

# Disproof of Bell's Theorem

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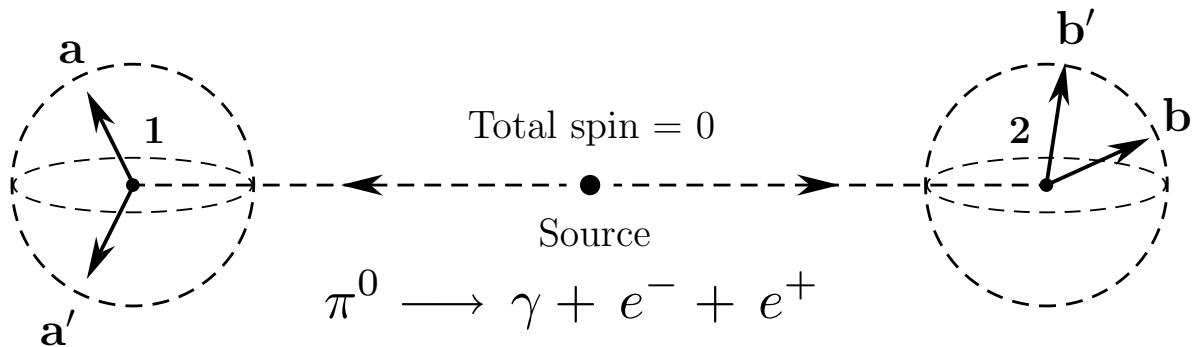
“I cannot seriously believe in [ quantum mechanics ] because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.”

Albert Einstein

Based on: (1) [arXiv:quant-ph/0703179](https://arxiv.org/abs/quant-ph/0703179)  
(2) [arXiv:quant-ph/0703244](https://arxiv.org/abs/quant-ph/0703244)  
(3) [arXiv:0707.1333](https://arxiv.org/abs/0707.1333)  
(4) [arXiv:0806.3078](https://arxiv.org/abs/0806.3078)  
and (5) [arXiv:0904.4259](https://arxiv.org/abs/0904.4259)

## Einstein, Podolsky, and Rosen (1935):

Pion decays into an electron-positron pair:



The electron-positron pair emerges from this decay in a rotationally invariant singlet state:

$$|\Psi_{\mathbf{n}}\rangle = \frac{1}{\sqrt{2}} \left\{ |\mathbf{n}, +\rangle_1 \otimes |\mathbf{n}, -\rangle_2 - |\mathbf{n}, -\rangle_1 \otimes |\mathbf{n}, +\rangle_2 \right\}.$$

## The Impeccable Logic Followed by EPR:

- (1) Criterion of Objective Reality
- + (2) Adherence to Local Causality
- + (3) Notion of a Complete Theory
- $\implies$  (4) QM is an Incomplete Theory

That is, entanglement is a cover for ignorance.

### Sufficient Criterion of Objective Reality:

*“If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”*

### Necessary Condition of Completeness:

*“Every element of the physical reality must have a counterpart in the physical theory.”*

★ The argument of EPR goes through for *all* components of spin, implying that the elements of reality of each of the two spins are points of a unit 2-sphere — i.e., the cardinality of the set of *all* elements of reality is that of a 2-sphere.

**Thus there are exactly 2-sphere worth of elements of reality to be accounted for by any complete physical theory.**

### Bell's Idea of a Complete Local Theory:

Bell takes the local-realistic expectation value of the product of spin measurements to be

$$\mathcal{E}_{L.R.}(\mathbf{a}, \mathbf{b}) = \int_{\Lambda} A_{\mathbf{a}}(\lambda) B_{\mathbf{b}}(\lambda) d\rho(\lambda),$$

where functions  $A_{\mathbf{a}}(\lambda)$  and  $B_{\mathbf{b}}(\lambda)$  are equal to  $\pm 1$ , and the “complete” state  $\lambda$  is supposed to specify all of the elements of reality of the spin.

