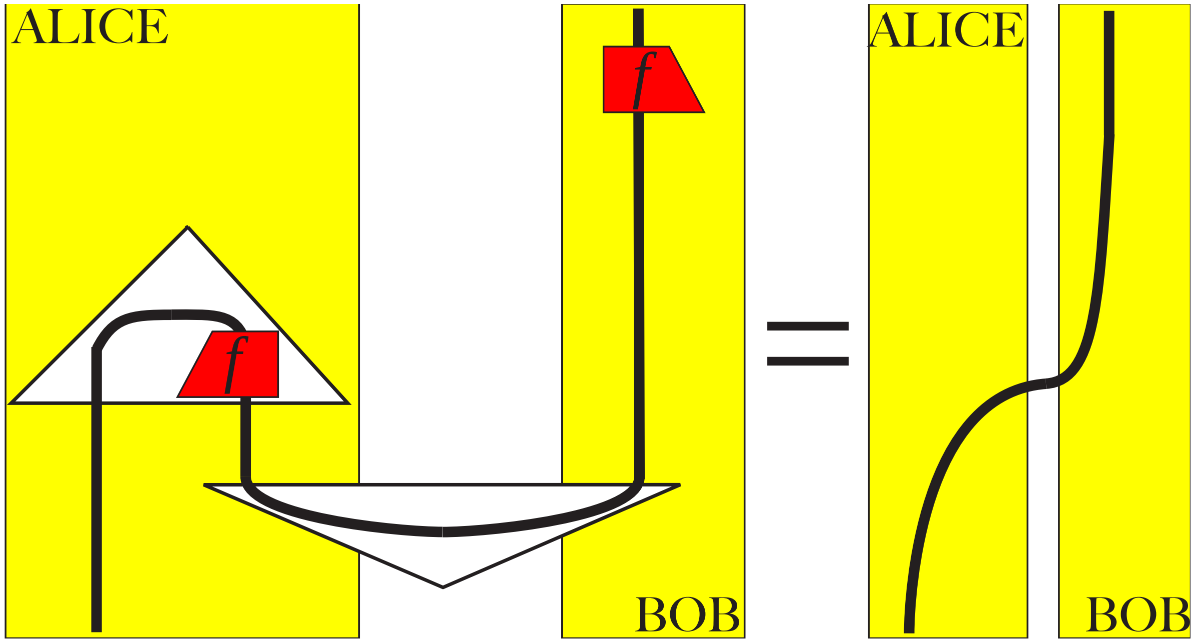
Quantum Picturalism

Bob Coecke – Oxford University



Funded by FQXi Large Grant: ***"The road to a new quantum formalism: Categories as a canvas for quantum foundations".***

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— *our new approach* —

Basic concept:

- Systems and processes
- Their composition: $-\circ-$ and $-\otimes-$

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- Monoidal categories + additional structure

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Beauty:

- Purely diagrammatic calculus

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Basic concept:

- Systems and processes
- Their composition: $- \circ -$ and $- \otimes -$

Algebra:

- Monoidal categories + additional structure

Beauty:

- Purely diagrammatic calculus

Application:

- Supports logic & automation e.g. `quantomatic`

— *data of a symmetric monoidal category* —

Systems:

$$A \qquad B \qquad C$$

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Systems:

$$A \qquad B \qquad C$$

Processes:

$$A \xrightarrow{f} A \qquad A \xrightarrow{g} B \qquad B \xrightarrow{h} C$$

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Compound systems:

$$A \otimes B \qquad \mathrm{I} \qquad A \otimes C \xrightarrow{f \otimes g} B \otimes D$$

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Compound systems:

$$A \otimes B \qquad \mathrm{I} \qquad A \otimes C \xrightarrow{f \otimes g} B \otimes D$$

Temporal composition:

$$A \xrightarrow{h \circ g} C \ := \ A \xrightarrow{g} B \xrightarrow{h} C \qquad A \xrightarrow{1_A} A$$

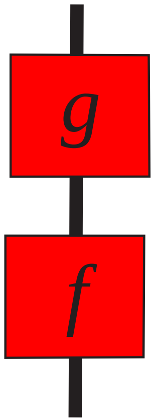
— *graphical formalism* —

$f \equiv$ 

$1_A \equiv$ 

— *graphical formalism* —

$$g \circ f \equiv$$



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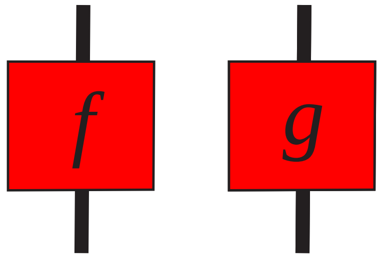
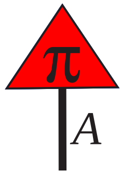
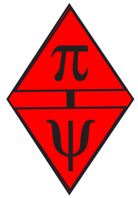


— *states, effects and quantities* —

$\psi : \mathrm{I} \to A$ $\qquad$ $\pi : A \to \mathrm{I}$ $\qquad$ $\pi \circ \psi : \mathrm{I} \to \mathrm{I}$





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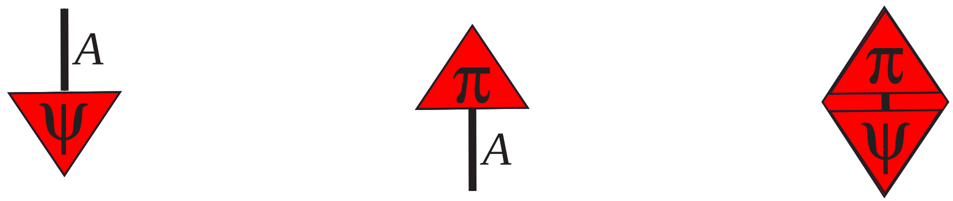

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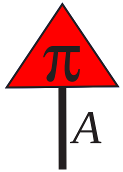

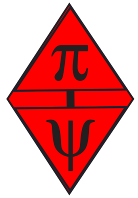

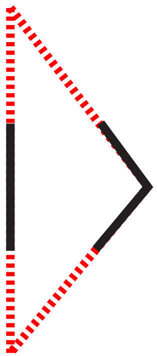

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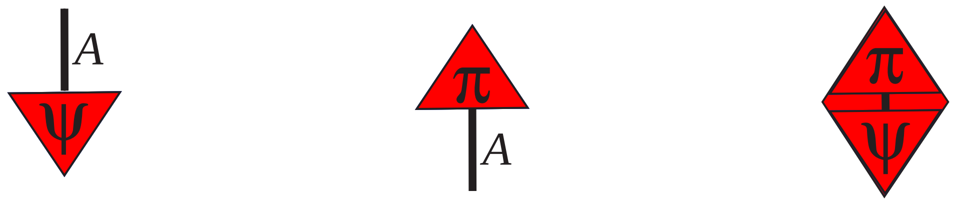

