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Quantum correlations with indefinite causal order

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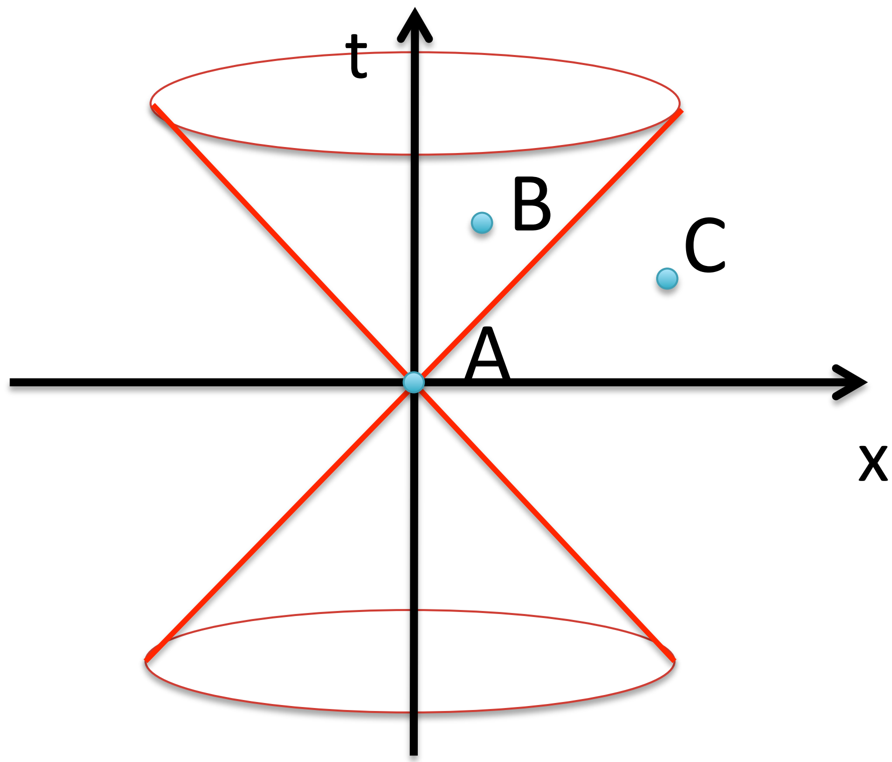
Nature Communications (2012), doi: 10.1038/ncomms2076
(See also News & Views: Nature Physics 8, 860–861 (2012))

FQXi Conference, Vieques, January 7th

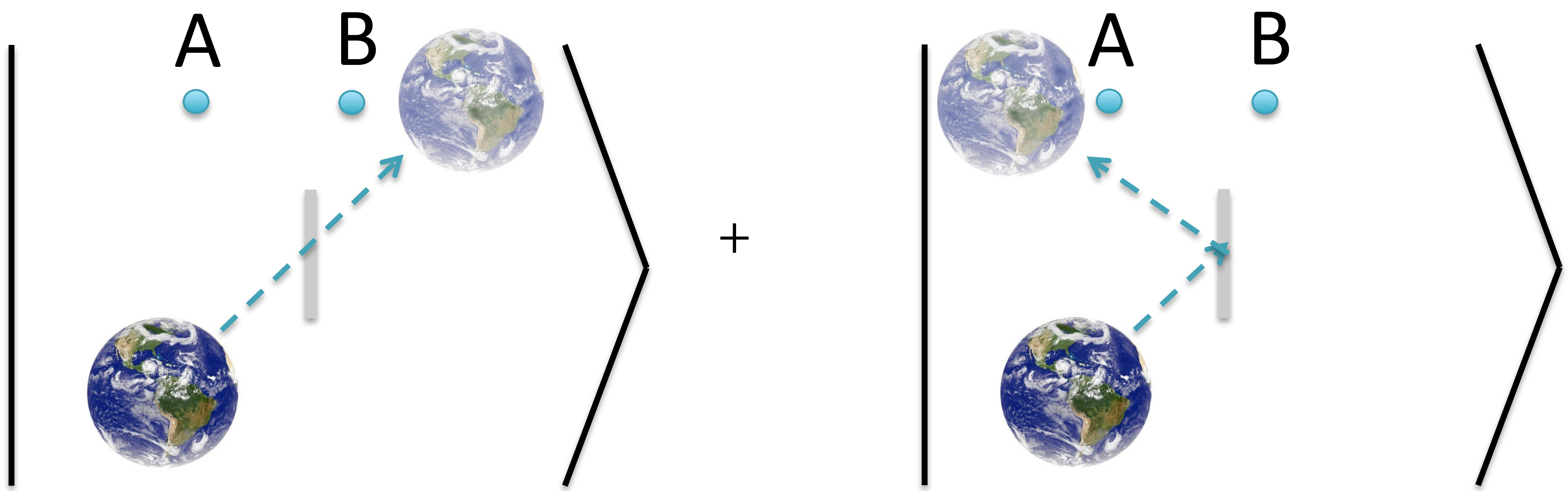
Causality & space time structure



$A \prec B$: A before B

$A \sim C$: A and C causally neutral

Quantum causal relations?



Space-time distance between A and B is not well-defined.

Outline

- Framework for quantum mechanics with no assumed global causal structure:

  includes all causally ordered (spatial and temporal) situations: shared states, channels, channels with memory, and probabilistic mixtures of these.

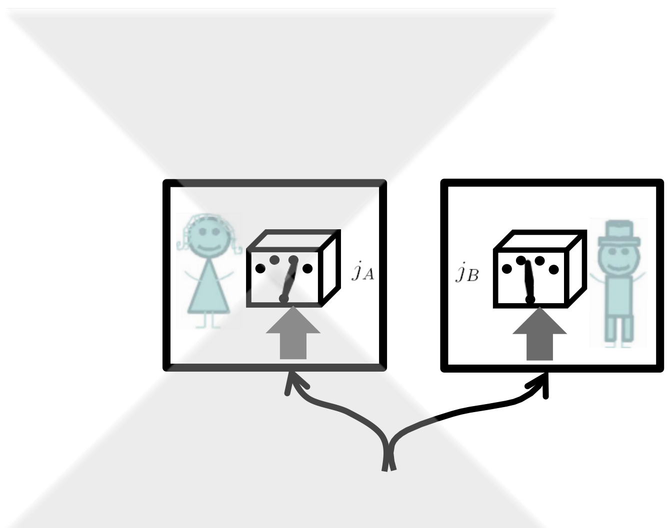
- Correlations that defy causal order:

  Violation of a "causal inequality" – a communication task that cannot be accomplished with causally ordered operations.

