uantum Fluctuatio
in de Sitter Space
(do not happen)

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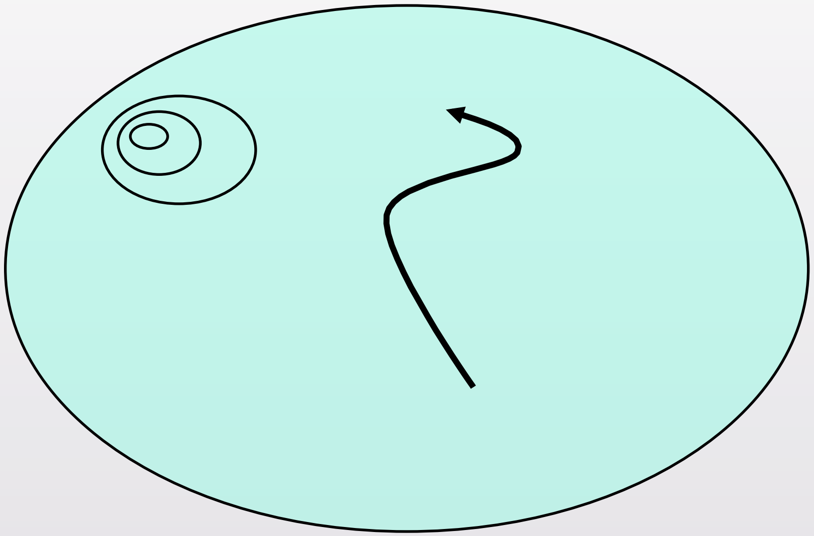
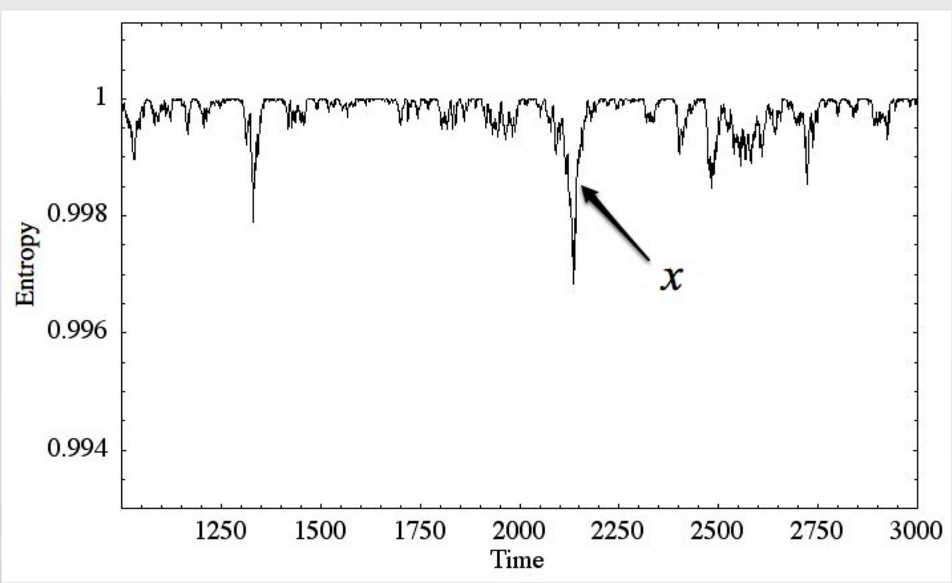
with Kim Boddy & Jason Pollack

Boltzmann (thermal) fluctuations in classical statistical mechanics

High-entropy equilibrium macrostate is static (independent of time).

But microstate is evolving.



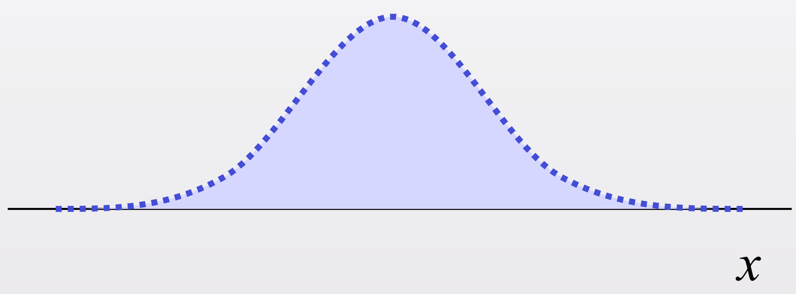


There will be rare fluctuations downward in entropy, with rate $\Gamma(\Delta S) \sim e^{-\Delta S}$.