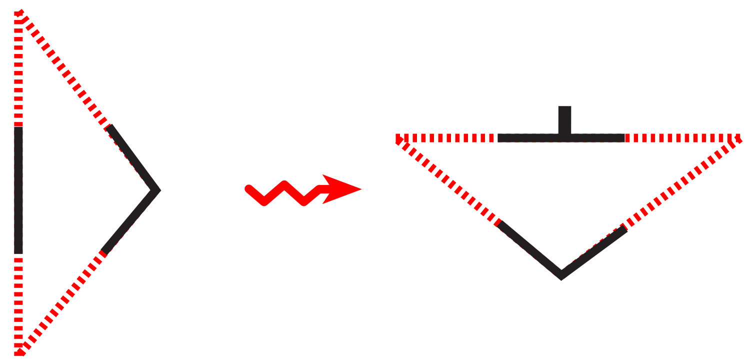

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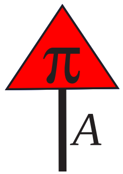

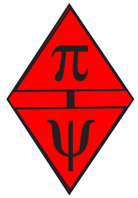

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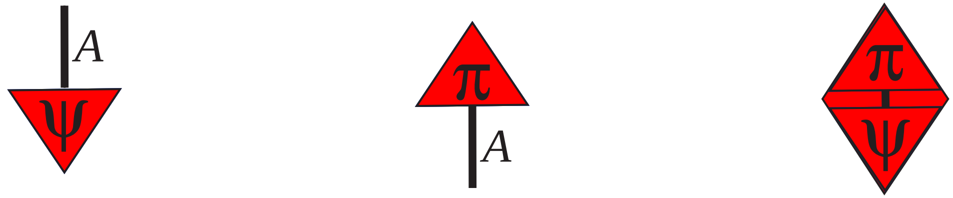

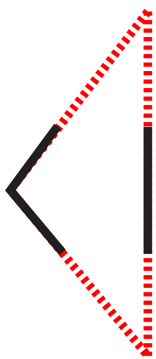

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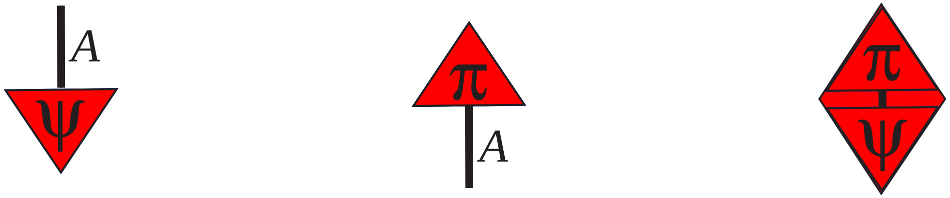

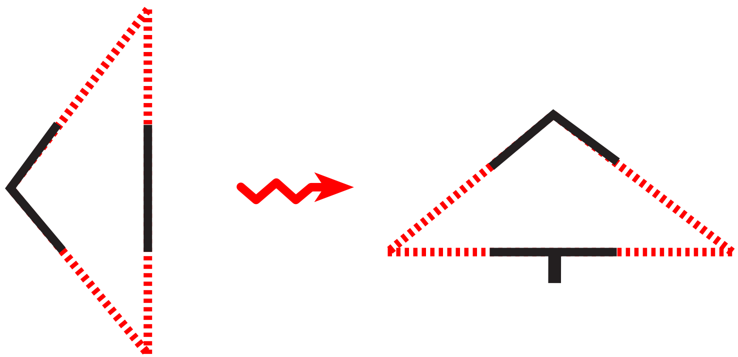

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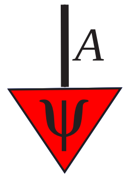

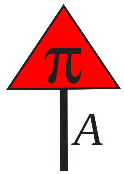

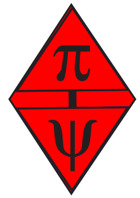

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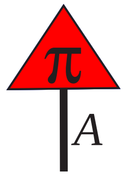

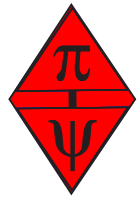

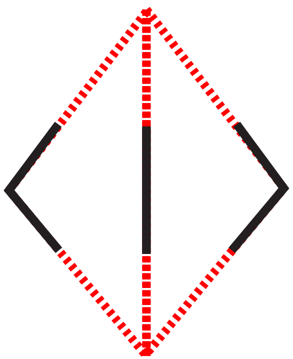

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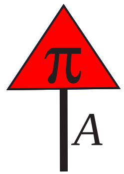
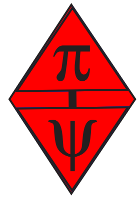
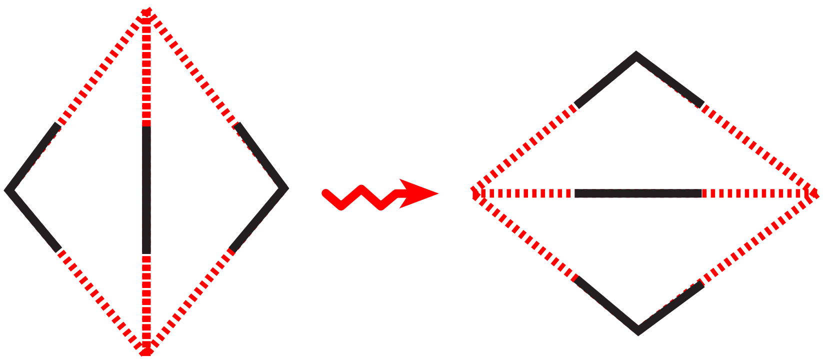