L. Hardy, arXiv:gr-qc/0509120, G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, arXiv:0912.0195v3, M. S. Leifer, R. W. Spekkens, arXiv:1107.5849

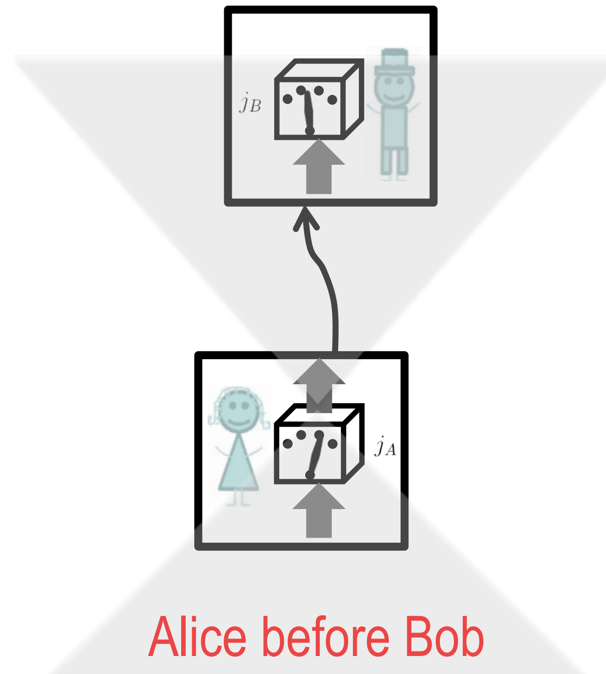
Causal order from space-time

No-signalling



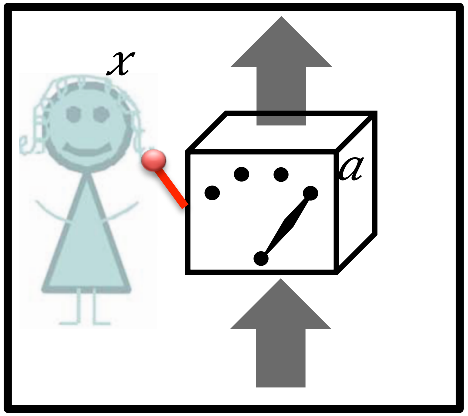
Space-like separated
Causally neutral

One-directional signalling

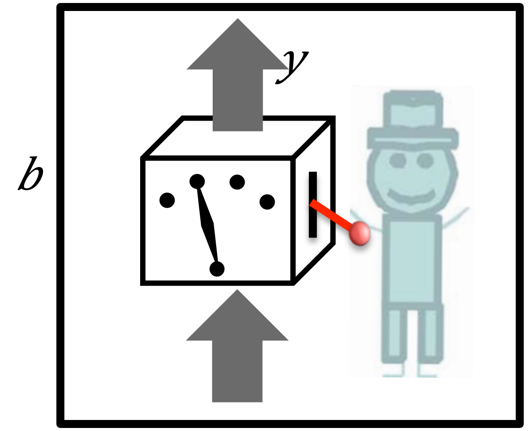


Time-like separated

Operational meaning of signalling



$$p(a,b|x,y)$$



No-signalling

Space-like separated

$$\sum_a p(a,b|x,y) = p(b|y)$$

$$\sum_b p(a,b|x,y) = p(a|x)$$

One-directional signalling

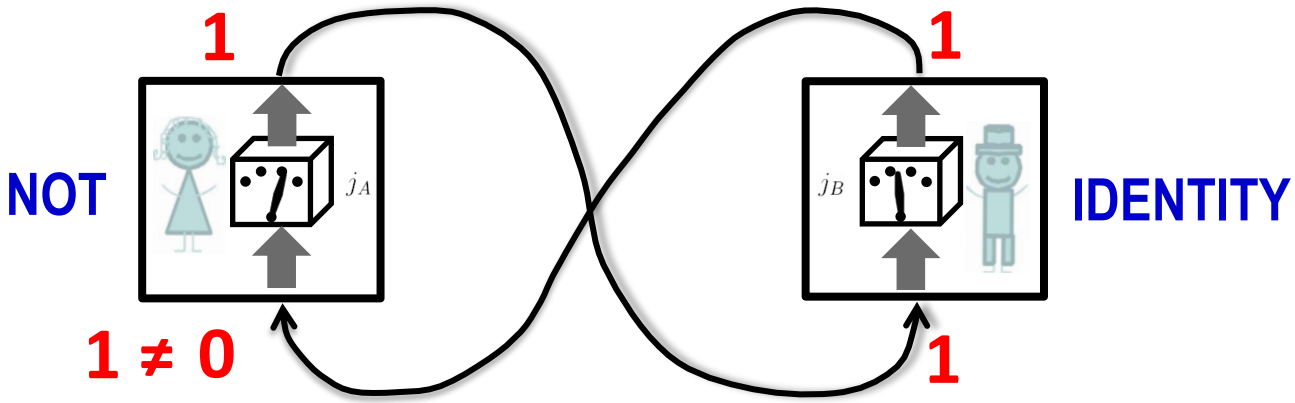
Time-like separated

$$\sum_a p(a,b|x,y) = p(b|y)$$

$$\sum_b p(a,b|x,y) = p(a|x,y)$$

Two-directional signalling?

Causal Loop



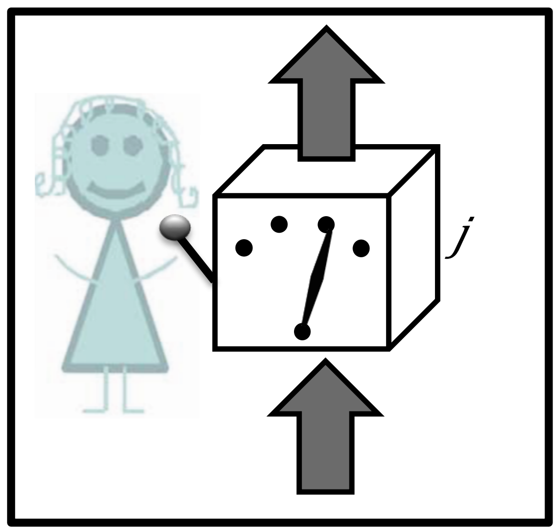
Grandfather paradox!

Gödel Universe: **Closed time-like curves** (CTC)

Proposed quantum solutions: Deutsch's or the Bennett-Schumacher-Svetlichny-Lloyd CTC-like structures are **non-linear** extensions of quantum theory

Linear structures, free of paradoxes?

