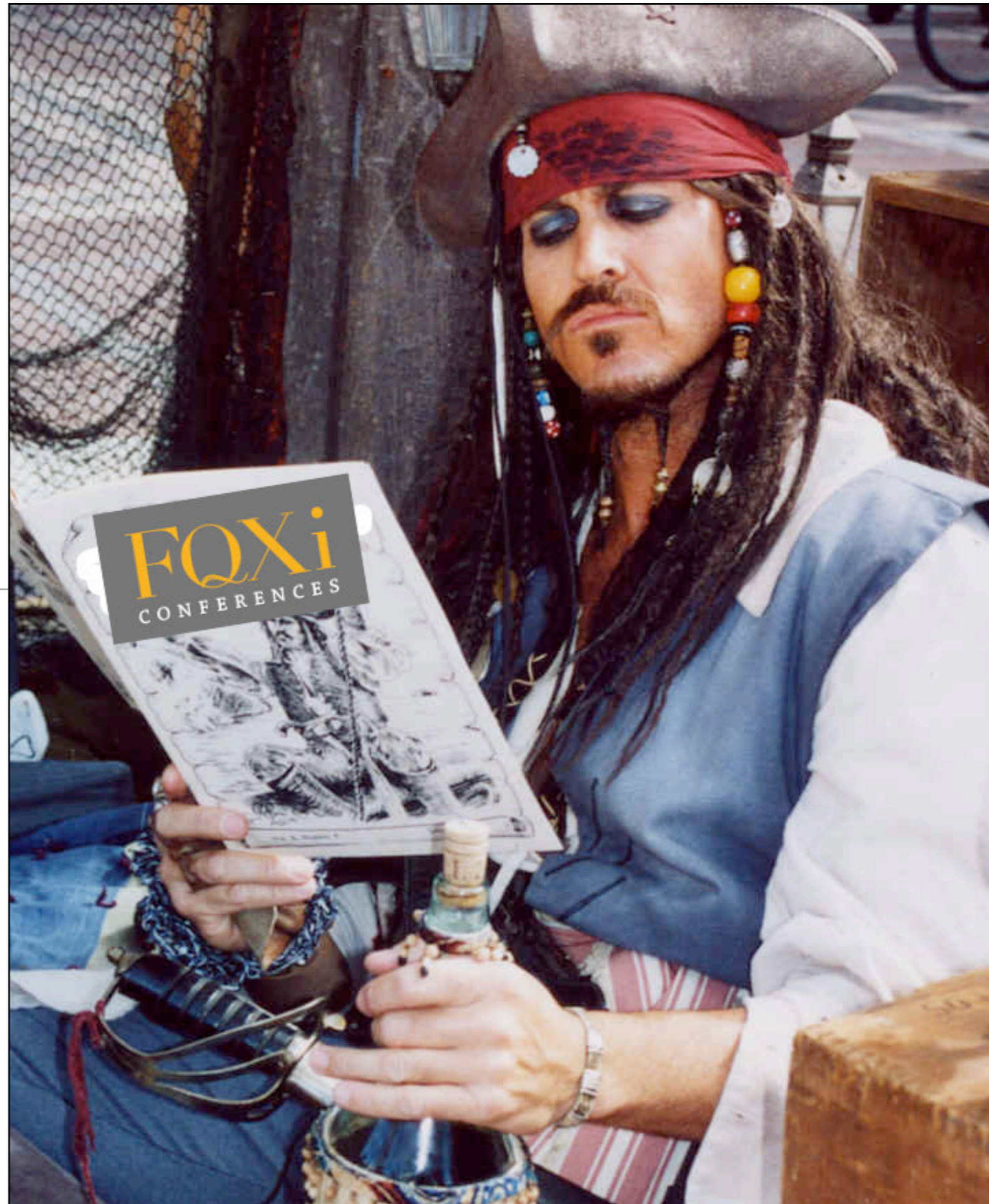


The Mysterious Thermodynamic Origin of Classical Gravity

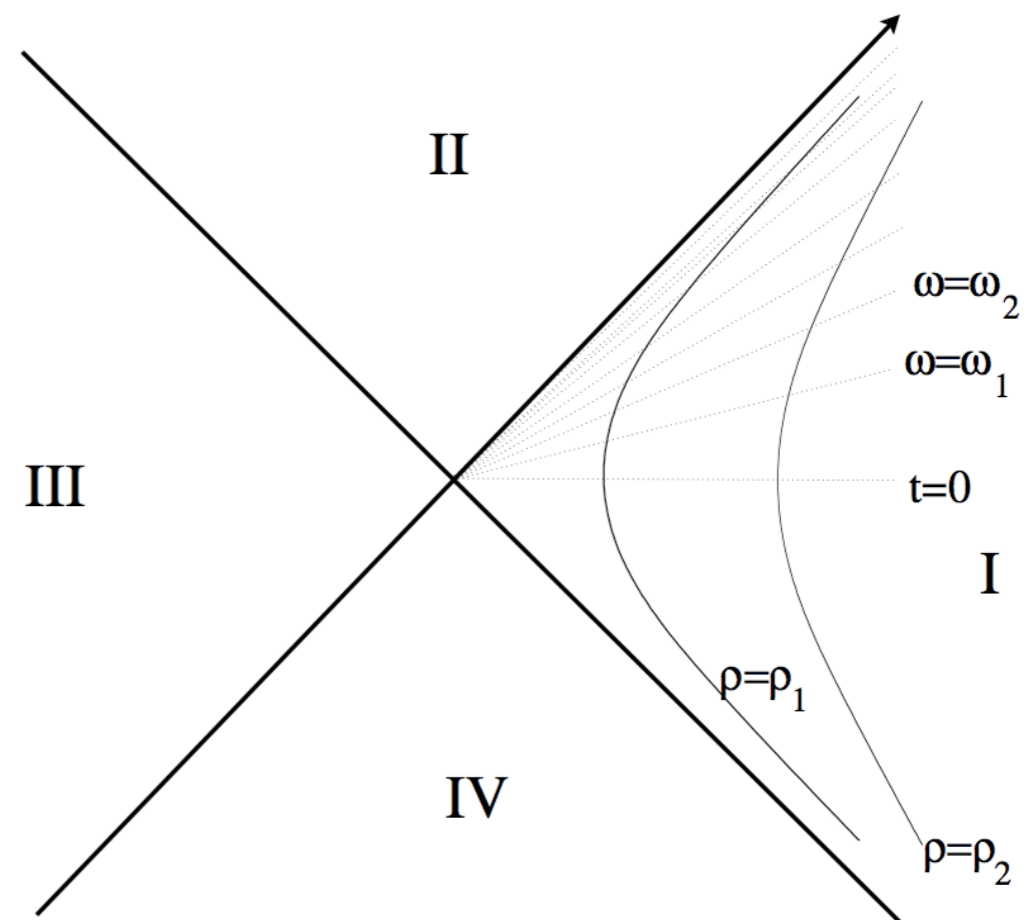