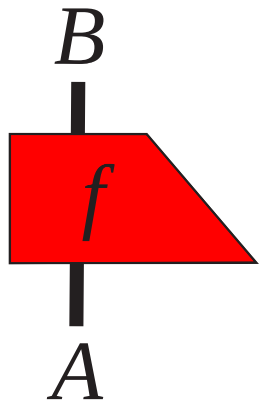




— *adjoint* $\Rightarrow$ *inner-product* $\Rightarrow$ *probabilities* —

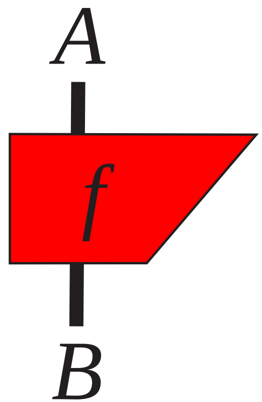
— adjoint ⇒ inner-product ⇒ probabilities —

$$f \; : A \longrightarrow B$$



— adjoint ⇒ inner-product ⇒ probabilities —

$$f^{\dagger} : B \longrightarrow A$$



— asserting entanglement: Bell structure —

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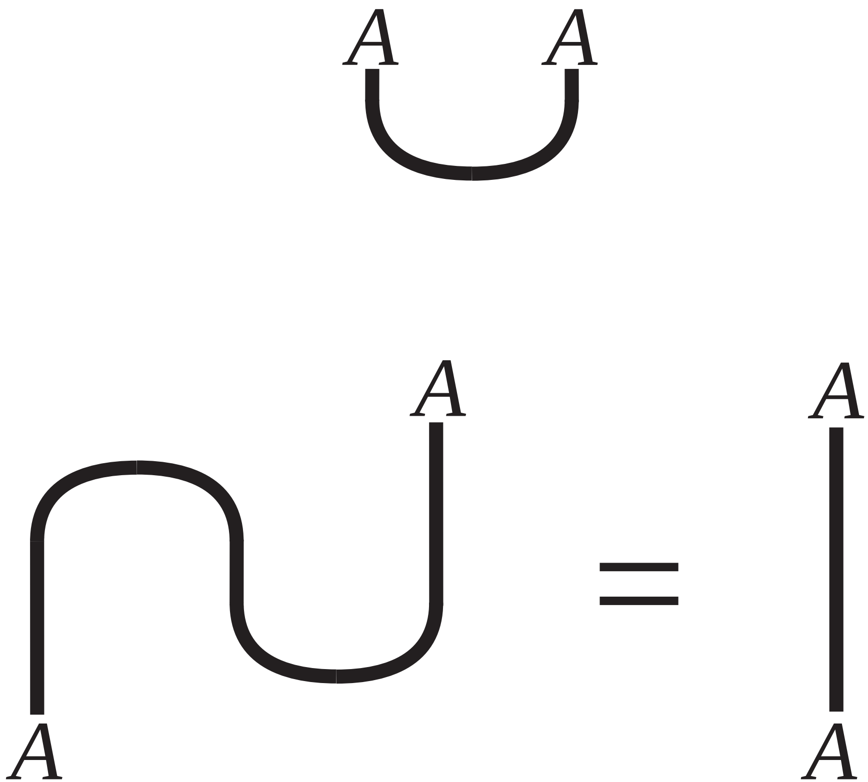
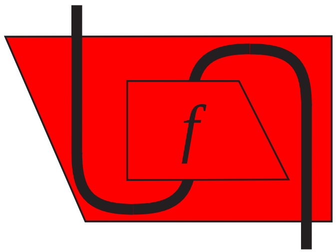


— *‘sliding’* —



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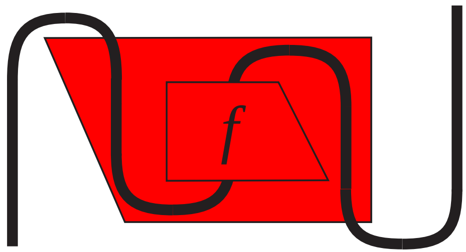

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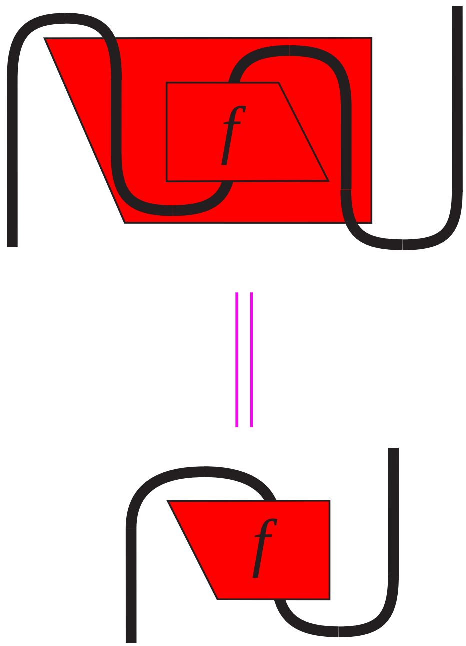

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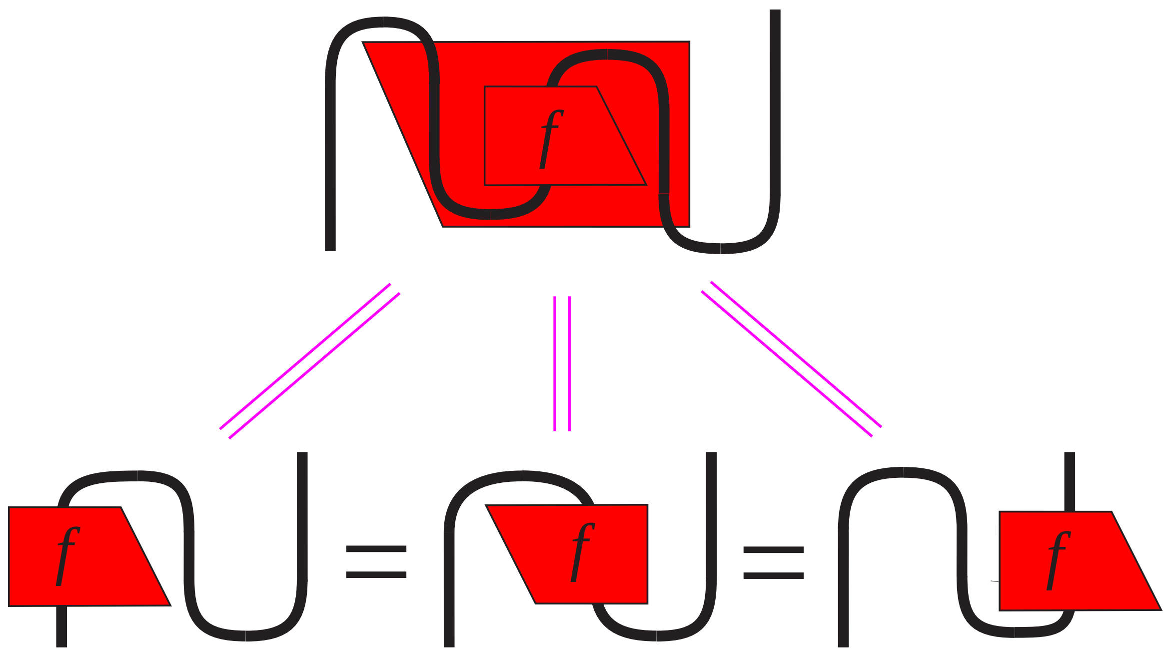

