Emergence of non-commutative space-time from spinfoam model

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Group field theories for spinfoams

A **group field theory** (GFT) is a (often non-local) scalar field theory over a (product of) Lie group or coset spaces.

$$\mathcal{S}(\phi) = \int [dg]\phi(g_i)\mathcal{K}(g_i, g_j)\phi(g_j) - \frac{\lambda}{n!}\int [dg]\phi(g_i)...\phi(g_j))\mathcal{V}(g_i, ..., g_j) \qquad G^{\times N} \ni g_i.$$

- *A GFT is a generalization of matrix models.*

- A GFT is a **tool** to generate spinfoams: each Feynman diagram is a spinfoam, ie a 2-complex decorated by representations of G

Example: GFT for 3d gravity

$$\text{Let } \phi : G^{\times 3} \to \mathbb{R}, \text{ such that } \phi(g_1, g_2, g_3) = \int_G [d\tilde{g}]\phi(g_1\tilde{g}, g_2\tilde{g}, g_3\tilde{g}),$$

Field theory on product of groups

$$\phi : G \times G \to \mathbb{R}$$

$$\mathcal{K}(g_1, g_2, g_3, g_1', g_2', g_3') = \delta(g_1' g_3^{-1})\delta(g_2' g_2^{-1})\delta(g_3' g_1^{-1})$$

$$\mathcal{V}_{BF}(g_i)\phi^4(g_i) \equiv \phi(g_1, g_2, g_3)\phi(g_3, g_4, g_5)\phi(g_5, g_2, g_6)\phi(g_6, g_4, g_1)$$

When dealing with GFT for spinfoams, we have a field defined either on

- **A group G**
- **A coset G/H**

If this is a field theory, study it as a field theory!

- **Symmetry analysis : quantum groups**
- Canonical quantization
- **Renormalisation analysis**
- ....

Key idea: Interpret the group or the coset as **momentum space or Fourier space**

Some field theory with a curved Fourier space can be interpreted as a field theory with ***deformed Poincaré symmetries.***

Dually the field theory will live in a **flat non-commutative space-time**

- Fourier space is a group: *a curved manifold*. **For example G=SU(2)≈ S^3**

- Fourier mode ***p*** is identified as coordinates on the group

$$G \ni g = e^{ip\cdot X}$$

We will have many patches in general!

- ***Fourier mode addition is given by the group multiplication***:

$$g_1 g_2 = e^{ip_1\cdot X}\, e^{ip_2\cdot X} = e^{i(p_1\oplus p_2)\cdot X}$$

It will be non-commutative in general!

- Use the Haar measure [dp] to construction an action invariant under a quantum group

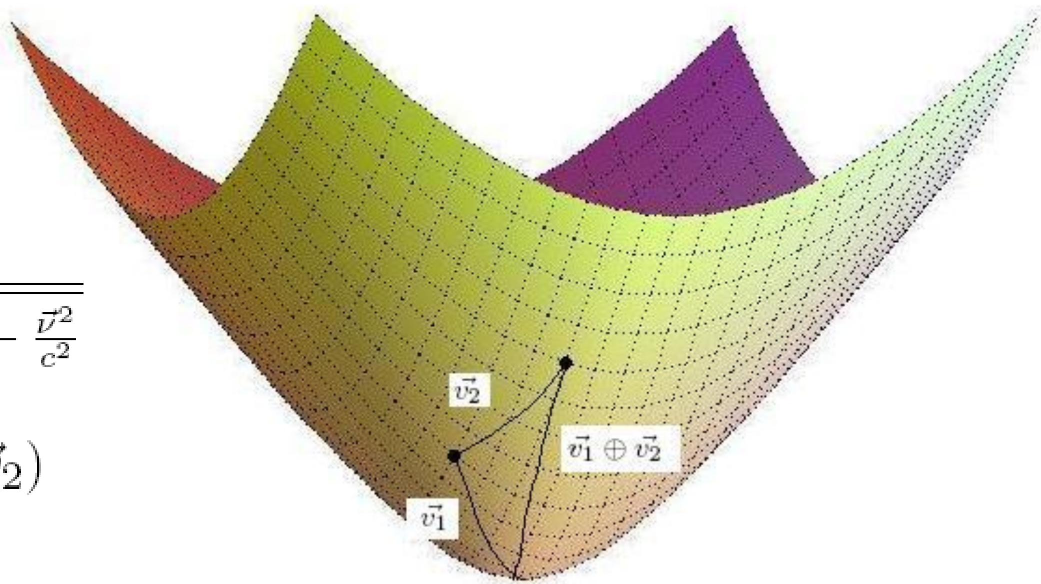
$$\mathcal{D}SU(2) \sim C(SU(2)) \rtimes k(SU(2))$$

$$\begin{aligned}\mathcal{S} \;&=\; \int_G [d^4p]\; (p^2 - m^2)\; \phi(p)\phi(\ominus p)\\ &\quad -\frac{\lambda}{4!}\int_{G^4}[d^4p]^4\, \delta(p_1 \oplus \ldots \oplus p_4)\, \phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)\end{aligned}$$

Same idea as in Special Relativity

v_μ : relativistic speed $\vec{\nu}$: 3d speed

$$\mathcal{H} = \{v_\mu \in \mathbb{R}^4 \mid -v_0^2 + \vec{v}^2 = -c^2\}$$



$$\vec{\nu} = \frac{\vec{v}}{v_0} = c \tanh \eta \, \vec{b} \qquad \gamma = v_0 = \frac{1}{\sqrt{1 - \frac{\vec{\nu}^2}{c^2}}}$$

$$e^{i\vec{v}_1 \cdot \vec{N}} e^{i\vec{v}_2 \cdot \vec{N}} = e^{i\overrightarrow{(v_1 \oplus_c v_2)} \cdot \vec{N}} R(\vec{v}_1, \vec{v}_2)$$