Quantum mechanics vs. classical fluctuations (in the Everett approach)

Energy eigenstates are static (up to a phase); they don't "fluctuate" at all.

$$\Psi(x,t) \propto e^{-iE_0 t} e^{-E_0 x^2}$$



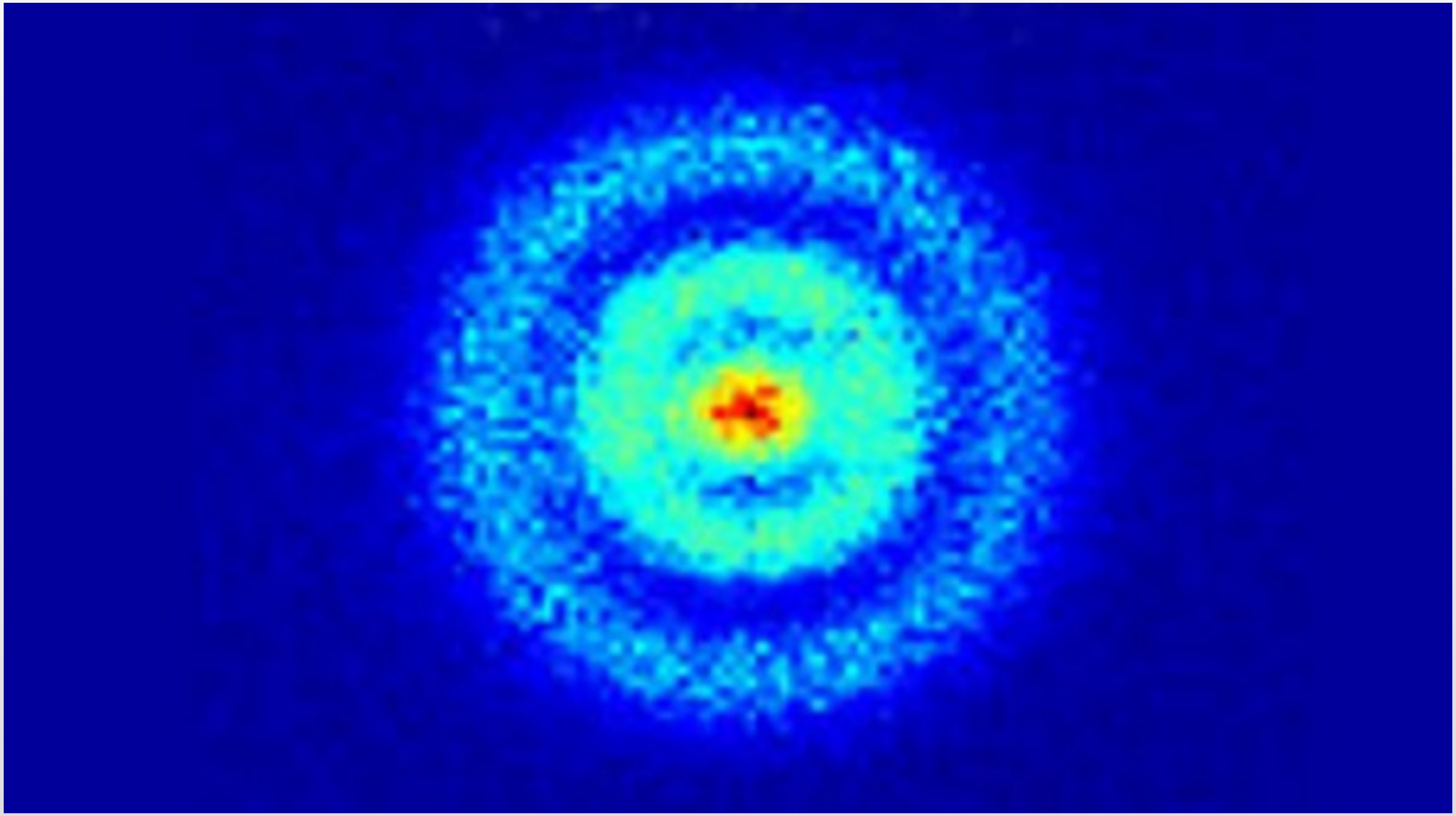
$$\rho = |\Psi(t)\rangle\langle\Psi(t)| = |\Psi_0\rangle\langle\Psi_0|$$

Density operator in an energy eigenstate is manifestly time-independent.