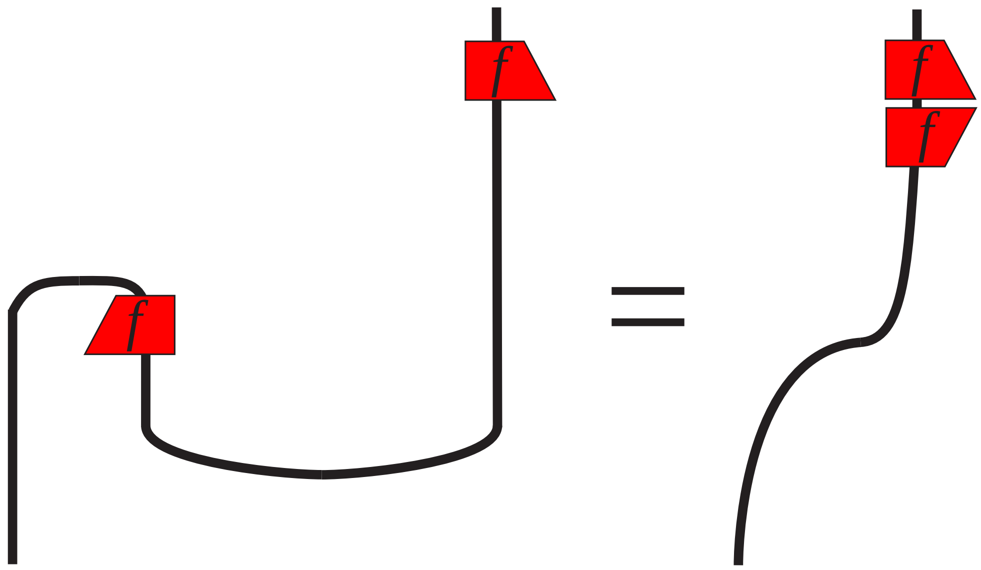
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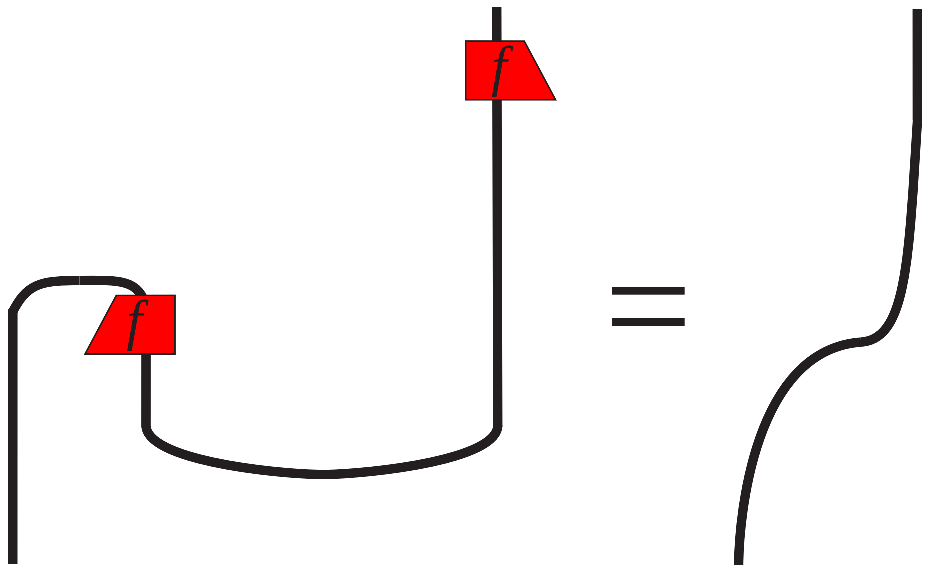
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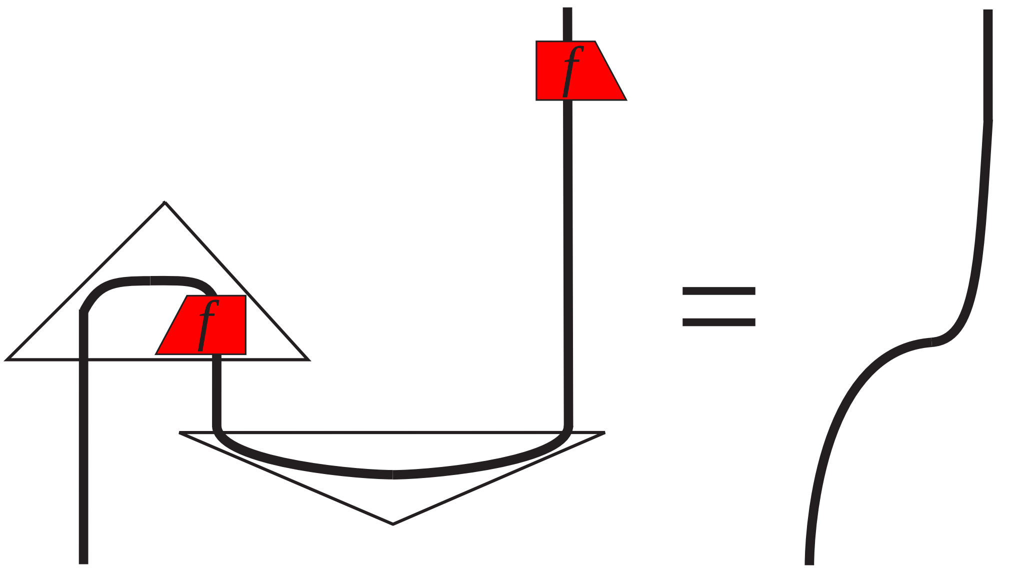