The framework: Closed laboratory



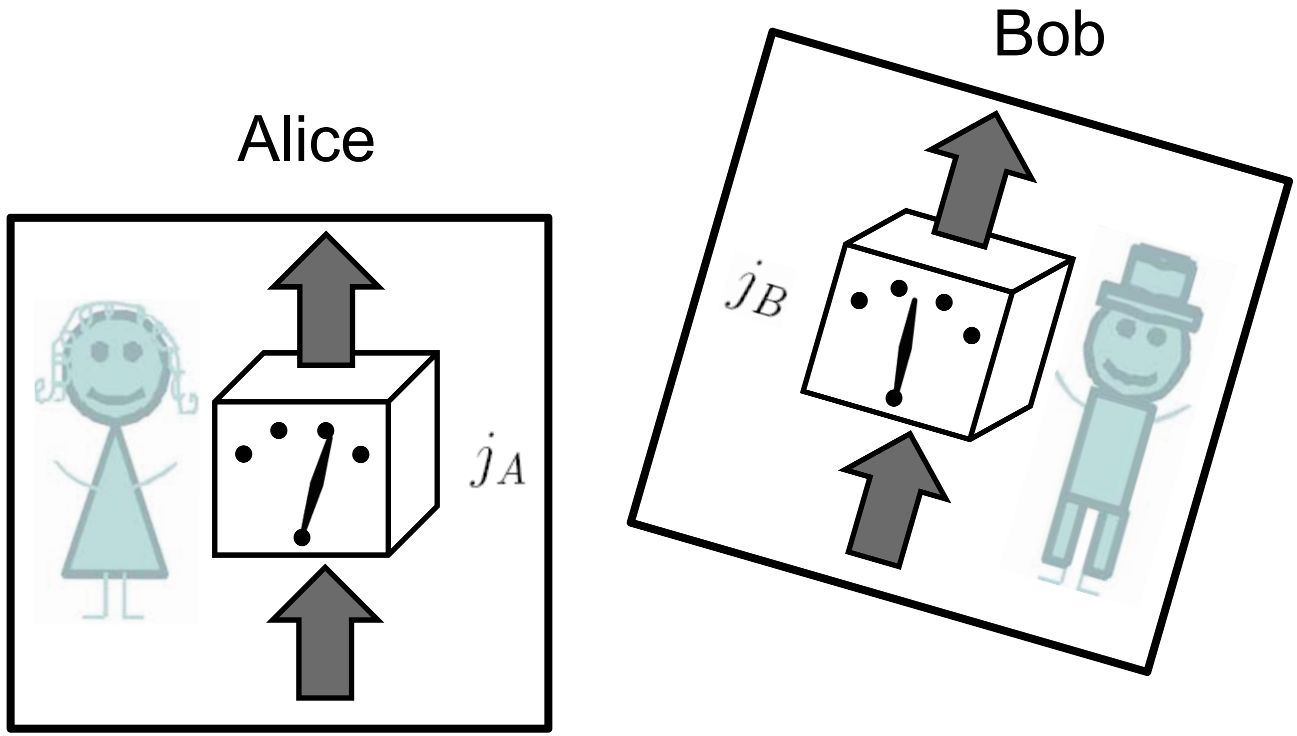
The system exits the lab.

A transformation is performed, and an outcome j is obtained.

A system enters the lab.

This is the **only** way how each party interacts with the “outside world”.

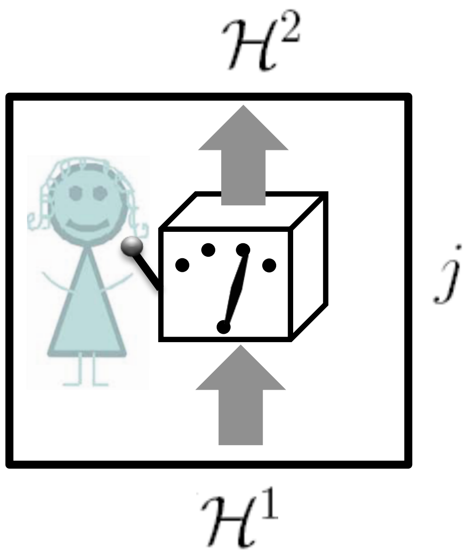
Correlations



No prior assumption of pre-existing causal structure, in particular of the pre-existing background time.

Main premise:

Local descriptions agree with quantum mechanics



Transformations = completely positive (CP) trace-nonincreasing maps

$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2)$$

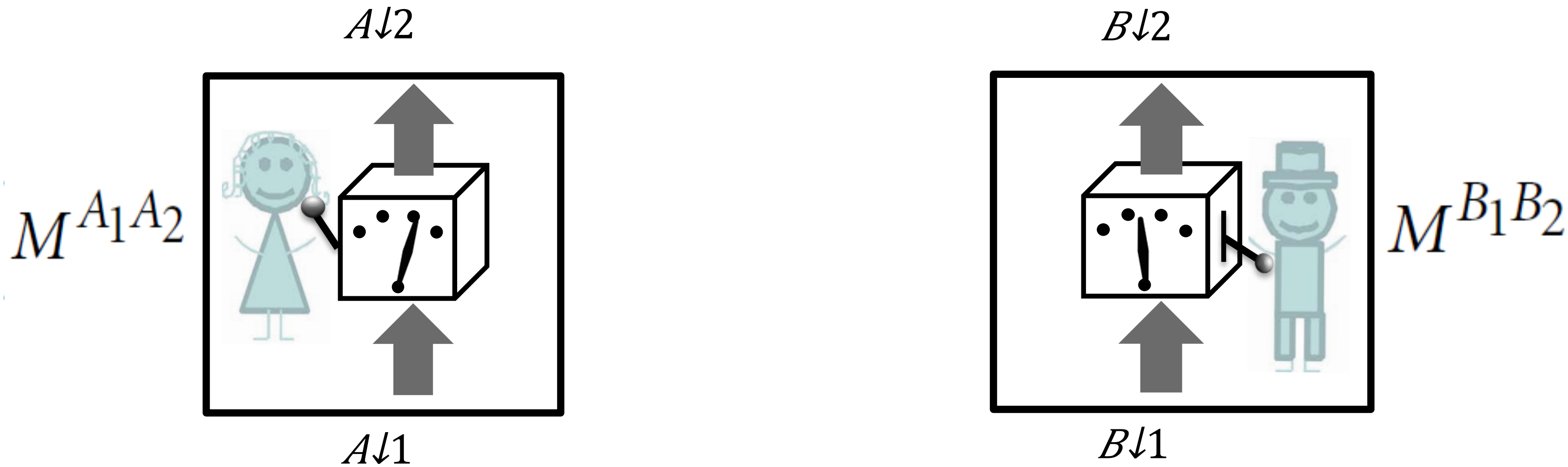
↕ **Choi-Jamiołkowski isomorphism**

$$M \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$$

$$\mathcal{M}_{CPTP} = \sum_j \mathcal{M}_j$$

Completely positive trace preserving (CPTP) map

Two (or more) parties



Probabilities are bilinear functions of the CP maps

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1A_2B_1B_2}\left(M^{A_1A_2} \otimes M^{B_1B_2}\right)\right]$$

„Process matrix“

Local CP maps

Goal: characterize the most general W

Bipartite probabilities

1. Nonnegative probabilities:

(ancillary entangled states do not fix causal order)

$$W^{A_1 A_2 B_1 B_2} \geq 0$$

2. Probability is 1 for all CPTP maps.

$$\mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2}\left(M_{\text{CPTP}}^{A_1 A_2} \otimes M_{\text{CPTP}}^{B_1 B_2}\right)\right] = 1,$$