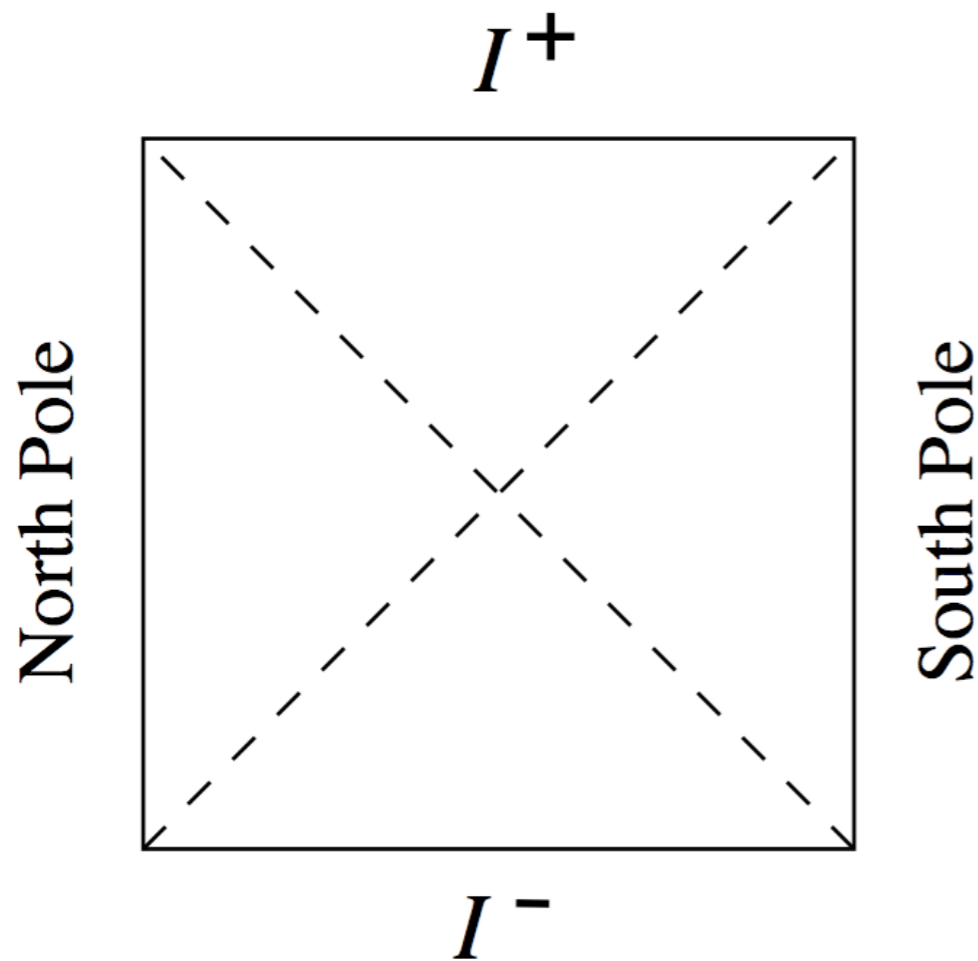
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FQXi 09, Azores



