

Geometric structure of the standard model and gravity

$$\begin{aligned}
 S_D &= \int d^4_{\sim}x |e| \{ \bar{\psi} \gamma^a (e_a)^\mu (\partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} + A_\mu^B T_B) \psi + \bar{\psi} \phi \psi \} \\
 &= \int d^4_{\sim}x |e| \{ \bar{\psi} \gamma^a (e_a)^\mu (\partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} + \frac{1}{2} A_\mu^{mn} \gamma_{mn} + \frac{1}{4} (e_\mu)^c \phi^m \gamma_{cm}) \psi \} \\
 &= \int \underline{e} \{ \bar{\psi} \underline{e} (\underline{d} + \underline{H}) \psi \} = \int \frac{1}{4!} \bar{\psi} \underline{e} \underline{e} \underline{e} \underline{e} \underline{D} \psi = \int \underline{e} \bar{\psi} \underline{D} \psi
 \end{aligned}$$

Gravity: $\underline{\omega} = \frac{1}{2} d\underline{x}^\mu \omega_\mu^{ab} \gamma_{ab} \in Cl(1,3)^2 = spin(1,3) \quad \underline{e} = d\underline{x}^\mu (e_\mu)^c \gamma_c \in Cl(1,3)^1 = 4$

GUT: $\underline{A} \in su(2)_L + u(1)_Y + su(3) \subset su(2)_L + su(2)_R + su(4) = spin(4) + spin(6) \subset spin(10)$

Fermions: $\psi \in 2 \times (2_L + 2_R) \times (1 + 3) = \mathbb{C}^{32} = \mathbb{R}^{64} \quad (\times 3)$

Higgs: $\phi = \phi^m \gamma_m \in Cl(4)^1 = 4 = \mathbb{C}^2 \text{ or } Cl(N)^1 = N$

ToE: $\underline{H} = \frac{1}{2} \underline{\omega} + \frac{1}{4} \underline{e} \phi + \underline{A} \in spin(1,3) + 4 \times 4 + spin(4) + spin(6)$

Curvature: $\underline{F} = \underline{dH} + \underline{H}\underline{H} = \frac{1}{2} (\underline{R} - \frac{1}{8} \underline{e} \underline{e} \phi^2) + \frac{1}{4} (\underline{T} \phi - \underline{e} \underline{D} \phi) + \underline{F}^A$

Superconnection: $\underline{A} = \underline{H} + \psi \in spin(1,7) + spin(6) + 64 \subset E8(8) = spin(1,7) + spin(7,1) + 8_v \times 8_v + 8_+ \times 8_+ + 8_- \times 8_-$

Supercurvature: $\underline{F} = \underline{dA} + \underline{A}\underline{A} = \underline{F} + \underline{D}\psi + \psi\psi$

$$S = \int \left\{ \underline{\bar{B}} \underline{F} - \frac{\pi G}{4} \underline{B} \underline{e} \underline{B} + g^2 \underline{B}' * \underline{B}' \right\} = \int \left\{ \underline{e} \bar{\psi} \underline{D} \psi + \frac{1}{16\pi G} \underline{e} (R - \frac{3}{2} \phi^2) \phi^2 + \frac{1}{4g^2} \underline{F}' * \underline{F}' \right\}$$