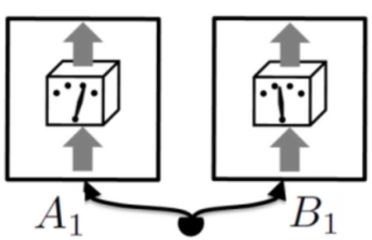
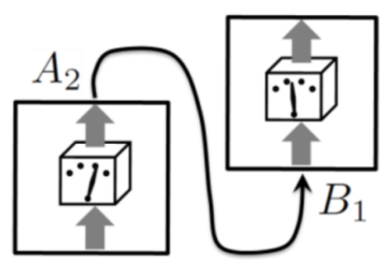
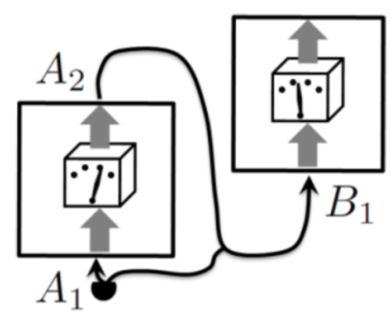
Terms appearing in process matrix

$$W^{A_1A_2B_1B_2} = \sum_{\mu_1,\ldots,\mu_4} a_{\mu_1\ldots\mu_4}\sigma_{\mu_1}^{A_1} \otimes \ldots \otimes \sigma_{\mu_4}^{B_2}$$

$\sigma_i^{A_1} \otimes \mathbb{1}^{rest}$ type A_1

$\sigma_i^{A_1} \otimes \sigma_j^{A_2} \otimes \mathbb{1}^{rest}$ type A_1A_2

...

A_1, B_1, A_1B_1	A_2B_1 A_1B_2	$A_1A_2B_1$ $A_1B_1B_2$
States	Channels	Channels with memory
		

Causal order between parties

$W^{B \not\leq A}$ Bob cannot signal to Alice

$W^{A \not\leq B}$ Alice cannot signal to Bob

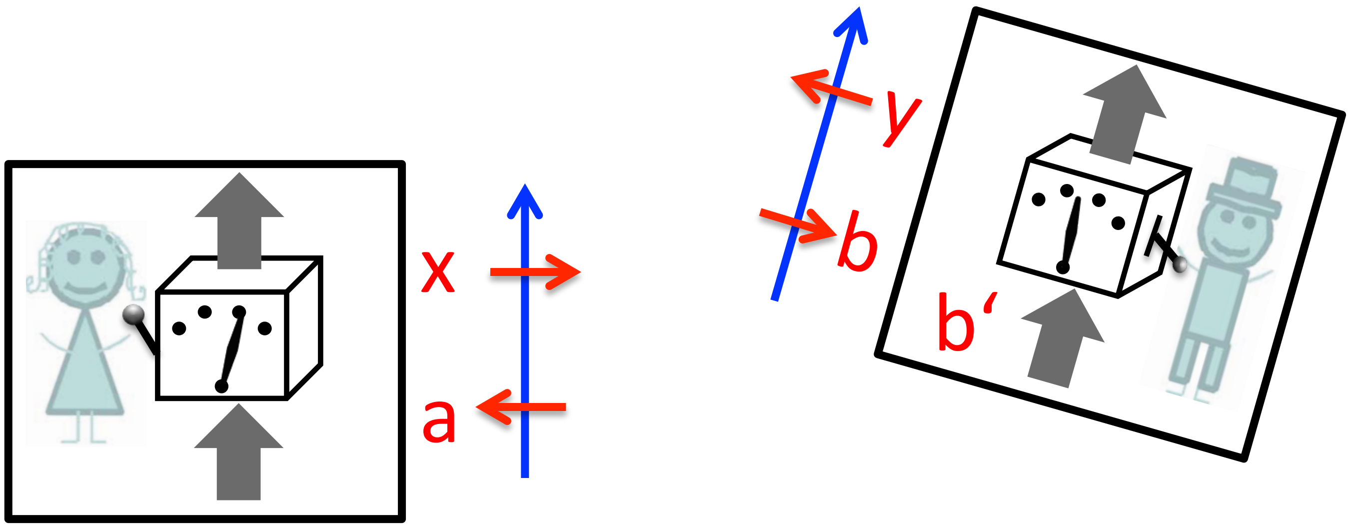
Probabilistic mixtures of ordered processes:

$$W^{A_1 A_2 B_1 B_2} = q W^{B \not\leq A} + (1-q) W^{A \not\leq B}$$

Are all *W* of that form?

No!