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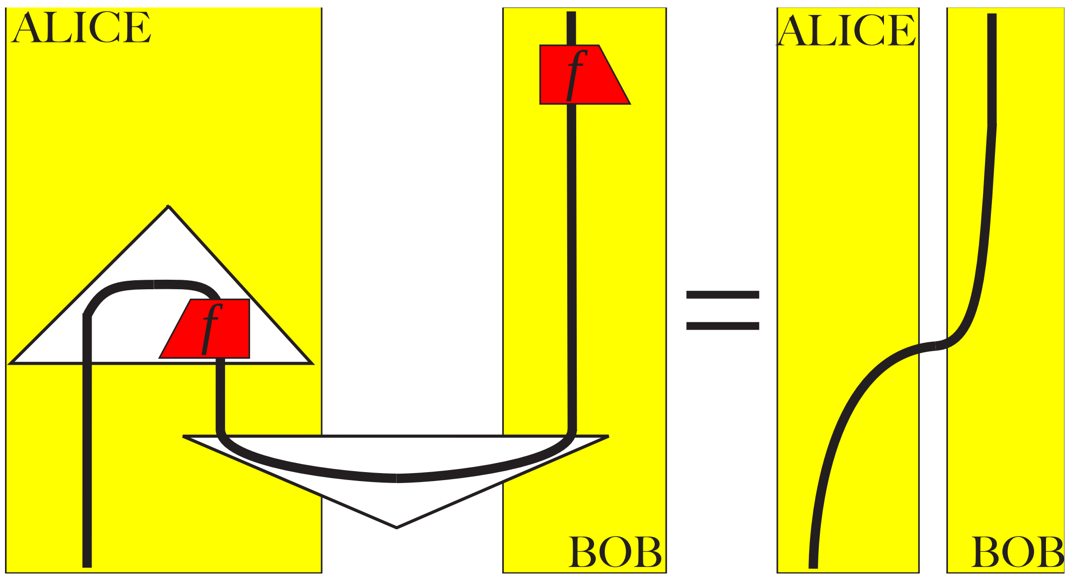
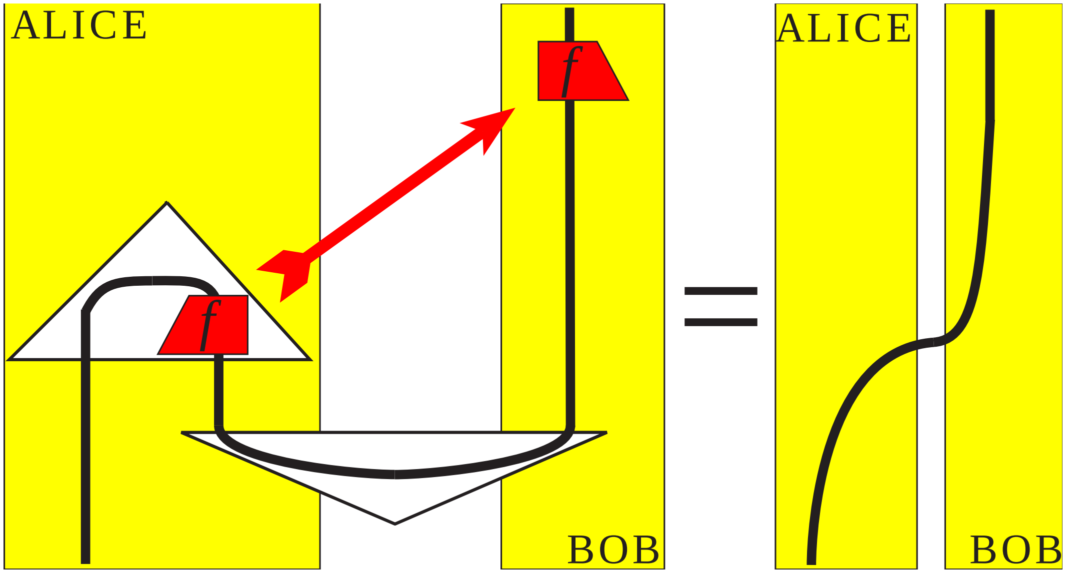


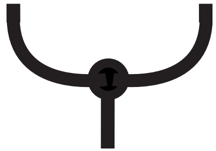
$\Rightarrow$ **quantum teleportation**

Classical data?



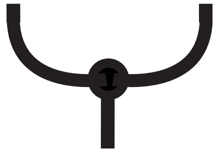
— ***observable*** **/** ***classical*** $:=$ ***copying*** **+** ***deleting*** —

— ***observable* / *classical*** := ***copying* + *deleting*** —

$A \xrightarrow{\delta} A \otimes A =$ 

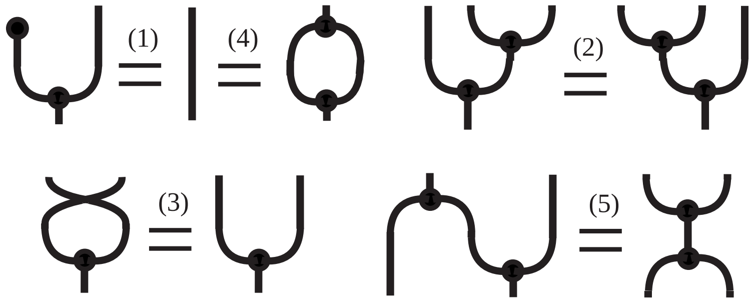
$A \xrightarrow{\epsilon} \mathrm{I} =$ 

— *observable* / *classical* := *copying* + *deleting* —

$$A \xrightarrow{\delta} A \otimes A =$$ 

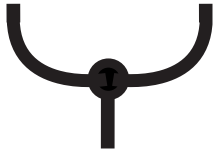
$$A \xrightarrow{\epsilon} \mathrm{I} =$$ 

such that:



$\Rightarrow$ it is a *commutative special †-Frobenius algebra*.

— *observable* / *classical* := *copying* + *deleting* —

$A \xrightarrow{\delta} A \otimes A =$  $A \xrightarrow{\epsilon} \mathrm{I} =$ 

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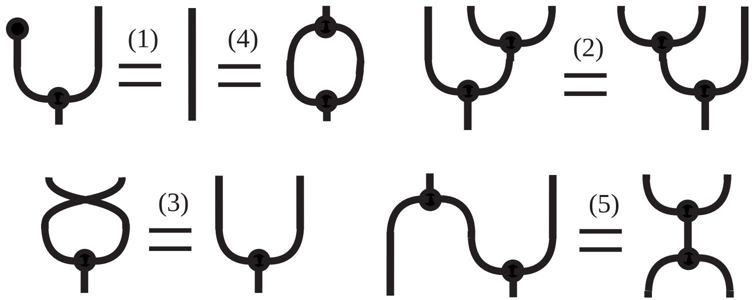
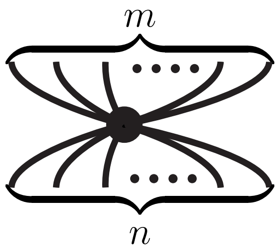


$\Rightarrow$ it is a *commutative special †-Frobenius algebra*.