What is crucial to note here is that the maps

$$A_{\mathbf{n}}(\lambda) \equiv A(\mathbf{n}, \lambda) : \mathbb{R}^3 \times \Lambda \longrightarrow S^0 \equiv \{-1, +1\}$$

are elements of a 0-sphere  $S^0$ , *not* 2-sphere  $S^2$ , with  $\mathbb{R}^3$  being a vector space and  $\Lambda$  the space of “complete” states  $\lambda$ , so that the products of these maps are also elements of a 0-sphere:

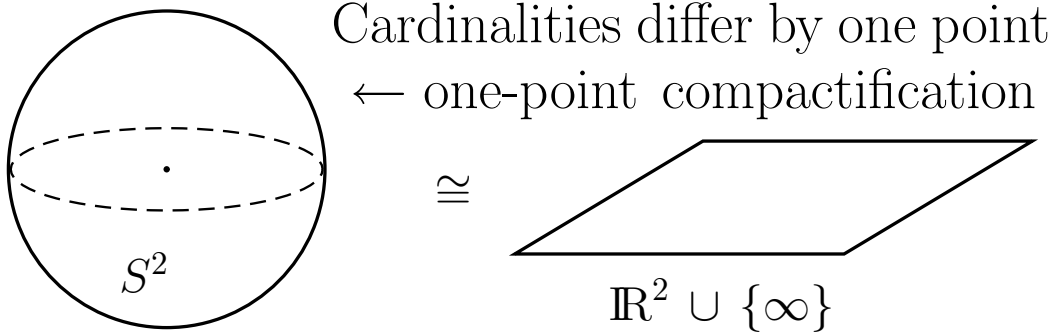
$$A_{\mathbf{a}}(\lambda)B_{\mathbf{b}}(\lambda) \in S^0 \times S^0 \cong S^0 \cong S^2 - S^1.$$

This allows Bell to impose his locality condition:

$$(A_{\mathbf{a}}B_{\mathbf{b}})(\lambda) = A_{\mathbf{a}}(\lambda)B_{\mathbf{b}}(\lambda) \in S^0, \quad \forall A, B \in S^0.$$

## Bell's Prescription Is NOT Complete:

EPR argument dictates that there are exactly  $S^2$  worth of elements of reality to be accounted for. But a Bell-type map  $A_{\mathbf{n}}(\lambda) : \mathbb{R}^3 \times \Lambda \rightarrow \Sigma$  can account for at most  $\mathbb{R}$  worth of elements, unless  $\Sigma$  is homeomorphic to  $S^2$ . Even  $\mathbb{R}^2 \not\cong S^2$ .



Hence Bell's prescription of reality is necessarily incomplete, unless the range  $\Sigma$  is taken to be  $S^2$ :

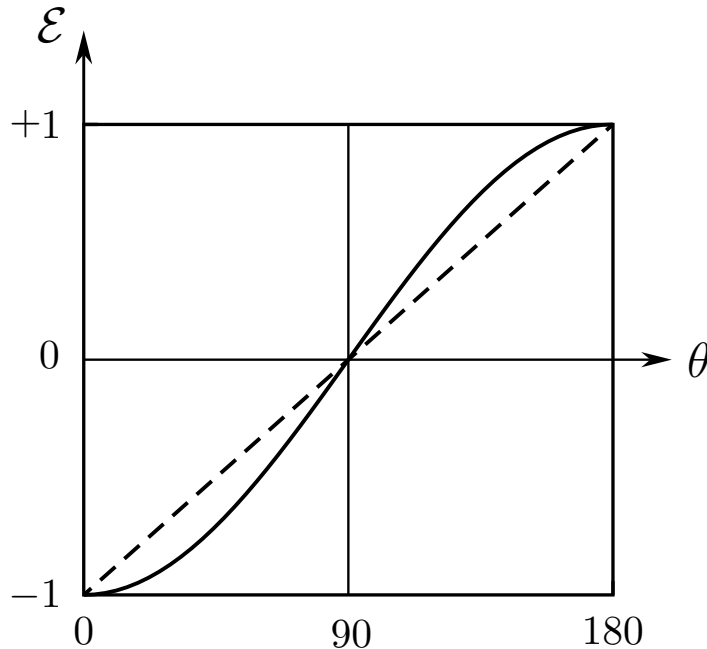
$$A_{\mathbf{n}}(\lambda) : \mathbb{R}^3 \times \Lambda \rightarrow S^2.$$

More generally, for arbitrary entangled states, the complete local maps must be of the form

$$A_{\mathbf{n}}(\lambda) : \mathbb{R}^3 \times \Lambda \rightarrow \Omega, \quad \text{with } A_{\mathbf{a}}(\lambda)B_{\mathbf{b}}(\lambda) \in \Omega,$$

where  $\Omega$  is a topological space of the elements of reality that is closed under multiplication.