Why Black Holes are of Wider Relevance

- The key feature that makes black holes interesting is the presence of a horizon.
- Two other types of horizons exist: cosmological horizons and acceleration horizons.



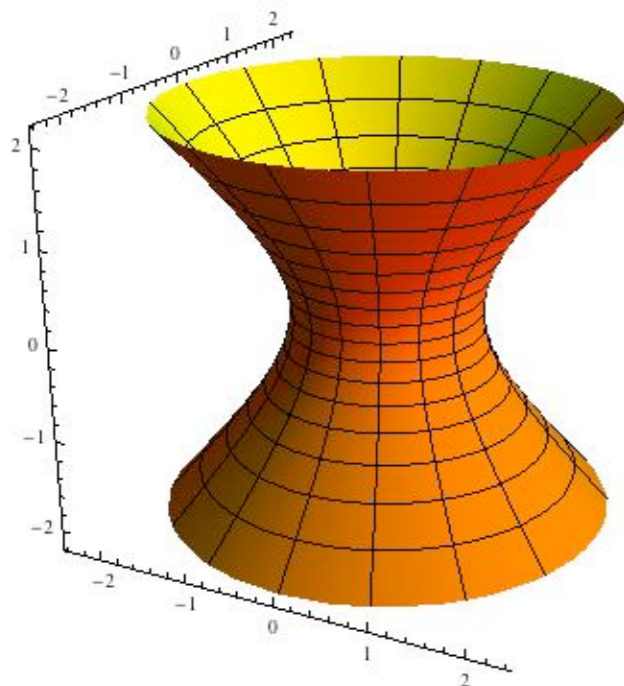
All Horizons are the Same

There is **no difference** between so-called observer-dependent horizons, like cosmological and acceleration horizons, and “true” black hole horizons.