In a static state, there is no statement *X* about the state such that "*X* is true at time t_0, but not at time t_1."

Image of an electron orbital in Hydrogen

[Stodolna et al. 2013]



There is not secretly an electron "orbiting" the nucleus a la Kepler; there is just a static quantum state.

Unlike classical mech, in QM a thermal state is truly static:

$$\rho = e^{-\beta H} = \sum_n e^{-\beta E_n} |E_n\rangle\langle E_n|$$

No hidden, evolving microstate. In that sense, QM exhibits *less* fluctuation than classical mechanics.

So why are we always talking about "quantum fluctuations"?

Observables generically have strictly-positive variance:

$$(\Delta\widehat{\mathcal{O}})^2 = \langle\widehat{\mathcal{O}}^2\rangle - \langle\widehat{\mathcal{O}}\rangle^2 > 0.$$

But "observables" matter when things are actually observed.

“Quantum fluctuations” require observations (decoherence)

Observation requires

(1) an apparatus in a “ready” state;

(2) a large, low-entropy environment;

(3) absence of initial entanglement between S/A/E.

ready state $$|\Psi\rangle = (|+\rangle_S + |-\rangle_S)|0\rangle_A|e_*\rangle_E$$

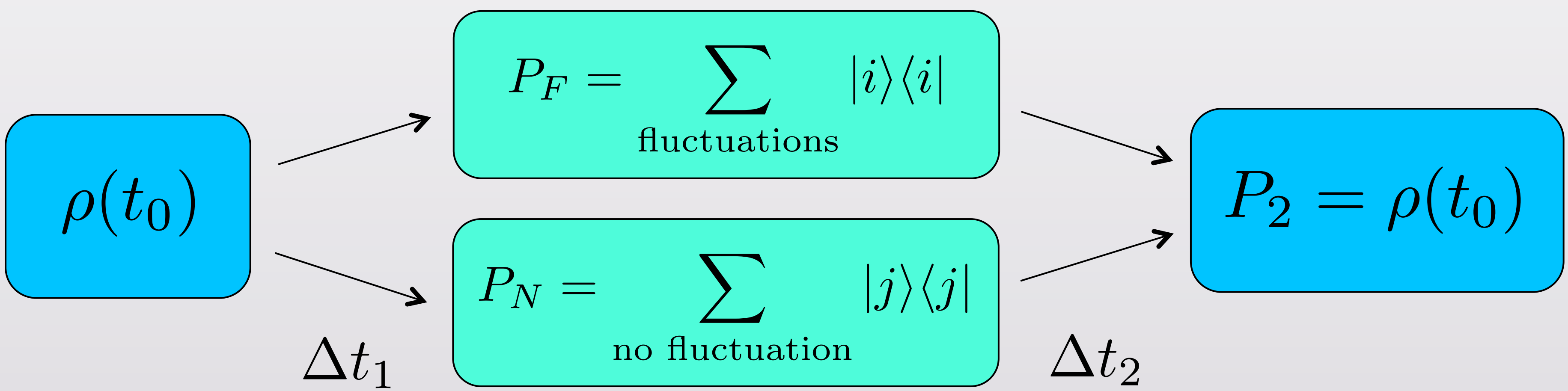
pre-measurement $$\rightarrow (|+\rangle_S|+\rangle_A + |-\rangle_S|-\rangle_A)|e_*\rangle_E$$

decoherence $$\rightarrow |+\rangle_S|+\rangle_A|e_+\rangle_E + |-\rangle_S|-\rangle_A|e_-\rangle_E$$

Quantum fluctuations are time-dependent histories of observations in specific branches.

Decoherent histories formalism

Start with density matrix $\rho(t_0)$. Consider projections onto either "fluctuation" P_F or "no fluctuation" P_N at t_1, and back to initial state at t_2.



The fluctuation "happens" if these two histories decohere, i.e. if their decoherence functional vanishes.