**Pointless  
The Most Profound Discovery of Science:**



**Violations of the Bell-CHSH Inequality:**

$$|\Delta| = |\mathcal{E}(\mathbf{a}, \mathbf{b}) + \mathcal{E}(\mathbf{a}, \mathbf{b}') + \mathcal{E}(\mathbf{a}', \mathbf{b}) - \mathcal{E}(\mathbf{a}', \mathbf{b}')|$$

$$= \int_{\Lambda} \sqrt{4 + [A_{\mathbf{a}}(\lambda), A_{\mathbf{a}'}(\lambda)][B_{\mathbf{b}'}(\lambda), B_{\mathbf{b}}(\lambda)]} d\rho(\lambda),$$

which implies  $\boxed{|\Delta| \leq 2}$  for  $A_{\mathbf{n}}(\lambda) \in \mathcal{I} \subseteq \mathbb{R}$  or  $\mathbb{R}^2$ .

Whereas the map  $A_{\mathbf{n}}(\lambda) : \mathbb{R}^3 \times \Lambda \rightarrow S^2$  leads to

$$\Delta = -\cos \theta_{\mathbf{a}\mathbf{b}} - \cos \theta_{\mathbf{a}\mathbf{b}'} - \cos \theta_{\mathbf{a}'\mathbf{b}} + \cos \theta_{\mathbf{a}'\mathbf{b}'},$$

i.e.  $\boxed{|\Delta| \leq 2\sqrt{2}}$ , which is what QM also predicts.

A local and realistic theory Can reproduce:

(1) Exact quantum mechanical expectation value for the Bell state:  $\mathcal{E}(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}$

(2) Exact violations of Bell inequalities:

$$|\mathcal{E}(\mathbf{a}, \mathbf{b}) + \mathcal{E}(\mathbf{a}, \mathbf{b}') + \mathcal{E}(\mathbf{a}', \mathbf{b}) - \mathcal{E}(\mathbf{a}', \mathbf{b}')| \leq 2\sqrt{2}$$

(3) All sixteen predictions of the Hardy state:

$$\langle \Psi_{\mathbf{z}} | \mathbf{a}', + \rangle_1 \otimes | \mathbf{b}, + \rangle_2 = 0,$$

$$\langle \Psi_{\mathbf{z}} | \mathbf{a}, + \rangle_1 \otimes | \mathbf{b}', + \rangle_2 = 0,$$

$$\langle \Psi_{\mathbf{z}} | \mathbf{a}, - \rangle_1 \otimes | \mathbf{b}, - \rangle_2 = 0,$$

$$\langle \Psi_{\mathbf{z}} | \mathbf{a}', + \rangle_1 \otimes | \mathbf{b}', + \rangle_2 = \frac{\sin \theta \cos^2 \theta}{\sqrt{1 + \cos^2 \theta}}$$

(4) Exact quantum mechanical expectation value for the three-particle GHZ state:

$$\mathcal{E}(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3) = \cos \alpha \cos \theta_1 \cos \theta_2 \cos \theta_3 + \sin \alpha \sin \theta_1 \sin \theta_2 \sin \theta_3 \cos (\phi_1 + \phi_2 + \phi_3 + \delta)$$

(5) Exact quantum mechanical expectation value for the four-particle GHZ state:

$$\mathcal{E}(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_4) = \cos \theta_1 \cos \theta_2 \cos \theta_3 \cos \theta_4 - \sin \theta_1 \sin \theta_2 \sin \theta_3 \sin \theta_4 \cos (\phi_1 + \phi_2 - \phi_3 - \phi_4)$$

(6)  $\mathcal{E}_{L.R.}(\Omega) = \mathcal{E}_{Q.M.}(\Omega)$  for *any* arbitrary state.