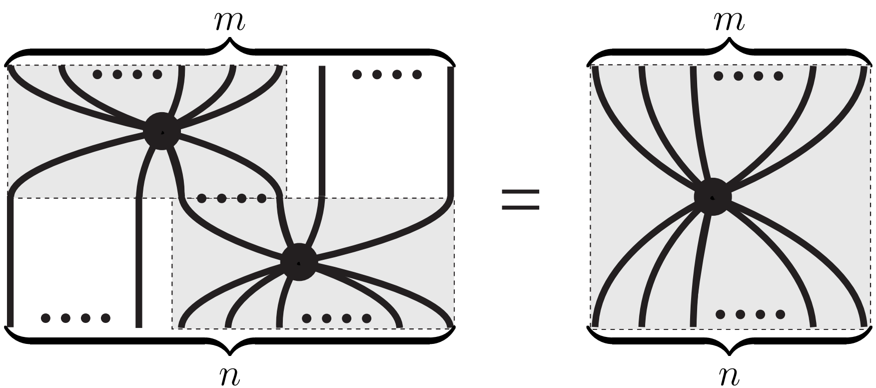
Thm. Observables in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.

— *observable* / *classical* := *copying* + *deleting* —

Diagrammatic calculus with 'spiders':

{  | $n, m \in \mathbb{N}$ }

which are such that:



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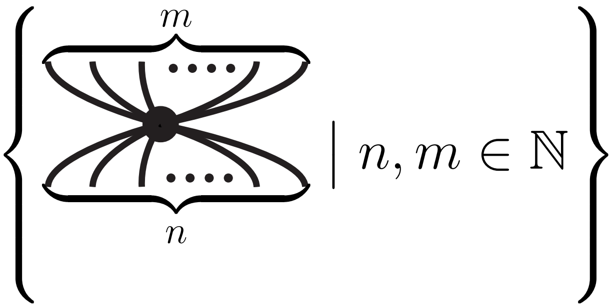

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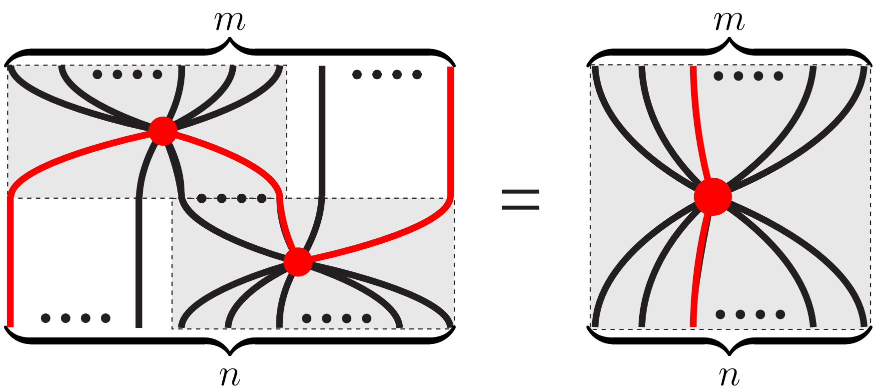
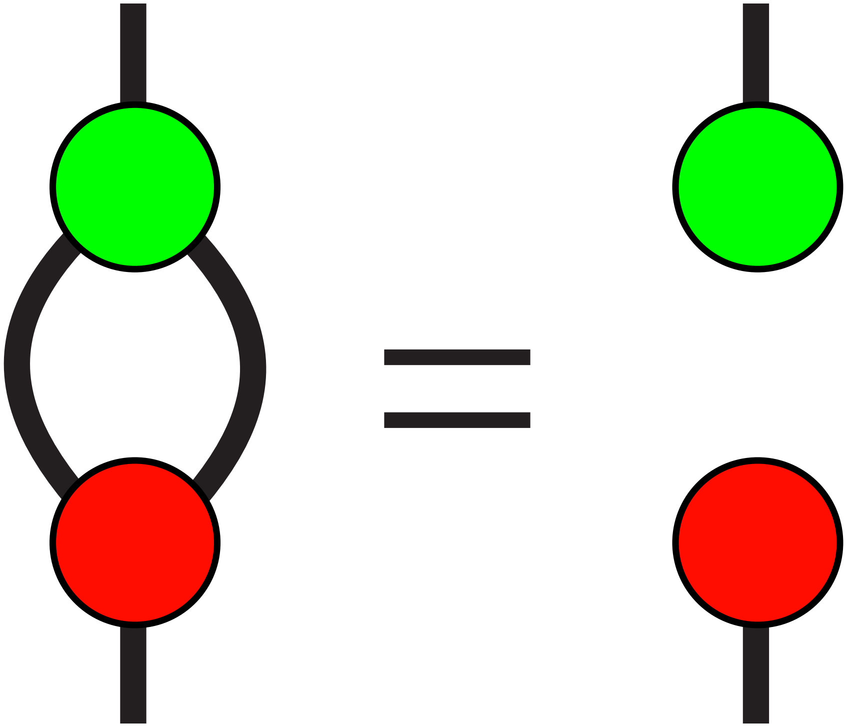


— *complementary observables* —

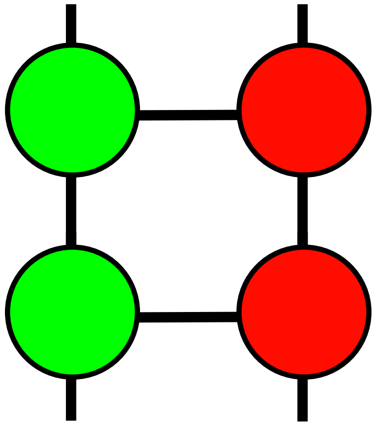
Thm. Observables are complementary if and only if:



— *two* **CX** *gates* —

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = ?$$

— *two* **CX** *gates* —



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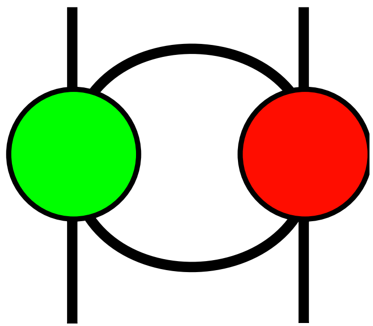

