Antigravitation

Sabine Hossenfelder

Perimeter Institute

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

SH, Phys.Rev.D 78:044015 (2008), arXiv:0807.2838v1

What do I mean with Anti-gravitating matter

- A second copy of the standard model, identical to the one we know, except for its graviational interaction.
- Both sorts of particles interact only graviationally.
- In particular, anti-matter has completely normal graviational properties.
- Disclaimer: This talk is unquantized.

How do we get the funny stuff to move differently?

- Covariant curves are defined via a connection. Yet this connection is unique only after requiring it to be torsion-free and metric-compatible.
- Need second derivative, throw out metric-compatibility.
- Introduce instead second metric $\underline{\mathbf{h}}$ with which the second connection ${}^{(h)}\underline{\nabla}$ is compatible.

$$\begin{array}{lcll} {}^{(g)}\nabla, {}^{(g)}\mathcal{R} & \text{with} & {}^{(g)}\nabla\mathbf{g} & = & 0 \\ {}^{(\underline{h})}\underline{\nabla}, {}^{(\underline{h})}\mathcal{R} & \text{with} & {}^{(\underline{h})}\underline{\nabla}\underline{\mathbf{h}} & = & 0 \end{array}$$

The Pullovers

- 2nd metric provides another interpretation of the manifold (same manifold, different distance measures) and results in different curves for particles. Its local coordinate basis doesn't normally coincide with ours.
- Two sorts of indices: ν raised/lowered with $\mathbf{g}$, $\underline{\nu}$ raised/lowered with $\underline{\mathbf{h}}$
- Pullovers to identify h-fields with observables for g-observer and vice versa: $P_g, P_{\underline{h}}$. Locally linear maps on tensor algebras.
- $\underline{\mathbf{h}}$ is then related to a two-tensor $\mathbf{h} = P_{\underline{h}}(\underline{\mathbf{h}})$ and vice versa $\underline{\mathbf{g}} = P_g(\mathbf{g})$.
- Induces pulled-over derivatives by metric-compatibility:

$$\begin{aligned} P_{\underline{h}}\left({}^{(\underline{h})}\underline{\nabla}\mathbf{A}\right) &= {}^{(h)}\nabla P_{\underline{h}}(\mathbf{A}) \\ P_g\left({}^{(g)}\nabla\mathbf{A}\right) &= {}^{(\underline{g})}\underline{\nabla} P_g(\mathbf{A}) \end{aligned}$$

Equations of Motion for Matter Fields

- Action for field

$$S = \int d^4x \sqrt{-h}\; P_{\underline{h}}\left(h^{\underline{\nu\kappa}}\; {}^{(\underline{h})}\underline{\nabla}_{\underline{\kappa}}\underline{\phi}\, {}^{(\underline{h})}\underline{\nabla}_{\underline{\nu}}\underline{\phi}\right)$$

- Leads to

$$P_{\underline{h}}\left({}^{(\underline{h})}\underline{\nabla}^{\underline{\alpha}}\, {}^{(\underline{h})}\underline{\nabla}_{\underline{\alpha}}\underline{\phi}\right) = 0$$

Same as

$${}^{(h)}\nabla^{\underline{\alpha}}\, {}^{(h)}\nabla_{\underline{\alpha}} P_{\underline{h}}(\underline{\phi}) = 0$$

Same as

$${}^{(\underline{h})}\underline{\nabla}^{\underline{\alpha}}\, {}^{(\underline{h})}\underline{\nabla}_{\underline{\alpha}}\underline{\phi} = 0$$

How to determine the second metric?

- For convenience

$$\begin{aligned} g_{\varepsilon\lambda} &= a_{\varepsilon}{}^{\nu} a_{\lambda}{}^{\kappa}\, h_{\nu\kappa} \quad , \quad a_{\underline{\varepsilon}}{}^{\underline{\nu}} = [P_g]^{\varepsilon}{}_{\underline{\varepsilon}}\, a_{\varepsilon}{}^{\nu}\, [P_{\underline{h}}]^{\underline{\nu}}{}_{\nu} \\ g_{\underline{\varepsilon\lambda}} &= a_{\underline{\varepsilon}}{}^{\underline{\nu}} a_{\underline{\lambda}}{}^{\underline{\kappa}}\, h_{\underline{\nu\kappa}} \quad , \quad g_{\varepsilon\lambda} = a_{\varepsilon}{}^{\underline{\nu}} a_{\lambda\underline{\nu}} \; , \; h_{\underline{\nu\kappa}} = a^{\varepsilon}{}_{\underline{\nu}} a_{\varepsilon\underline{\kappa}} \end{aligned}$$

- The a's are not independent: $\delta a^{\nu\kappa} = \delta a^{\underline{\nu\kappa}} = 0$.
- Now use symmetry principle

$$\begin{aligned} {}^{(g)}R_{\kappa\nu} - \frac{1}{2} g_{\kappa\nu}{}^{(g)}R &= T_{\kappa\nu} - |P_{\underline{h}}| \sqrt{\frac{\underline{h}}{g}}\, a_{\nu}{}^{\underline{\nu}} a_{\kappa}{}^{\underline{\kappa}}\, \underline{T}_{\underline{\nu\kappa}} \\ {}^{(\underline{h})}R_{\underline{\nu\kappa}} - \frac{1}{2} h_{\underline{\nu\kappa}}{}^{(\underline{h})}R &= \underline{T}_{\underline{\nu\kappa}} - |P_g| \sqrt{\frac{g}{\underline{h}}}\, a^{\kappa}_{\underline{\kappa}} a^{\nu}_{\underline{\nu}}\, T_{\kappa\nu} \end{aligned}$$

with

$$T_{\mu\nu} = -\frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} + \frac{1}{2} g_{\mu\nu} \mathcal{L} \quad , \quad \underline{T}_{\underline{\nu\kappa}} = -\frac{1}{\sqrt{-\underline{h}}} \frac{\delta \underline{\mathcal{L}}}{\delta h^{\underline{\nu\kappa}}} + \frac{1}{2} h_{\underline{\nu\kappa}} \underline{\mathcal{L}}$$

- Degrees of freedom in pull-overs needed to fulfill Bianchi identities.

Action

- Full action

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left({}^{(g)}R/8\pi G + \mathcal{L}(\psi)\right) + \sqrt{-h}P_{\underline{h}}(\underline{\mathcal{L}}(\underline{\phi})) \\ &+ \int d^4x\sqrt{-\underline{h}}\left({}^{(\underline{h})}R/8\pi G + \underline{\mathcal{L}}(\underline{\phi})\right) + \sqrt{-\underline{g}}P_g(\mathcal{L}(\psi)) \quad , \end{aligned}$$

- Dynamical variables $\mathbf{g}$ and $\underline{\mathbf{h}}$, ψ and $\underline{\phi}$.
- Variation of $g_{\varepsilon\lambda}h_{\kappa\nu}a^{\varepsilon\kappa}a^{\mu\nu} = \delta^{\mu}{}_{\lambda}$ with $\delta a^{\varepsilon\kappa} = 0$ yields

$$\delta h_{\kappa\lambda} = -[a^{-1}]^{\mu}{}_{\kappa}[a^{-1}]^{\nu}{}_{\lambda}\delta g_{\mu\nu}$$

- Note: There is no negative kinetic energy in the action. Variation over fields (for 'inertial' stress-energy) has no change of sign. Sign change only for source term of field equations.
- Both graviational AND inertial mass (energy) are conserved, thus no vacuum decay possible.