Causal game: Guess partner's input

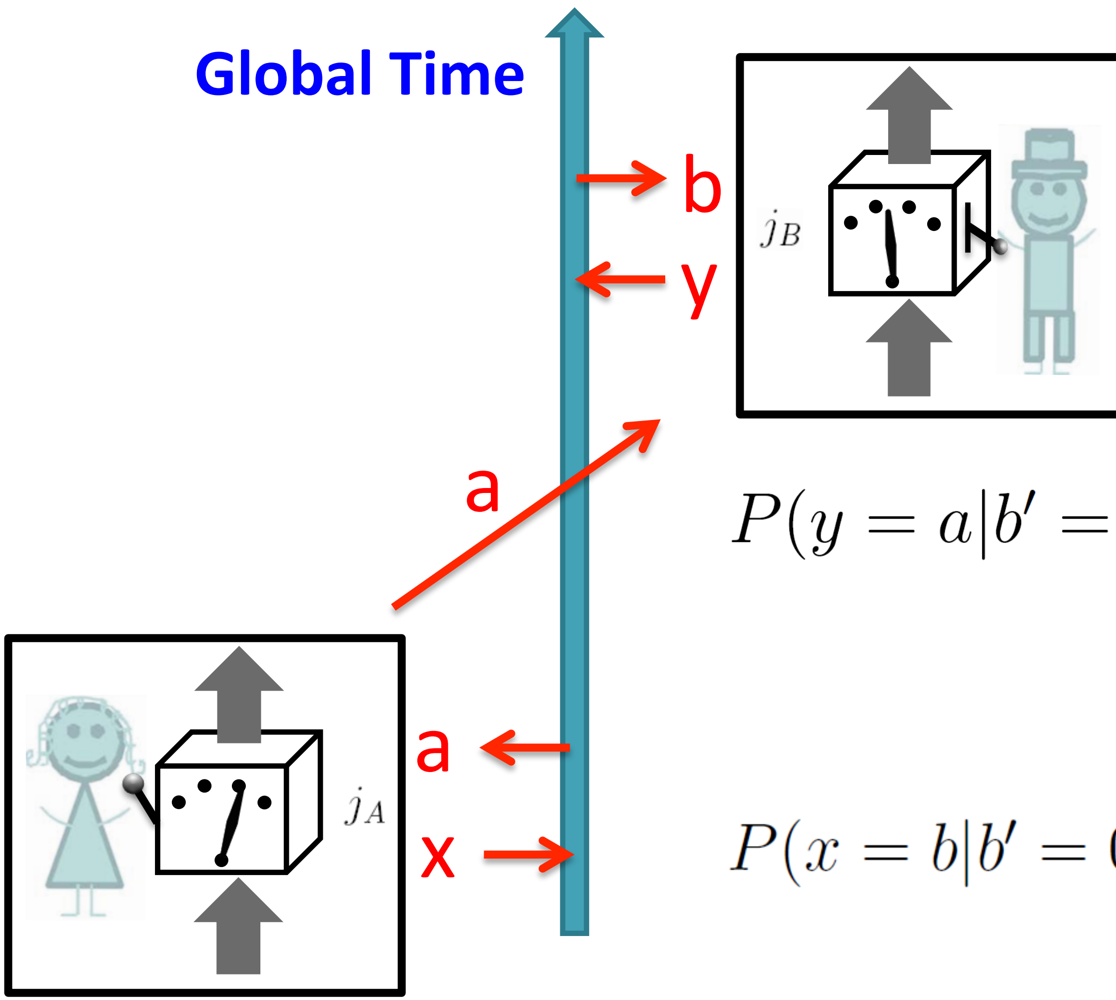


- Alice is given bit **a** and Bob bit **b**.
- Alice produces **x** and Bob **y**, which are their best guesses for the value of the bit given to the other.
- Bob is given an additional bit **b'** that tells him whether he should guess her bit (**b'=1**) or she should guess his bit (**b'=0**).
- The goal is to maximize the probability for correct guess:

$$p_{succ} := \frac{1}{2}\left[P(x = b|b' = 0) + P(y = a|b' = 1)\right]$$

Causally ordered situation

Case: $B \nprec A$



$$P(y = a|b' = 1) = 1$$

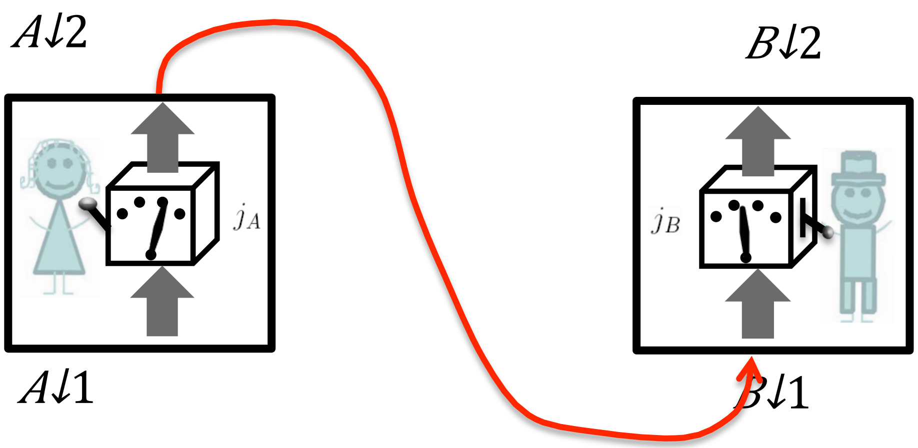
$$P(x = b|b' = 0) = 1/2$$

$$p_{succ} := \frac{1}{2}\left[P(x = b|b' = 0) + P(y = a|b' = 1)\right] \leq \frac{3}{4}$$

Indefinite Causal Structures

two-level systems

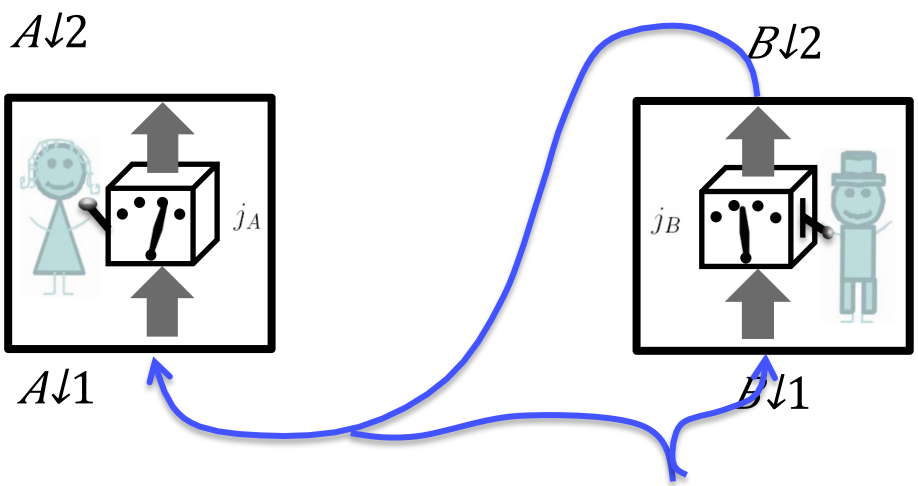
$$W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1}^{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2} \right) \right]$$



b'=1: Bob measures $\sigma_z^{B_1}$

Indefinite Causal Structures

$$W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1}^{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$

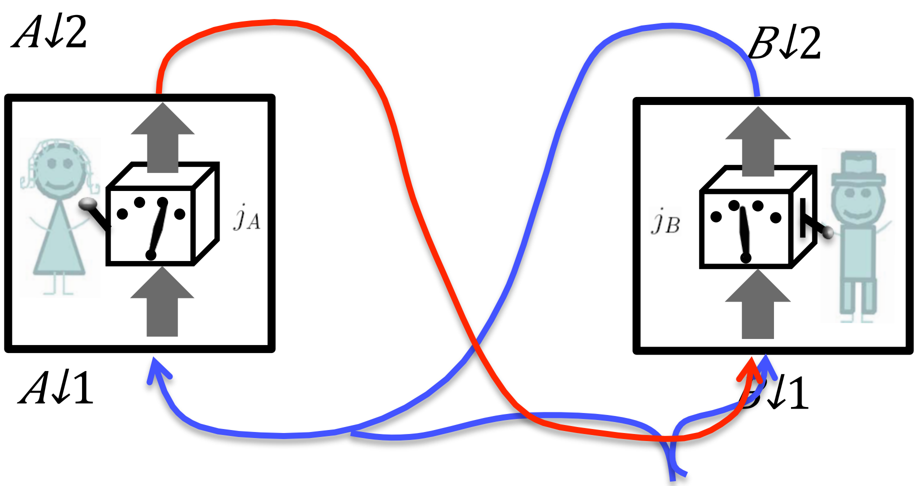


b'=1: Bob measures $\sigma_z^{B_1}$

b'=0: Bob measures $\sigma_x^{B_1}$

Indefinite Causal Structures

$$W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1}^{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$



b'=1: Bob measures $\sigma_z^{B_1}$

b'=0: Bob measures $\sigma_x^{B_1}$

Depending on his choice Bob can end up after or before Alice with probability $\sqrt{2}/2$

The probability of success is: $p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$

Progress

- $\frac{2+\sqrt{2}}{4}$ is the Tsirleson‘s bound for quantum correlations with indefinite causal order
- One has causally non-separable process matrices that do not violate causal inequality (analogous to the relation between nonlocality and entanglement)
- „GHZ-type“ of correlations for three observers [arxiv: 1312.5916 Ämin Baumeier, Stefan Wolf]

Summary

- Unified framework for signalling (time-like) and no-signalling (space-like) quantum correlations.