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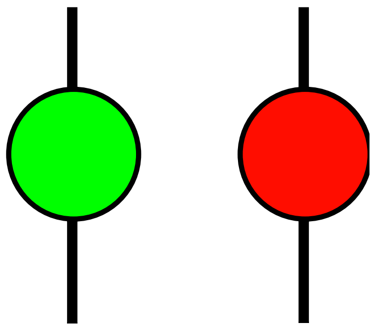
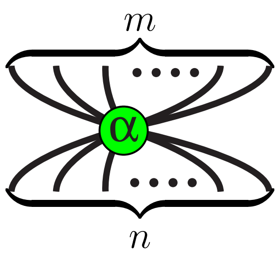


— *two* CX *gates* —

— *phases* *for observables ...* —

... result in 'decorated spiders':

{ 

| $n, m \in \mathbb{N}$ }

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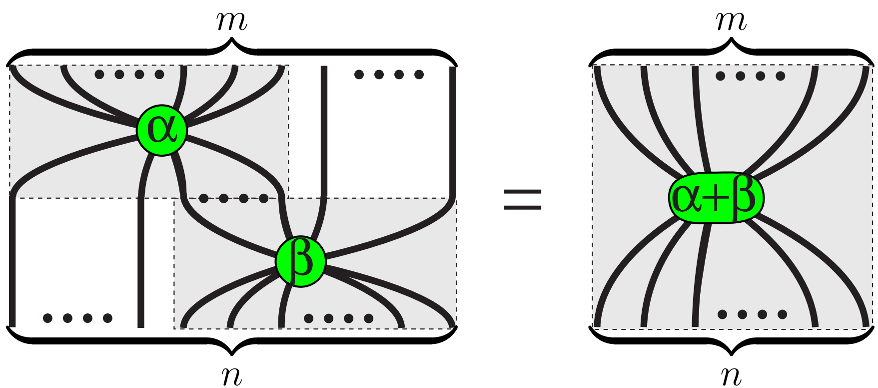
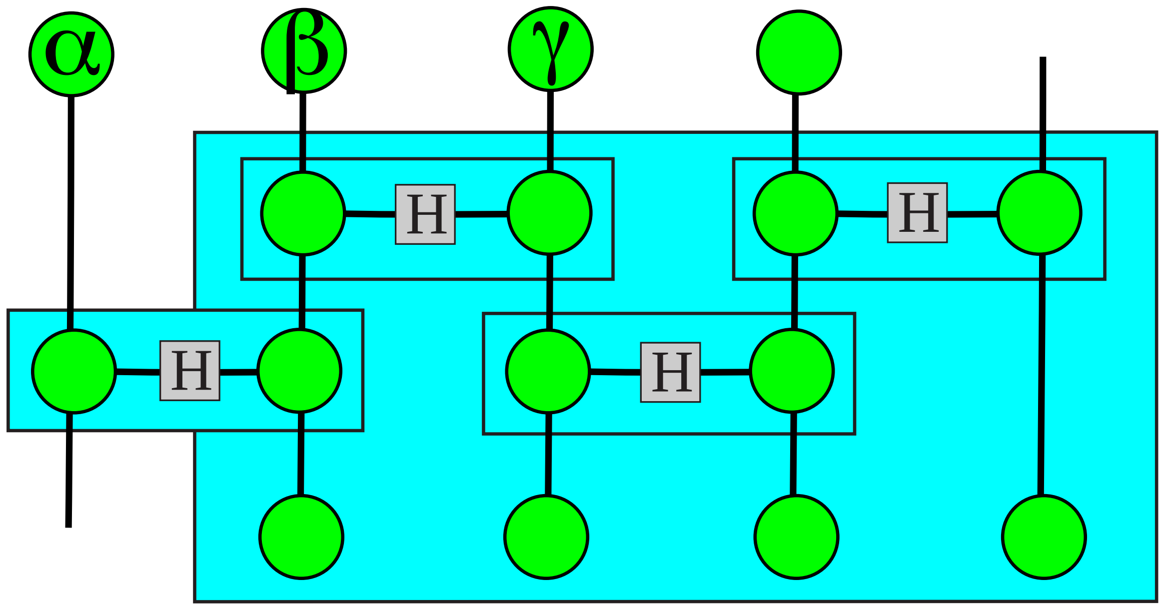