Thomas precession

$$\overrightarrow{\nu_1} \oplus \overrightarrow{\nu_2} = \frac{1}{1 + \frac{\overrightarrow{\nu_1}.\overrightarrow{\nu_2}}{c^2}} \left(\overrightarrow{\nu_1} + \frac{1}{\gamma_1}\overrightarrow{\nu_2} + \frac{1}{c^2}\frac{\gamma_1}{1+\gamma_1}(\overrightarrow{\nu_1} \cdot \overrightarrow{\nu_2})\overrightarrow{\nu_1} \right)$$

This addition is associative only when vectors are colinenar!

$$\overrightarrow{\nu} = \frac{\overrightarrow{\nu_1} + \overrightarrow{\nu_2}}{1 + \frac{1}{c^2}\overrightarrow{\nu_1}.\overrightarrow{\nu_2}}$$

We introduce the **plane-wave** and a **star product** between plane-waves to encode the non-trivial momentum addition

$$e^{ix\cdot p} = e^{ix\cdot p(a)}$$

$$e^{ix\cdot p_1} * e^{ix\cdot p_2} \equiv e^{ix\cdot(p_1 \oplus_c p_2)}$$

Using the measure on G, we define the Fourier transform

$$\hat{\phi}(x) \equiv \int [da]\, e^{ix\cdot p(a)} \phi(a)$$

$$x_i * x_j - x_j * x_i = \frac{1}{\kappa} C_{ij}^k x_k$$

Structure constants for Lie G

Using the Fourier transform, we have the scalar field action defined in the non-commutative space-time

$$S(\phi) = \int d^3x\, \partial^\mu \phi * \partial_\mu \phi(x) - m^2 \phi * \phi(x) - \frac{\lambda}{4!} \phi * \ldots * \phi(x)$$

For example, field theory on kappa Minkowski, ie field theory in **Deformed Special Relativity**

G=AN(3) ⟷ $[x_0, x_i] = -\frac{1}{\kappa} x_i \qquad [x_i, x_j] = 0$

Can we generate the non-commutative scalar field theory from the GFT behind Quantum Gravity models?

Yes, use intuition from Analog model for Gravity:

Non-commutative space-time is « emerging ».

Emergence of curved metric in Bose-Einstein Condensates

Atoms $\hat{\Psi}$ of mass m described by non-linear Schrodinger equation (non relativistic!):

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\hat{\Psi} - \mu\hat{\Psi} + \kappa|\hat{\Psi}|^2\hat{\Psi}.$$

Mean-field approximation:

$$\hat{\Psi} = \psi\mathbb{I} + \hat{\chi}, \text{ with } \langle\Omega|\hat{\Psi}|\Omega\rangle = \psi.$$

Analog model for gravity result:

The massless perturbations $\hat{\chi}$ (phonons), in some regime, are propagating in a Lorentzian metric *depending on the condensate* ψ.

$$ds^2 = \frac{n_c}{m c_s}\left[-\left(c_s^2 - \vec{v}^2\right)dt^2 - 2v_i\, dt dx^i + \delta_{ij}\, dx^i dx^j\right]$$

with

$$\psi = \sqrt{n_c}e^{i\theta}, \qquad c_s = \frac{\kappa n_c}{m}, \qquad \vec{v} = \frac{1}{m}\vec{\nabla}\theta.$$

Emerging a non-commutative space from a spinfoam GFT

$$S_{4d} = \frac{1}{2}\int [dg]^3\, \phi(g_1, g_2, g_3, g_4)\phi(g_4, g_3, g_2, g_1) - \frac{\lambda}{5!}\int [dg]^9 \phi(1,2,3,4)\phi(4,5,6,7)\phi(7,3,8,9)\phi(9,6,2,10)\phi(10,8,5,1)$$

G=SO(4,1)

- Look at the perturbations around the solution of the equation of motion

$$\phi = \phi_0 + \psi$$

- Identify the solution such that the perturbations ψ describe a scalar field theory on kappa Minkowski

$$\phi_0(g_i) = \sqrt[3]{\frac{4!}{\lambda}} \int_{SO(4,1)} dg\, \delta(g_1 g) F(g_2 g)\tilde{F}(g_3 g)\delta(g_4 g), \qquad \int F\tilde{F} = 1$$

$$\phi(g_1, g_2, g_3, g_4) = \phi_0(g_1, g_2, g_3, g_4) + \psi(g_1, g_4)$$

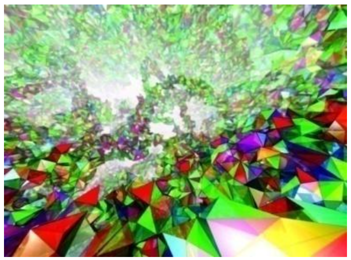
$$S_{eff}[\psi] = S(\phi_0 + \psi) - S(\phi_0)$$

ψ **lives on AN(3) , subgroup of SO(4,1)**

Girelli, Livine, Oriti arXiv: 0903.3475

Common (mathematical) framework between Quantum Gravity and Non-Commutative Geometry!

LOOP QUANTUM GRAVITY SPINFOAM ⟷ **NON-COMMUTATIVE GEOMETRY**

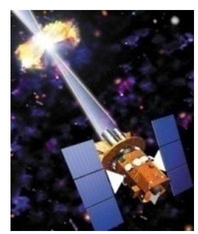


GROUP FIELD THEORY

Fuzzy spheres,

***Flat semi-classical space-time*:**
Kappa-Minkowski, Snyder…

PHENOMENOLOGY



Fermi/GLAST telescope…

Nice illustration of the concept of emergence of (non-commutative) space-time