- Situations where the causal order between laboratory operations is not definite → global causal order need not be a necessary element of quantum theory.

Outlook

- Can we realize non-causal processes in the lab?
- A generalization of concept of space-time?
- Principles that select the generally signalling correlations allowed by QM?
- A new resource for quantum information processing?

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Thank you for your attention

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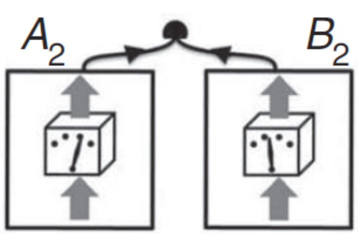
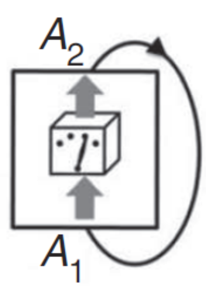
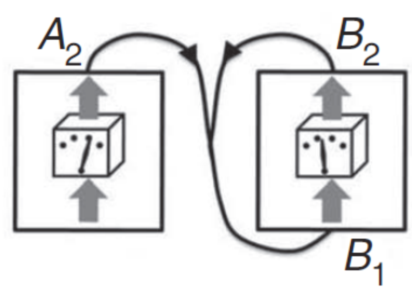
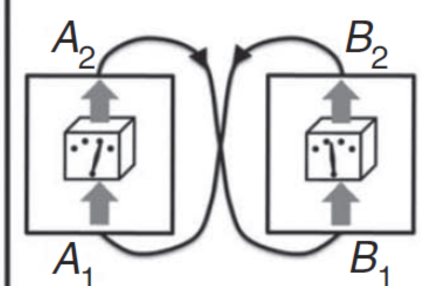
Terms **not** appearing in process matrix

$$W^{A_1 A_2 B_1 B_2} = \sum_{\mu_1,\dots,\mu_4} a_{\mu_1 \dots \mu_4} \sigma^{A_1}_{\mu_1} \otimes \dots \otimes \sigma^{B_2}_{\mu_4}$$

$\sigma^{A_1}_i \otimes \mathbb{1}^{rest}$ type A_1

$\sigma^{A_1}_i \otimes \sigma^{A_2}_j \otimes \mathbb{1}^{rest}$ type $A_1 A_2$

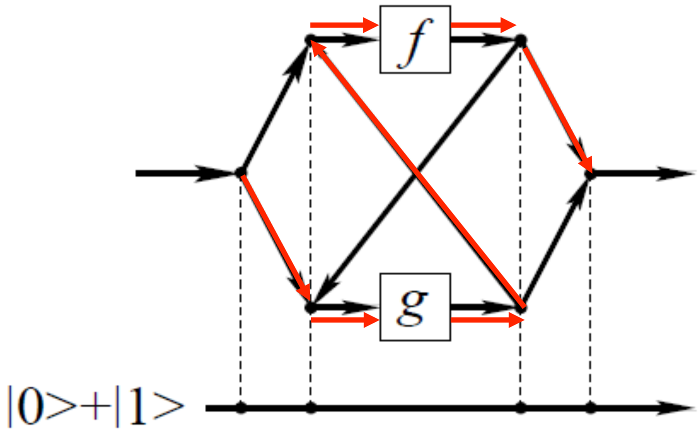
...

A_2, B_2, $A_2 B_2$	$A_1 A_2$, $B_1 B_2$	$A_1 A_2 B_2$, $A_2 B_1 B_2$	$A_1 A_2 B_1 B_2$
Postselection	Local loops	Channels with local loops	Global loops
			

Where to look for non-causal processes?

1. Within standard quantum mechanics? It may be possible to create causally non-separable situations making clever use of superposition and entanglement.

Pavia group,
Chriribella et. al



Conclusions

- [Not shown]: In the classical limit all correlations are causally ordered
- Unified framework for both signalling ("time-like") and non-signalling ("space-like") quantum correlations
- Situations where a causal ordering between laboratory operations is not definite → Suggests that causal ordering might not be a necessary element of quantum theory

Choi-Jamiołkowski isomorphism

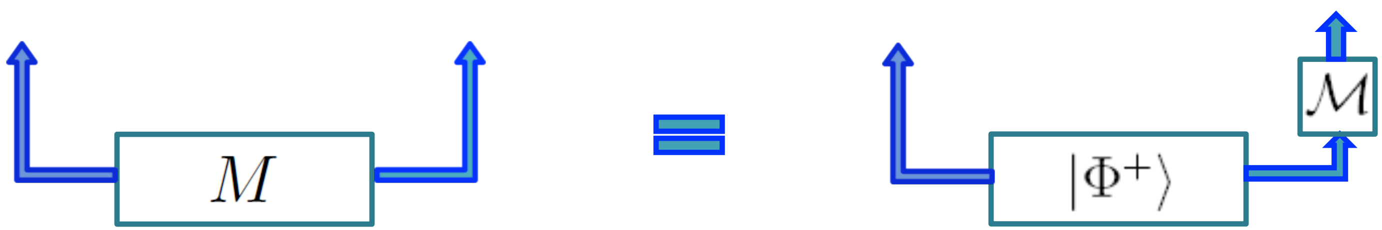
CP maps

$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2)$

$\longleftrightarrow$

Positive matrices

$M \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$



Maximally entangled state

$$M^{12} := \left[\mathcal{I} \otimes \mathcal{M}\left(|\Phi^+\rangle\langle\Phi^+|\right)\right]^{\mathrm{T}}$$

$$|\Phi^+\rangle = \sum_i |i\rangle|i\rangle$$

$$|i\rangle \in \mathcal{H}^1$$

Outlook

- Can we realize non-causal processes in the lab?
- A generalization of concept of space-time?
- Principles that select the generally signalling correlations allowed by QM?
- Is $\frac{2+\sqrt{2}}{4}$ a "Tsirelson bound for non-causal correlations"?
- A new resource for quantum information processing?

