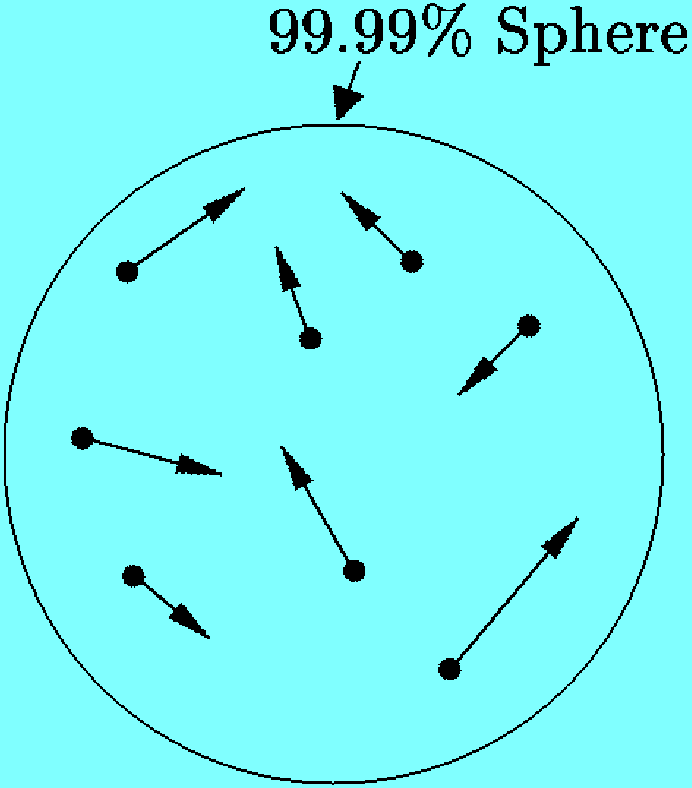
If a random number generator could generate real numbers with uniform probability, suppose I generated two real numbers in succession, A and B. Then

- $p(A, B) = p(B, A) = p(A)p(B)$.
- Once A is generated, what is the probability P that $|B| > |A|$?
  1) $P = \frac{1}{2}$, since order is irrelevant.
  2) $P = 1$, since only a finite range of numbers have magnitude $< A$, while an infinite range have magnitude larger.

— I learned this argument from Aron Wall.

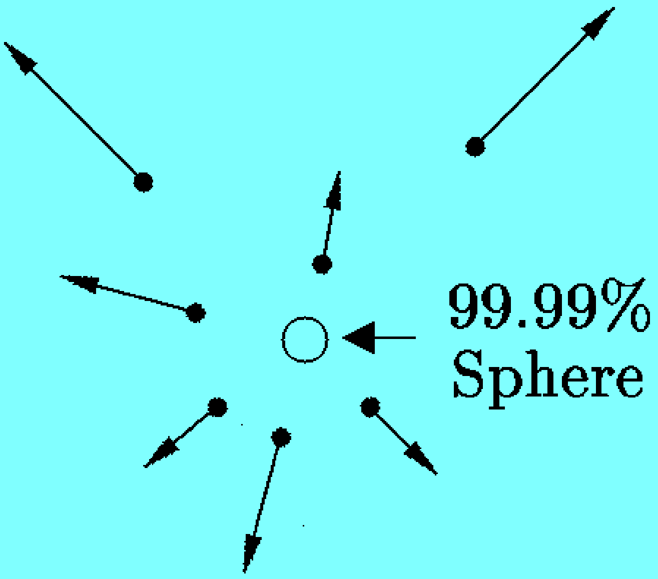
Toy Model: Initial Conditions

- ☆ Normalizability implies that the distribution of particles is localized — it must be possible to draw a sphere that is big enough so that that 99.99% (or any number you choose) of all particles are expected to be in the sphere.
- ☆ Initially some particles are moving in, others are moving out, entropy might be going up or down. For a while we do not expect an arrow of time.



Toy Model: Behavior at Late Times

- Let the system evolve a long time.
- Particles have fixed velocities, so after a long time they will have moved a large distance, far outside the 99.99% sphere.
- The picture will look like the diagram, with a visually clear arrow of time. Coarse-grained entropy will grow indefinitely as the gas spreads out through the infinite space.



- If we evolved the system backwards in time, it would behave the same way, but at large negative times the arrow of time would point the other way!

- Bottom line: for a finite amount of time near the starting point, there is no arrow of time. But for infinite periods of time in the future and in the past, the arrow of time — the monotonic change in coarse-grained entropy with time — is well-defined.

- The ever-expanding gas is a metaphor for (eternal) inflation. In the semiclassical picture, the eternally inflating multiverse can expand forever into an infinite phase space, just like the toy-model gas. But it is certainly not clear if we should include entropy from regions beyond the horizon.

- The global picture is time-symmetric, with an infinitely expanding multiverse in the future, and a time-reversed infinitely expanding multiverse in the past.

Can fine-tuning be explained in the context of Hamiltonian evolution?

Example of a Hamiltonian leading to fine-tuning:

$$\boxed{H = -pq \ .}$$

Then

$$\dot{p} = -\frac{\partial H}{\partial q} = p \qquad \dot{q} = \frac{\partial H}{\partial p} = -q \ .$$

So, as time passes, q is fine-tuned to become arbitrarily close to 0. (Similarly q can be fine-tuned to any value, and any function of p and q can be fine-tuned.)

If we assume that the universe has a finite available phase space, do we get into trouble?

- ☆ If the universe has a finite available phase space, then presumably it will approach the maximum entropy state, the state of thermal equilibrium.
- ☆ Life in the form of Boltzmann brains — brains that form as thermal fluctuations and rapidly dissipate — would continue to exist in the thermal equilibrium phase.
- ☆ Since the thermal equilibrium phase would continue forever, essentially all living beings would find themselves to be Boltzmann brains living in the thermal equilibrium phase.
- ☆ But in the thermal equilibrium phase, the probabilities for observations are determined by state counting. In our world, probabilities are determined by history. These pictures are not compatible.

Infinite Phase Space and the Two-Headed Arrow of Time

1) Can an arrow of time develop in a system with time-reversal invariant laws, and no special initial conditions?

   YES: if there is an infinite available phase space.

2) Can fine-tuning be explained in the context of Hamiltonian evolution?

   YES: if there is an infinite available phase space.

3) If we assume that the universe has a finite available phase space, do we get into trouble?

   YES: We would predict that the world we see should be described by state-counting statistics, and not by an historical picture.