Decoherence functional:

$$D(F,N) =$$

$$\mathrm{Tr}\big(P_2 e^{-iH\Delta t_2} P_F e^{-iH\Delta t_1} \rho(t_0) e^{iH\Delta t_1} P_N e^{iH\Delta t_2} P_2\big)$$

For evolving initial/final states:

$$D(F,N) = \sum_{ijk\ell} c_{ijk\ell} e^{-i(E_i - E_j)\Delta t_2 + i(E_k - E_\ell)\Delta t_1}$$

If dim($\mathcal{H}$) is finite, eventuallys add to zero. For static states:

$$D(F,N) = \sum_{ijk\ell} d_{ijk\ell} = \text{const.}$$

Fluctuating histories never decohere from non-fluctuations.

de Sitter space

Energy density in vacuum
(cosmological constant):

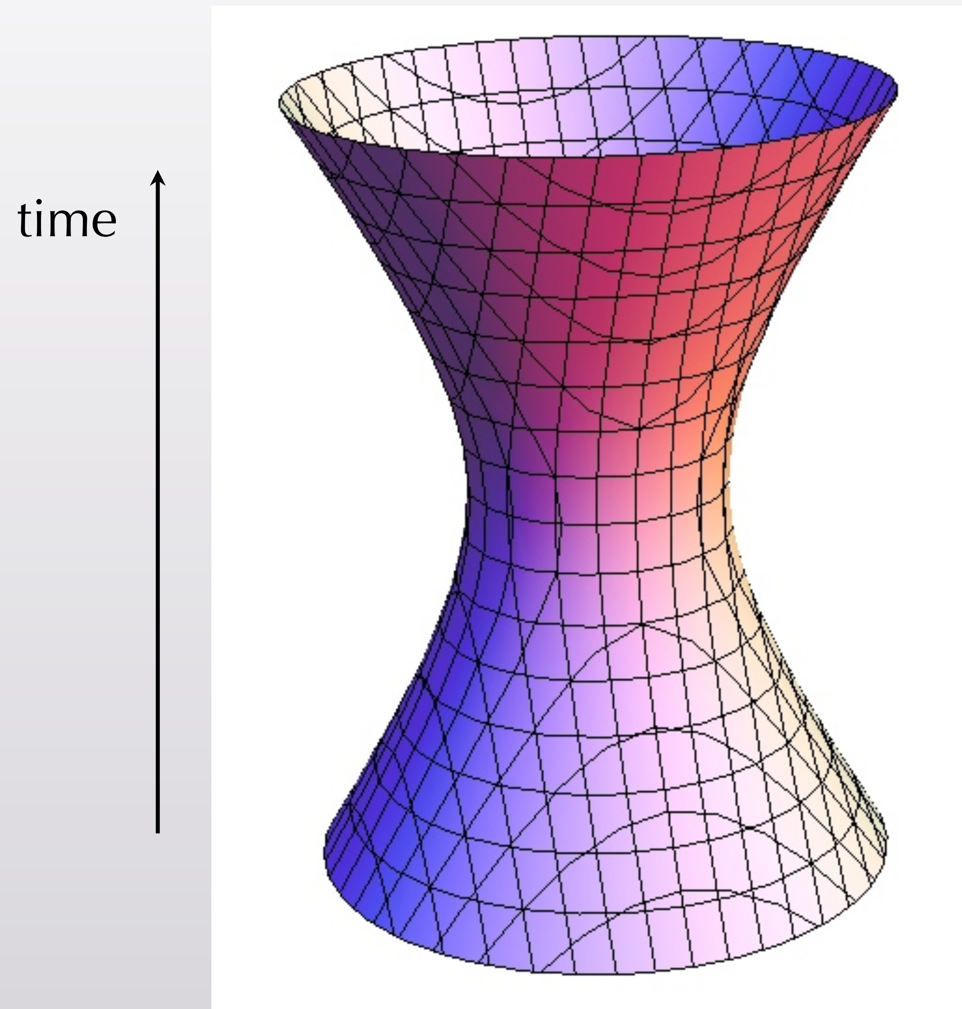
$$\rho = \rho_{\text{vac}} = E_{\text{vac}}^4$$

$$\Lambda = \rho_{\text{vac}}/8\pi G$$

Accelerated expansion:

$$a(t) = a_0 \cosh(Ht)$$

$$H = \sqrt{\Lambda/3}$$



QFT in de Sitter

- There is a global vacuum state (Bunch-Davies).
- Particle detectors observe thermal radiation at the Gibbons-Hawking temperature:

$$T = H/2\pi \sim \sqrt{\Lambda}$$

- Each horizon-sized patch has an entropy:

$$S = A_{\text{horizon}}/4G \sim 1/G\Lambda$$

- Described by a finite-dimensional Hilbert space:

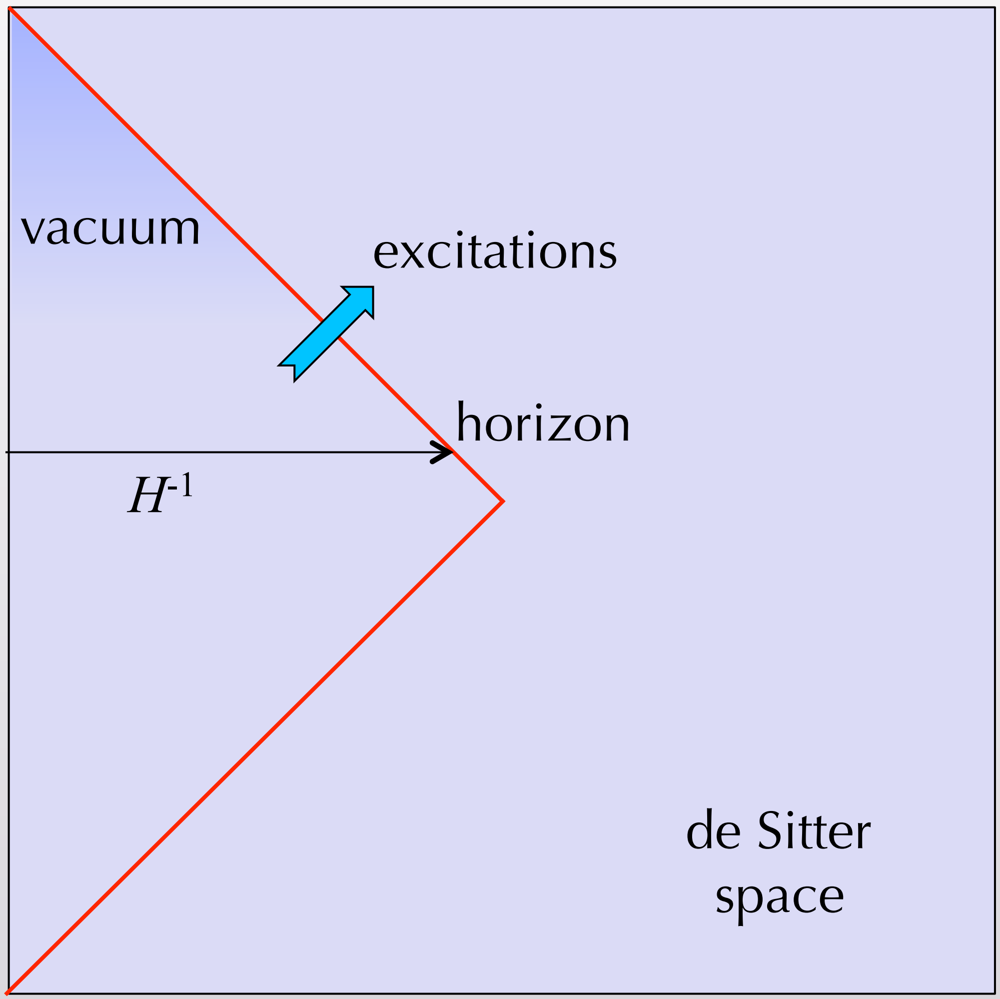
$$\dim \mathcal{H} = e^{S} = e^{10^{120}}$$

Cosmic no-hair theorem

An expanding universe with Λ>0 will come to be totally Λ-dominated, asymptotically approaching de Sitter (if it doesn't collapse first).

Quantum version: state approaches the Bunch-Davies vacuum.

[Marolf, Morrison; Hollands]