In fact, Hawking and de Sitter radiation can both be viewed as Unruh radiation in higher-dimensional Minkowski space.

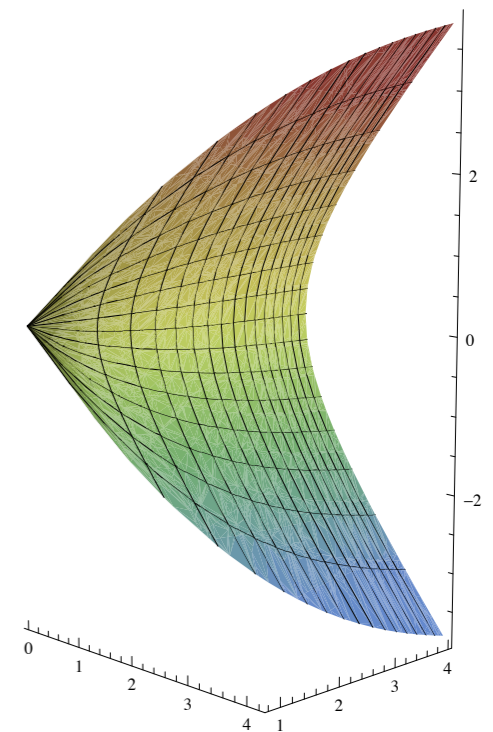
De Sitter space

$$-T^2 + X_1^2 + \dots + X_4^2 = +L^2$$



Schwarzschild black hole

$$\begin{aligned} T &= 4M \sqrt{1 - 2M/r} \sinh(t/4M) \\ X_1 &= 4M \sqrt{1 - 2M/r} \cosh(t/4M) \\ X_2 &= dr \sqrt{(2Mr^2 + 4M^2r + 8M^3)/r^3} \\ X_3 &= r \sin \theta \sin \phi \\ X_4 &= r \sin \theta \cos \phi \\ X_5 &= r \cos \theta \end{aligned}$$



A Puzzling Aspect of Black Hole Mechanics

Bardeen, Carter and Hawking (1973) proved that classical black holes obey four mechanical laws that are analogous to the four laws of thermodynamics:

0) at equilibrium, the surface gravity, κ , is constant over the horizon

$$1) dM = \frac{\kappa}{2\pi} d\left(\frac{A}{4G}\right) - \Omega dJ$$

2) horizon area always increases: $dA \geq 0$

3) in no physical process can κ go to zero

These analogies turned into identities once Hawking radiation was discovered.

But ... how did the classical laws “know” about temperature and entropy?

$$T_H = \frac{\hbar\kappa}{2\pi} \qquad S_{BH} = \frac{A}{4G\hbar}$$

The Einstein Equation of State

Jacobson (1995) made a bold attempt to solve this puzzle.

By assuming, rather than deriving the thermodynamic properties of black holes, and then applying them to local Rindler horizons, classical gravitational equations remarkably appear as a consequence of thermodynamic equations.

$$dQ = TdS \Rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The Einstein equation thus re-appeared as an equation of state!

General Theories of Gravity

Consider an arbitrary relativistic diffeomorphism-invariant theory of gravity:

$$I = \frac{1}{16\pi} \int d^D x \sqrt{-g} L(g_{ab}, R_{abcd}) + I_{\text{matter}}$$

Define

$$P^{abcd} = \frac{\partial L}{\partial R_{abcd}}$$

Then the equation of motion of classical gravity is

$$P_a{}^{cde} R_{bcde} - 2\nabla^c \nabla^d P_{acdb} - \frac{1}{2} L g_{ab} = 8\pi T_{ab}$$