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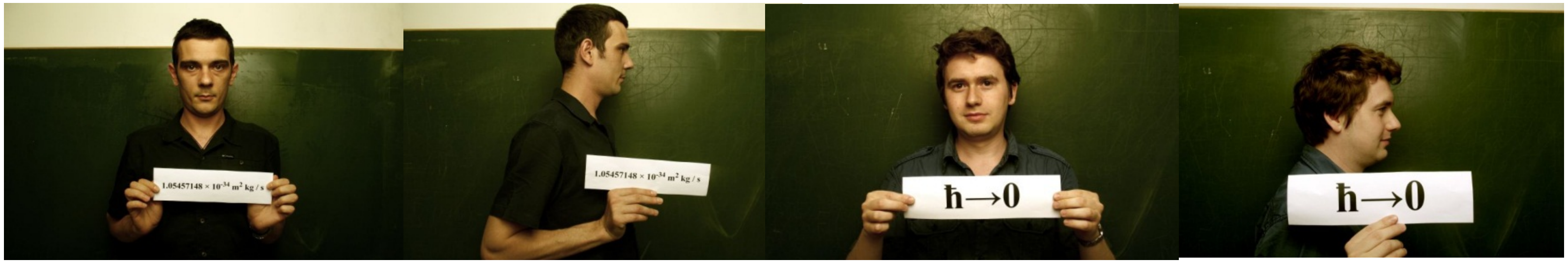
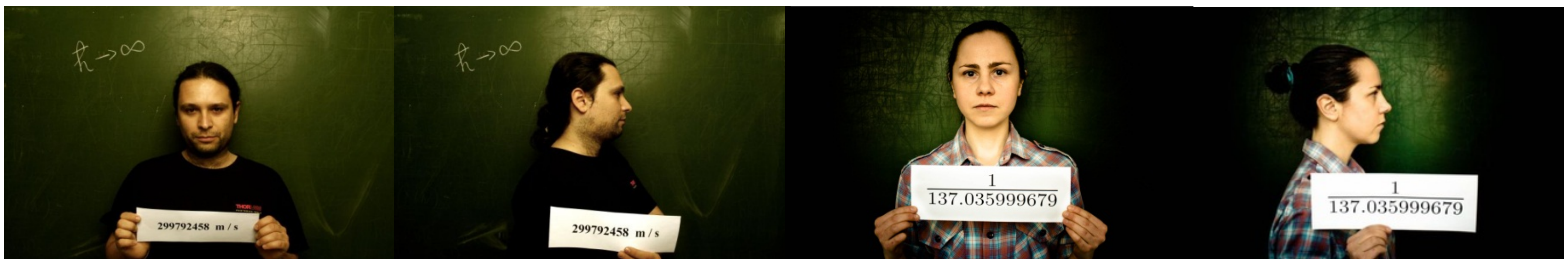


Borivoje Dakic   Igor Pikovski



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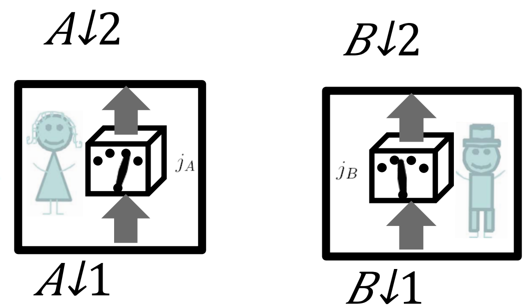
Bipartite probabilities

Bilinear functions of the CP maps: $M^{A_1A_2}$



$M^{B_1B_2}$

Representation

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1A_2B_1B_2}\left(M^{A_1A_2} \otimes M^{B_1B_2}\right)\right]$$

„Process matrix“

Choi-Jamilkowski representation of CP maps

Goal: characterize the most general W

Where to look for non-causal processes?

1. Within standard quantum mechanics? It may be possible to create causally non-separable situations making clever use of superposition and entanglement.

Pavia group,
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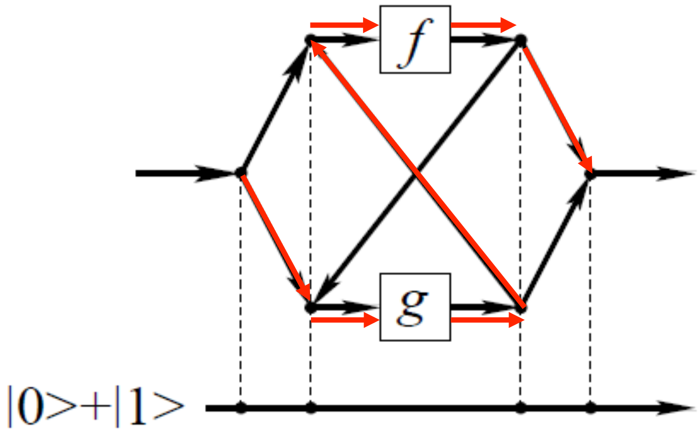
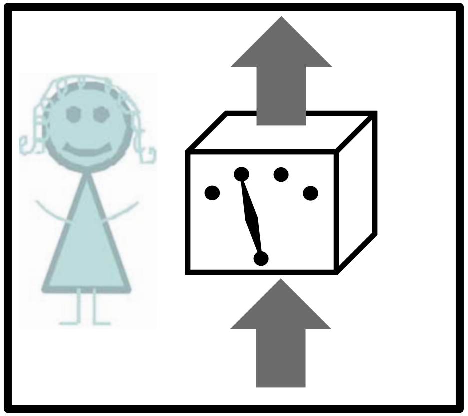
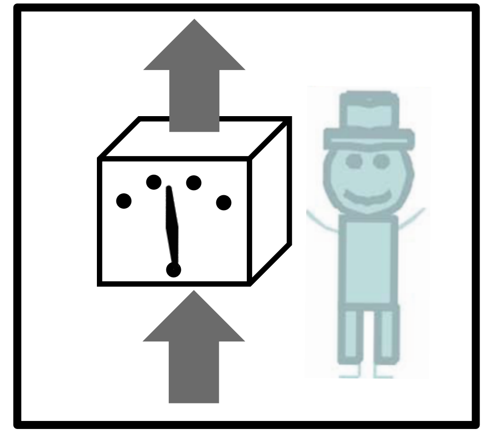


Causal order from correlations?





"In summer with a large amount of ice-cream consumption there are lot of sun-burn cases."

⇨ "Ice consumption causes sun-burn."

Motivation

- Can one formulate physical theories without the assumption of background space-time or causal structure?

  Using tools of quantum information to address problems that traditionally have been considered within quantum gravity

- Quantum correlations are the crucial resource for performing computational tasks that are impossible classically.