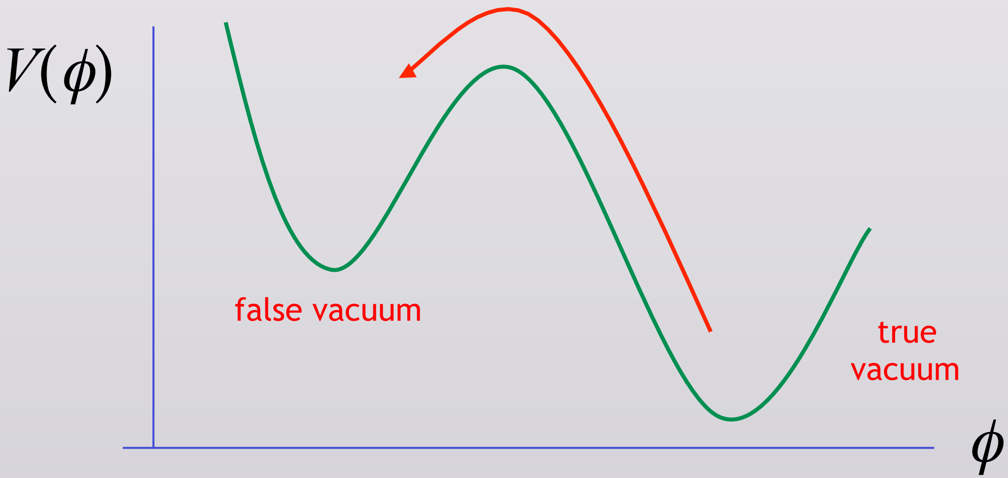


de Sitter fluctuations: conventional wisdom

An horizon-sized patch of de Sitter has a finite entropy and a finite temperature. Therefore, there are fluctuations into lower-entropy states with rate $P(\Delta S) \sim e^{-\Delta S}$.

These fluctuations can produce Boltzmann Brains: freak observers with unreliable memories/beliefs, inducing cognitive instability.

[Dyson, Kleban & Susskind; Albrecht & Sorbo; Page; Bousso & Freivogel; Linde]



Also play a crucial role in "up-tunneling" from true to false vacua in landscape cosmology.

[Lee & Weinberg;
Aguirre, Carroll & Johnson]

Conventional wisdom on de Sitter fluctuations is wrong

Density matrix in a horizon patch is (asymptotically) thermal – diagonal in the energy eigenbasis, therefore static.

$$\rho = e^{-\beta H} = \sum_n e^{-\beta E_n} |E_n\rangle\langle E_n|$$

Hence, the de Sitter vacuum is free from fluctuations.

Boltzmann Brains are not produced.

Low-energy vacua do not up-tunnel to high-energy vacua.

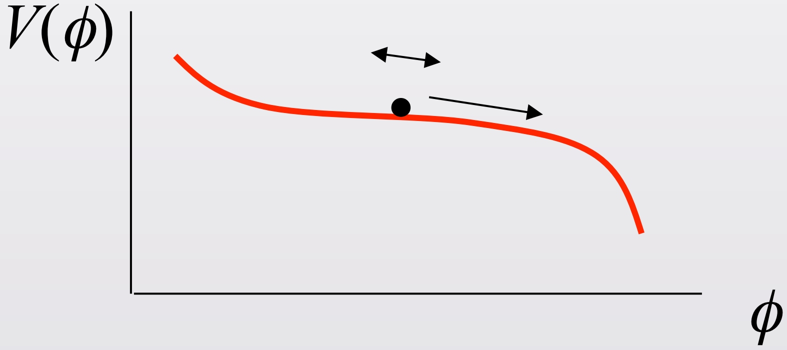
Loopholes

- Horizon Complementarity tells us to think locally, not globally. If the true vacuum is de Sitter, the *total* Hilbert space is finite-dimensional, and we expect Boltzmann fluctuations. But if there is a Minkowski vacuum, the Hilbert space is infinite-dimensional, and you should relax to a static state (before tunneling to the true vacuum).

- We've assumed the Everett formulation of QM. In a dynamical-collapse model, collapses render states non-static. In a hidden-variables model, the situation is more like classical stat mech, and we expect Boltzmann fluctuations.

Inflation?

Inflation still produces density/tensor fluctuations.
Inflaton quantum state is "observed" at reheating, which generates entropy and induces decoherence.



Slow-roll eternal inflation is problematic. Our analysis isn't strictly applicable, since there is true time-dependence.
But there is very little entropy generation, therefore no decoherence, and the state evolves slowly.

We should be very skeptical of "stochastic fluctuations" upward in the potential.

Conclusions

- Quantum variables are not like classical stochastic variables. A closed quantum system in a static state does not fluctuate.
- Boltzmann Brains can be avoided in de Sitter space if you asymptote to the vacuum (i.e. if Hilbert space is infinite-dimensional).
- Up-tunneling is also avoided, making it harder to populate a landscape.
- Inflationary perturbations are fine – but eternal inflation needs to be re-thought.