— *MBQC example* —



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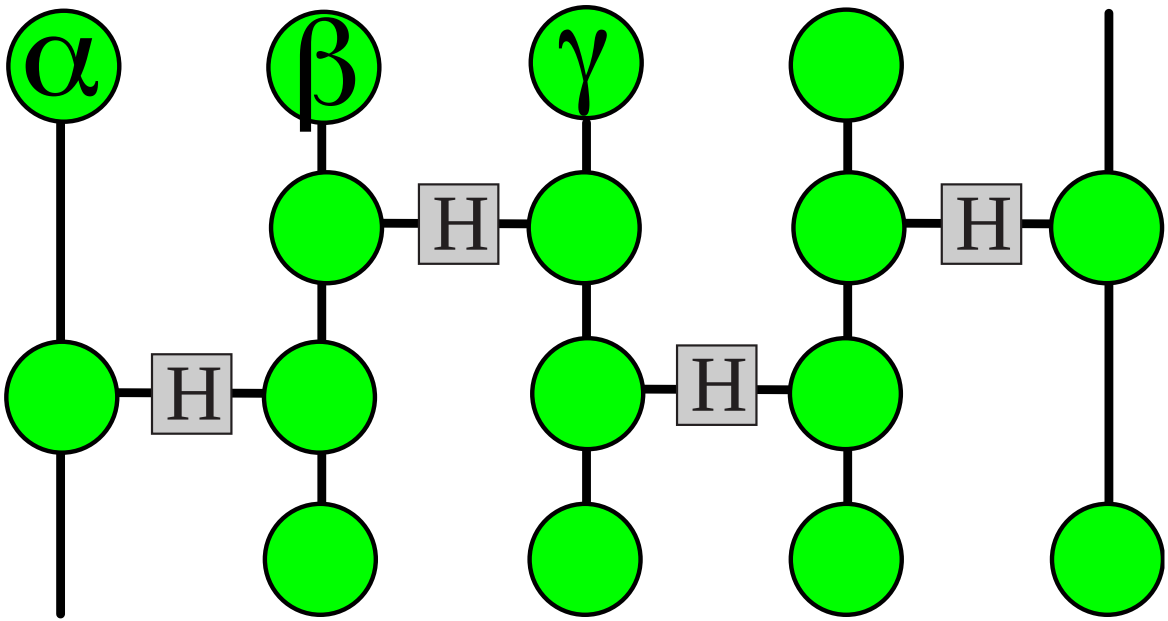
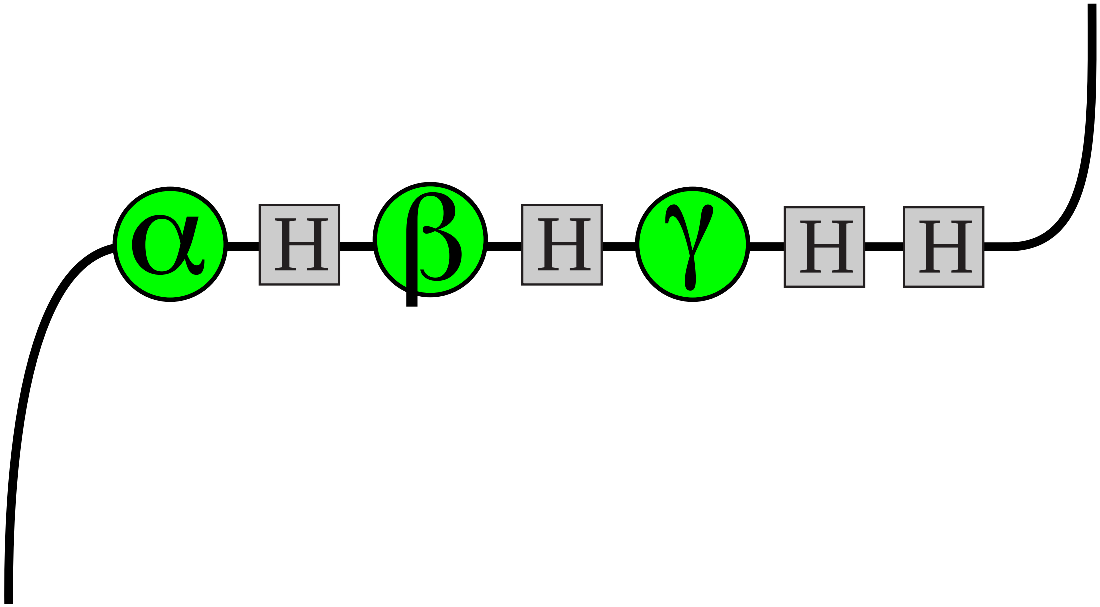
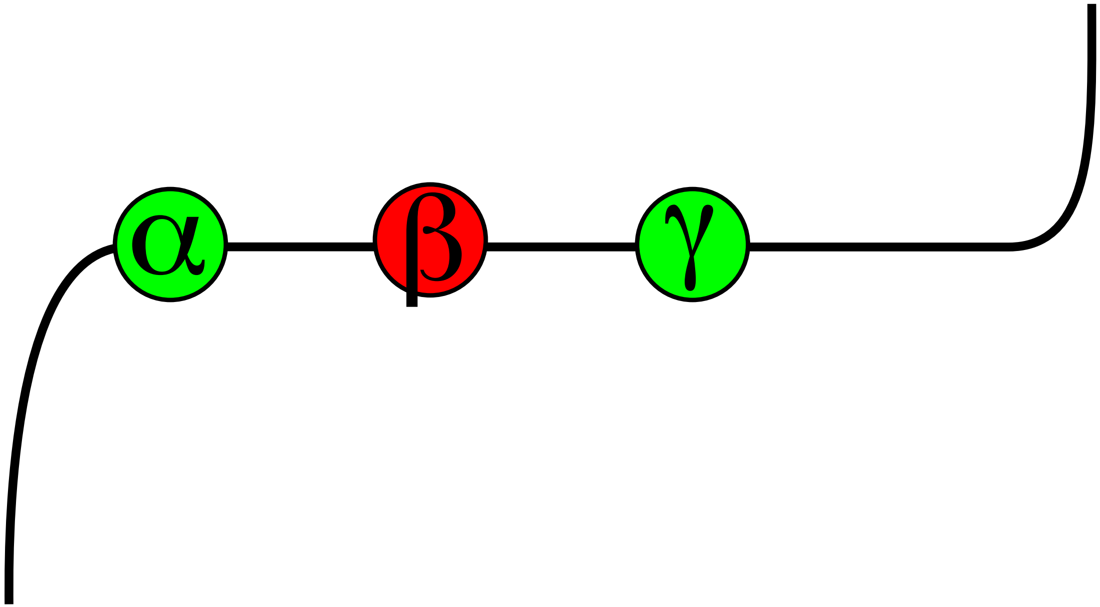


— *MBQC example* —



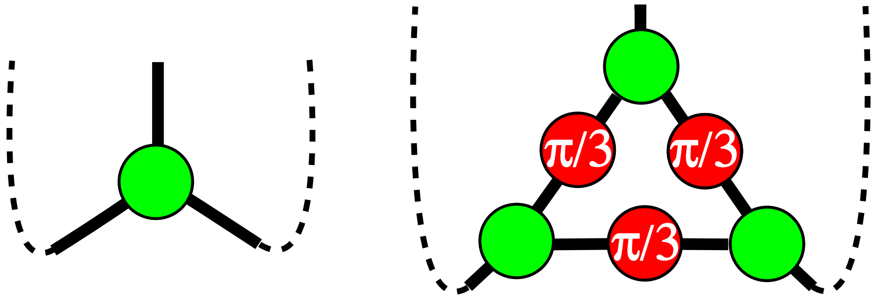
— MBQC example —



⇒ Euler decomposition of one-qubit unitary

Thm. Any linear map can be expressed in terms of a pair of complementary observables and their phases, i.e., red and green decorated spiders.

Example:



GHZ W

Diagrammatic QM introductions:

Appetizer: ***Kindergarten Quantum Mechanics.*** **arXiv:quant-ph/0510032.**

Survey: ***Quantum picturalism.*** **Soon on arXiv. (ask me for copy)**

Categories for physicists:

Appetizer: ***Introducing categories to the practicing physicist.*** **arXiv:0808.1032.**

Tutorial: ***Categories for the practicing physicist.*** **arXiv:0905.3010.**

Some technical papers:

C., Edwards and Spekkens: ***The group theoretic origin of non-locality for qubits.*** **web.comlab.ox.ac.uk/publications/publication3026-abstract.html.**

C. & Duncan: ***Interacting quantum observables.*** **arXiv:0906.4725.**

C., Paquette & Pavlovic: ***Classical and quantum structuralism.*** **arXiv:0904.1997.**