For example, when $L = f(R)$ we have $P^{abcd} = \frac{1}{2} f'(R) (g^{ac} g^{bd} - g^{ad} g^{bc})$ and thus

$$f'(R) R_{ab} - \nabla_a \nabla_b f'(R) + \left(\square f'(R) - \frac{1}{2} f(R) \right) g_{ab} = 8\pi T_{ab}$$

Wald Entropy

Each of these different theories of gravity has its own black hole solutions with their own entropy formulas.

f(R) gravity: $\mathcal{S}_f = f'(R) \frac{A}{4}$

Gauss-Bonnet gravity: $\mathcal{S}_{\text{G-B}} = \frac{1}{2} \int d^{D-2}x \sqrt{\sigma} R_{(D-2)}$

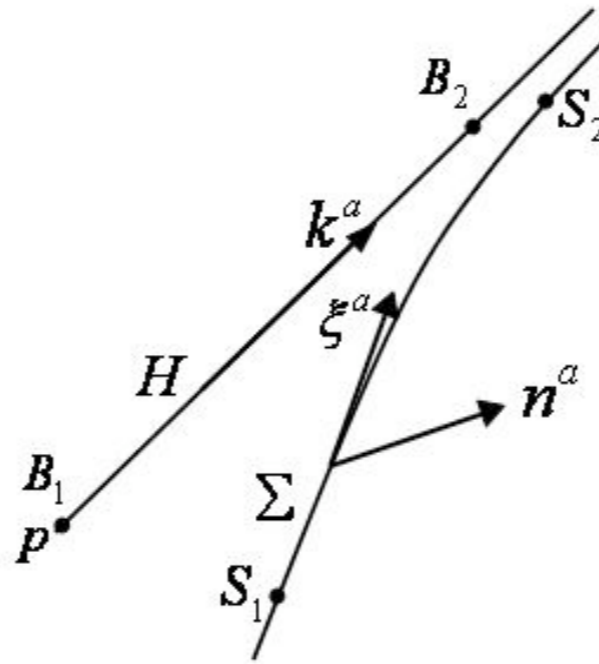
The general formula was given by Wald: $\mathcal{S} = \frac{1}{8\kappa} \int_S dS_{ab} J^{ab}$

where $J^{ab} = -2P^{abcd} \nabla_c \xi_d + 4\xi_d (\nabla_c P^{abcd})$ is the “Noether potential”

for a time-like Killing vector ξ^a .

Local Acceleration Horizons

Take an arbitrary point \mathbf{p} in an arbitrary spacetime and choose a null vector k^a :



Near \mathbf{p} the metric can be written as locally flat: $g_{ab} = \eta_{ab} - \frac{1}{3}R_{acbd}x^c x^d + \dots$

In this patch, there is an approximate boost Killing vector $\xi^a = x\partial_t^a + t\partial_x^a$

which asymptotes to the chosen k^a and which defines a stretched horizon, Σ .

Variation of Wald Entropy

We want to derive $P_a{}^{cde}R_{bcde} - 2\nabla^c\nabla^d P_{acdb} - \frac{1}{2}Lg_{ab} = 8\pi T_{ab}$ from $dQ = TdS$

Assume Wald entropy formula for local Rindler stretched horizon:

$$\mathcal{S} = \frac{1}{8\kappa} \int_S dS_{ab} J^{ab} = -\frac{1}{4\kappa} \int_{S(\tau)} dS_{ab} (P^{abcd} \nabla_c \xi_d - 2\xi_d \nabla_c P^{abcd})$$

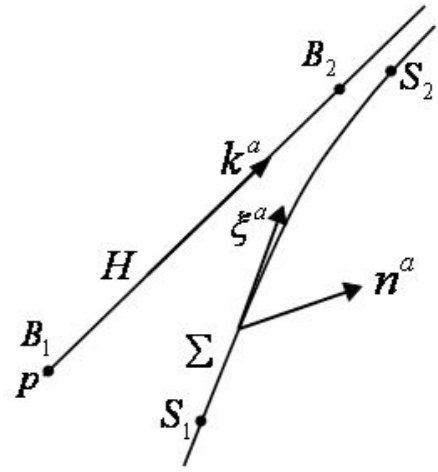
Then

$$\delta\mathcal{S} = \mathcal{S}(\tau_2) - \mathcal{S}(\tau_1) = +\frac{1}{4\kappa} \int_{\Sigma} d\Sigma_a \nabla_b (P^{abcd} \nabla_c \xi_d - 2\xi_d \nabla_c P^{abcd})$$