  "Superpositions of quantum cuircits"

L. Hardy, arXiv:gr-qc/0509120, G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, arXiv:0912.0195v3, M. S. Leifer, R. W. Spekkens, arXiv:1107.5849

Indefinite Causal Structures

Alice always measures in the z basis and encodes the bit in the *z* basis

Alice's CP map: $|z_x\rangle\langle z_x|^{A_1} \otimes |z_a\rangle\langle z_a|^{A_2} \qquad x, a = \pm 1$

If Bob wants to receive (**b'=1**), he measures in the *z* basis

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$

$$\langle z_\pm|\sigma_x|z_\pm\rangle = 0$$

Channel from Alice to Bob

Not seen by Bob

Bob receives the state $\widetilde{W}^{B_1 B_2} = \frac{1}{2}(\mathbb{1} + a\frac{1}{\sqrt{2}}\sigma_z^{B_1})$

He can read Alice's bit with probability

$$P(y = a|b' = 1) = \frac{2+\sqrt{2}}{4}$$

Indefinite Causal Structures

If Bob wants to send (***b'* = 0**), he measures in the *x* basis and encodes in the *z* basis conditioned on his outcome

Bob's CP map: $|x_y\rangle\langle x_y|^{B_1} \otimes |z_{by}\rangle\langle z_{by}|^{B_2}$ $\quad y, b = \pm 1$

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$

$$\langle x_\pm|\sigma_z|x_\pm\rangle = 0$$

Not seen by Bob

Channel from Bob to Alice, correlated with Bob's outcome

Alice receives the state $\widetilde{W}^{A_1 A_2} = \frac{1}{2}(\mathbb{1} + b\frac{1}{\sqrt{2}}\sigma_z^{A_1})$

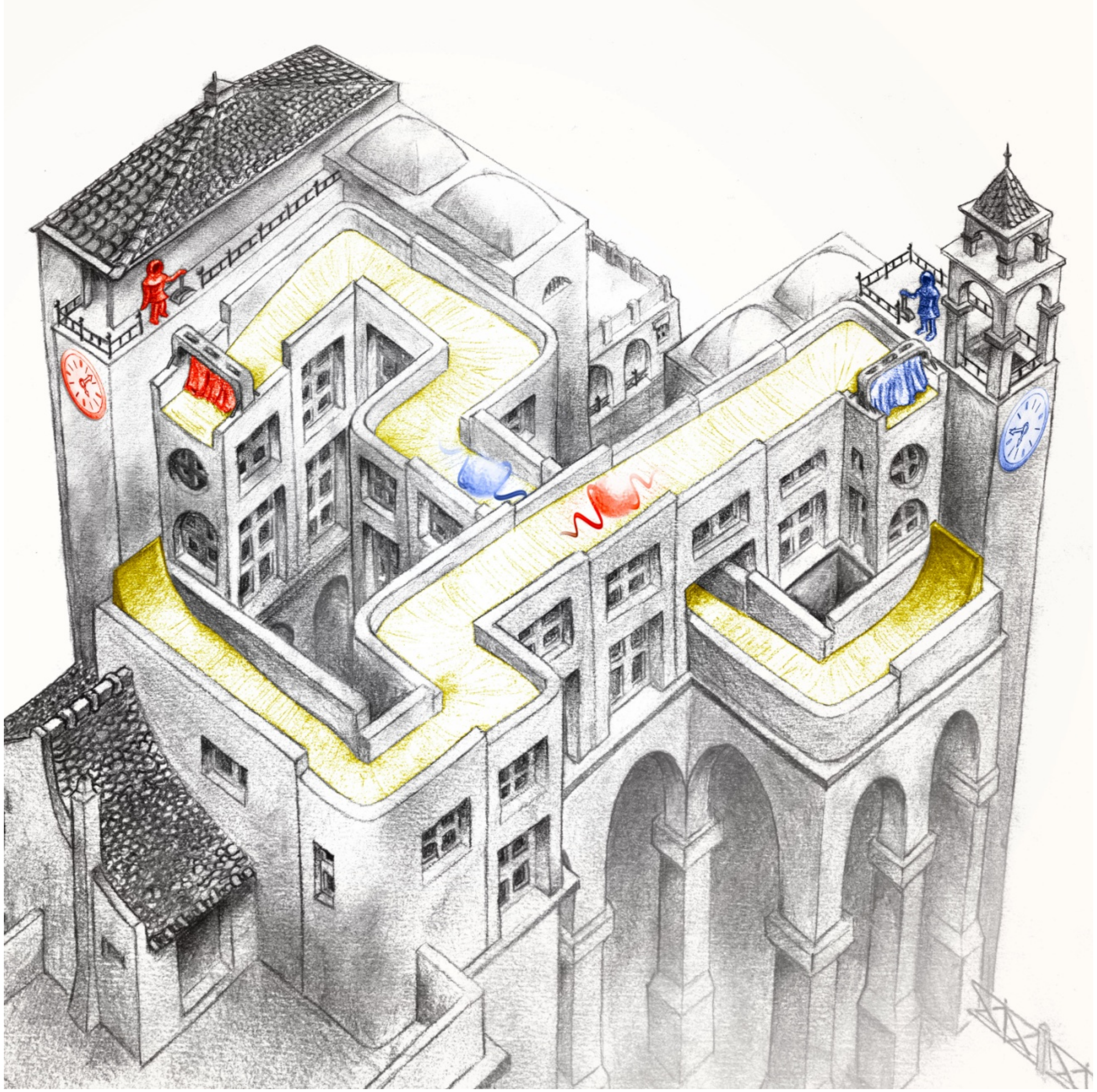
She can read Bob's bit with probability

$$P(x = b|b' = 0) = \frac{2+\sqrt{2}}{4}$$

Indefinite Causal Structures

$$W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1}^{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$

There are quantum processes that **cannot** be understood as probabilistic mixtures of causally ordered situations!



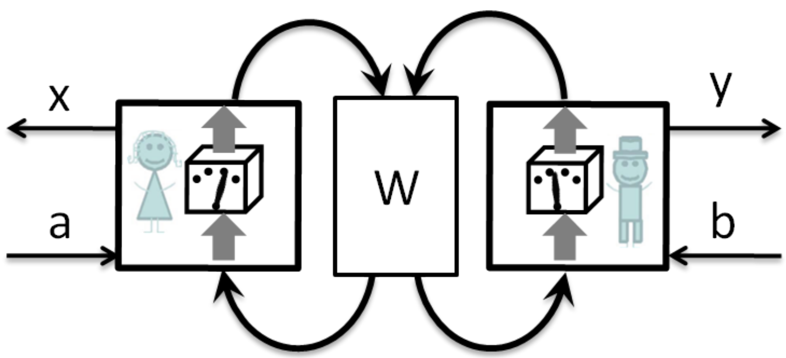
Artistic view on `Superpositions of causal orders‘ inspired by M. C. Escher’s *Ascending and Descending*, 1960

Jonas Schmöle, University of Vienna

Where to look for non-causal processes?

1. Within standard quantum mechanics? It may be possible to create causally non-separable situations similarly to mixtures, making clever use of superposition and entanglement.

2. Closed time-like curves? Every W can be seen as a noisy channel back in time.



Grandfather paradox is avoided.

This CTC-like structure is linear, unlike Deutsch's or Bennet's CTC models.

Where to look for non-causal processes?

3. "Superposition of causal orders"? Every W contains terms that correspond to situations with definite causal order, yet is not a classical mixture of those.

[**Not shown**]: In the classical limit all processes are causally separable, i.e., global causal order arises!

Space-time may emerge in a quantum-to-classical transition.