$$\delta\mathcal{S} = \frac{1}{4\kappa} \int_{\Sigma} [-\nabla_b (P^{adbc} + P^{acbd}) \nabla_c \xi_d + P^{abcd} \nabla_b \nabla_c \xi_d - 2\xi_d \nabla_b \nabla_c P^{abcd}] d\Sigma_a$$

$$T_{\text{local}} = \frac{\kappa}{2\pi\alpha}$$

$$T_{\text{local}} \delta\mathcal{S} = \frac{1}{8\pi\alpha} \int_{\Sigma} (P^{abcd} R_{dcbe} \xi^e - 2\xi_d \nabla_b \nabla_c P^{abcd}) n_a d\tau dA$$



The Thermodynamics of Spacetime

Heat flow $\delta Q = + \int_{\Sigma} d\Sigma_a T_e^a u^e = \frac{1}{\alpha} \int_{\Sigma} dA d\tau n_a T_e^a \xi^e$

Equate to $T_{\text{local}} \delta S = \frac{1}{8\pi\alpha} \int_{\Sigma} (P^{abcd} R_{dcbe} \xi^e - 2\xi_d \nabla_b \nabla_c P^{abcd}) n_a d\tau dA$

Take null limit $(P_a{}^{cde} R_{bcde} - 2\nabla^c \nabla^d P_{acdb}) k^a k^b = 8\pi T_{ab} k^a k^b$

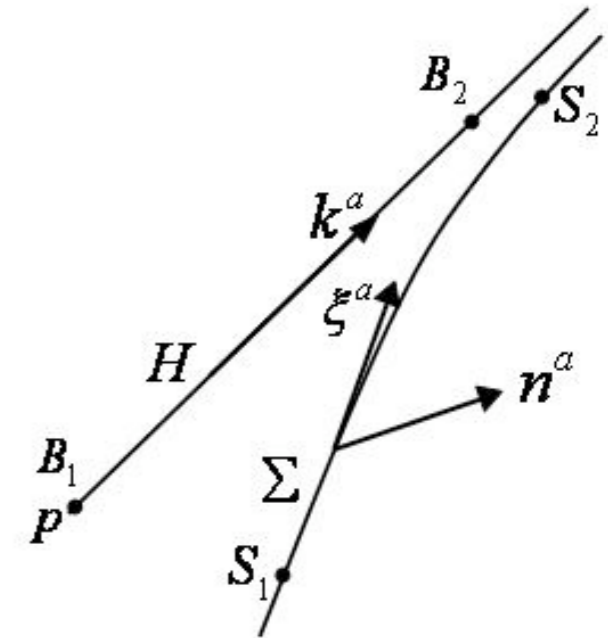
This holds for any choice of null vector k^a . Hence

$$P_a{}^{cde} R_{bcde} - 2\nabla^c \nabla^d P_{acdb} + \varphi g_{ab} = 8\pi T_{ab}$$

Imposing the Bianchi identities gives

$$P_a{}^{cde} R_{bcde} - 2\nabla^c \nabla^d P_{acdb} - \frac{1}{2} L g_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$$

The equations of gravity appear as a consequence of $dQ = TdS$!



The “Atoms” of Spacetime?

Does gravity arise from the thermodynamics of the “atoms” of spacetime?

Speculation: In string theory, we can calculate black hole entropy in a dual picture which has no manifest spacetime structure to it.

Using the entropy obtained from microstate counting of these “atoms” of spacetime, we can apply the Clausius relation locally, thereby obtaining the gravitational equations of motion.